6DOF Vision-Aided Inertial Localization

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Overview

- Desired localization accuracy
- Available sensor inputs (inertial measurement unit, camera)
- Types of environments
- State propagation
- State augmentation
- Feature extraction and matching
- State update
- Some results
- Concluding remarks

Our goals

- Estimate the 3D position and 3D orientation (3D pose) of a moving robot in real-time.

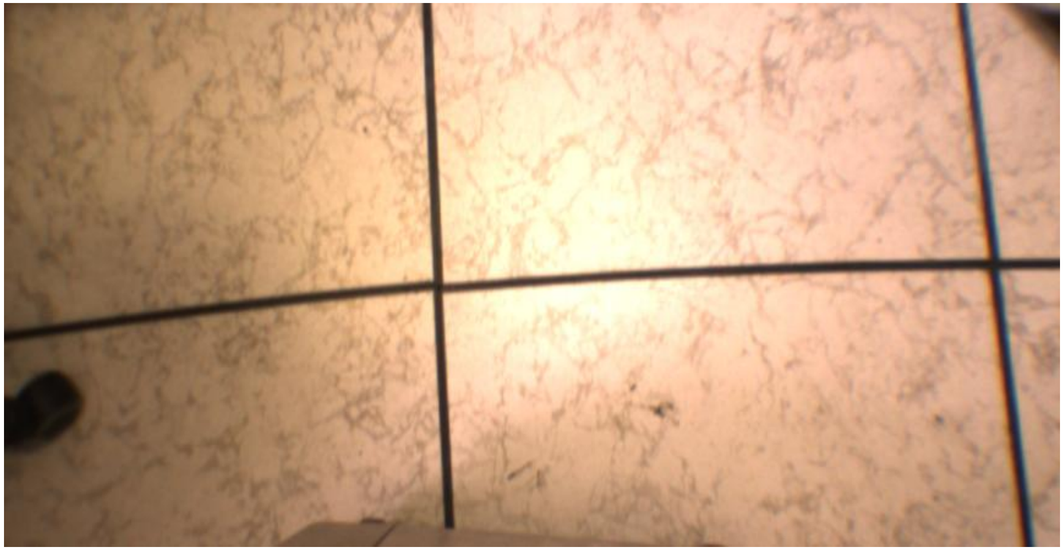
- Ideally, we'd like position error $\leq 10\%$ of the distance traveled.

- Unfortunately, this is hard to guarantee.

Visual input in the lab

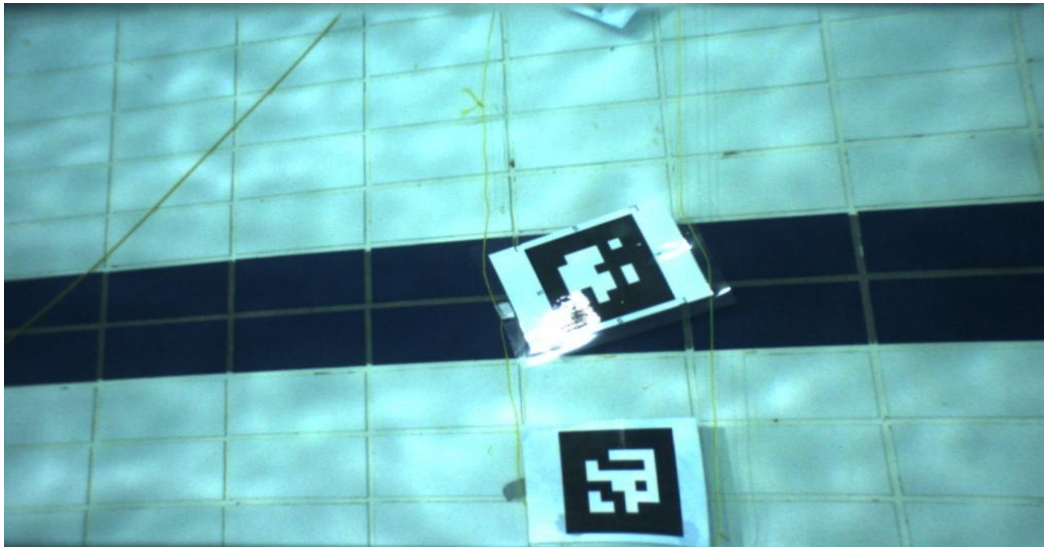
Good

Not so good

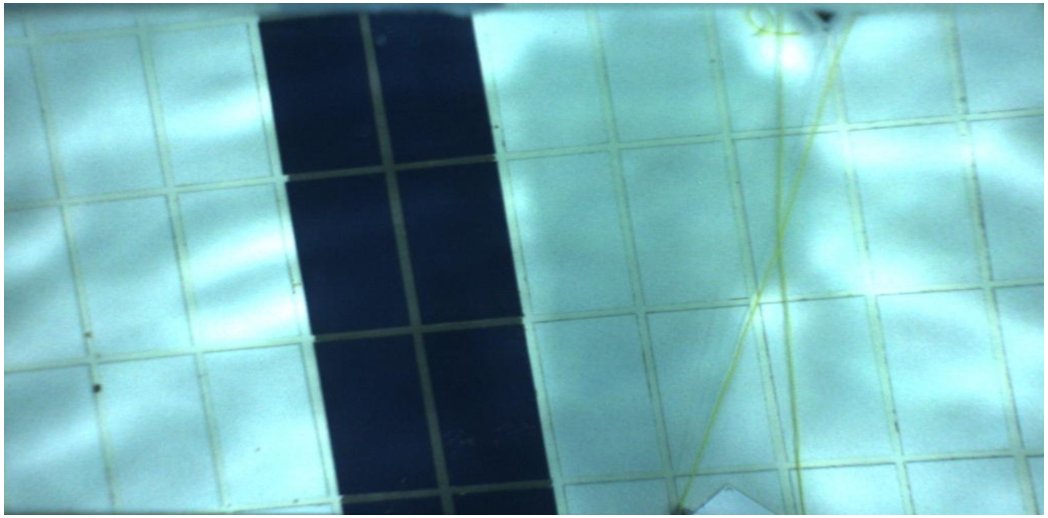


Visual input in the pool

Good

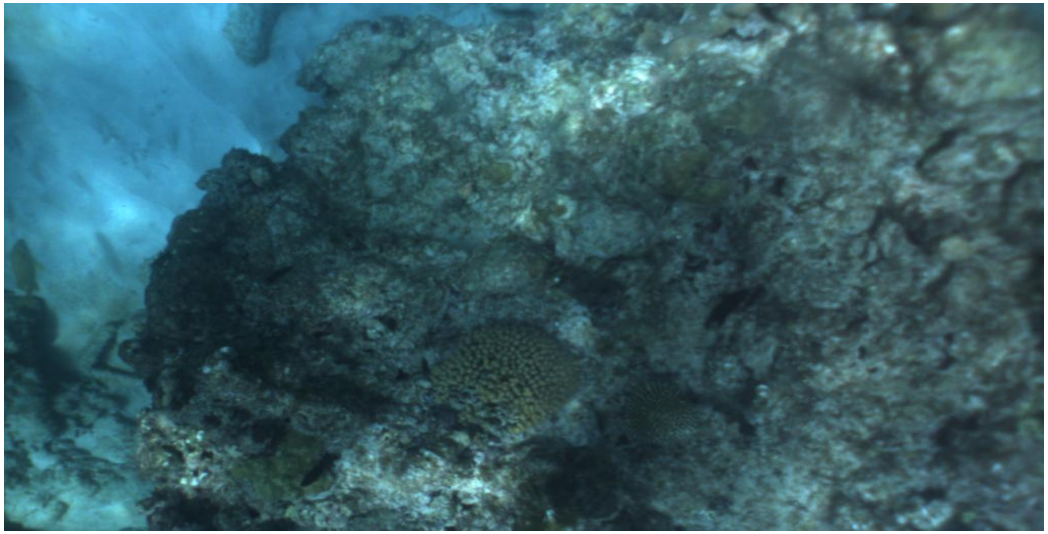


Not so good



Visual input in the ocean

Good



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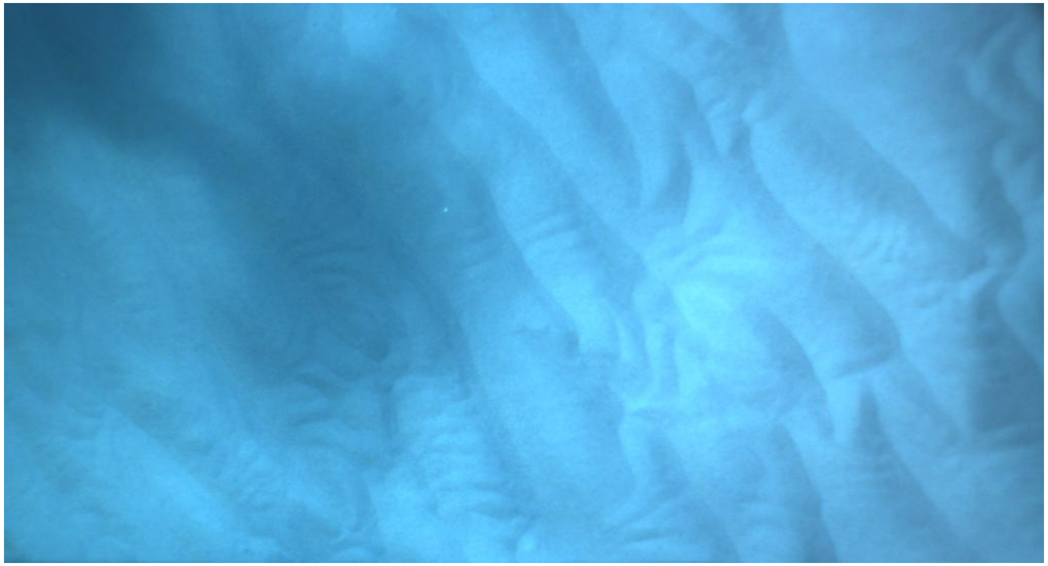


Typical SLAM approaches

- Try to estimate the robot's 3D pose **and** the 3D positions of the observed landmarks

$$\mathbf{x} = [\underbrace{x \;\; y \;\; z \;\; \theta \;\; \phi \;\; \psi}_{R} \;\; \underbrace{x \;\; y \;\; z}_{L_1} \;\; \ldots \;\; \underbrace{x \;\; y \;\; z}_{L_N}]$$

- Correlations between landmarks are also estimated.

- The size of the state becomes significantly big, very quickly. Each update step of EKF-SLAM takes $O(N^2)$.

- Need to remove landmarks from the state.

Proposed approach

- Heavily based on work by A. Mourikis and S. Roumeliotis [1]
- Uses input from a single camera and an inertial measurement unit (IMU).
- Combines these inputs through an Extended Kalman Filter.
- Each update step takes $O(N)$ where N is the number of landmarks.
- [1] "A multi-state constrained Kalman filter for vision-aided inertial navigation," ICRA 2007

Inertial Measurement Units

- Think of an IMU as a gyroscope and an accelerometer.
- If $[x \;\; y \;\; z \;\; \theta \;\; \phi \;\; \psi]$ is the robot's pose then the IMU gives us noisy measurements of $[\ddot{x} \;\; \ddot{y} \;\; \ddot{z} \;\; \dot{\theta} \;\; \dot{\phi} \;\; \dot{\psi}]$
- Let $\mathbf{a} = [\ddot{x} \;\; \ddot{y} \;\; \ddot{z}]$ be the true linear acceleration and $\boldsymbol{\omega} = [\dot{\theta} \;\; \dot{\phi} \;\; \dot{\psi}]$ be the true angular velocity. IMU measurements are typically modeled as:

$$\mathbf{a}_m = c\mathbf{a} + \mathbf{b}_a + \mathbf{n}_a \qquad \mathbf{n}_a \sim N(0, \boldsymbol{\sigma}_{an}) \text{ and } \dot{\mathbf{b}}_a \sim N(0, \boldsymbol{\sigma}_{ab})$$

$$\boldsymbol{\omega}_m = \boldsymbol{\omega} + \mathbf{b}_g + \mathbf{n}_g \qquad \mathbf{n}_g \sim N(0, \boldsymbol{\sigma}_{gn}) \text{ and } \dot{\mathbf{b}}_g \sim N(0, \boldsymbol{\sigma}_{gb})$$

IMU noise modeling

- The IMU measurement noise $\mathbf{n}_a \sim N(0, \boldsymbol{\sigma}_{an})$ and $\mathbf{n}_g \sim N(0, \boldsymbol{\sigma}_{gn})$ is typically assumed to be fixed. It is estimated offline while keeping the robot still.

- The accelerometer bias is initialized as the average offset from [0 0 -g] when the robot is left still with zero pitch and roll. The noise parameters of $\dot{\mathbf{b}}_a \sim N(0, \boldsymbol{\sigma}_a)$ are estimated offline. $\mathbf{b}_a$ is estimated online.

- Similarly for the gyroscope bias.

- Note: In the proposed algorithm we do not estimate the accelerometer scaling factor. We fix it to $c = 1$.

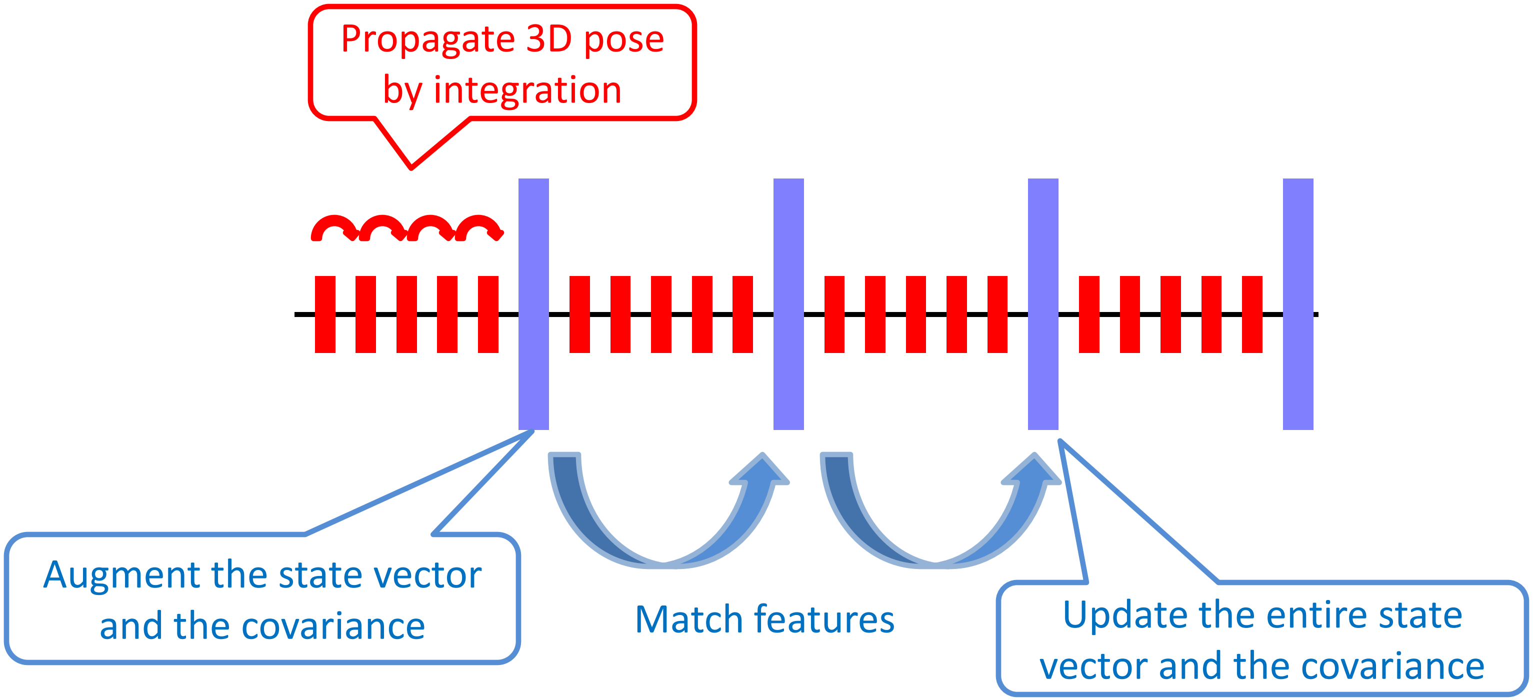
IMU integration drift

- “Why don’t we just integrate the IMU measurements to get a pose estimate?”

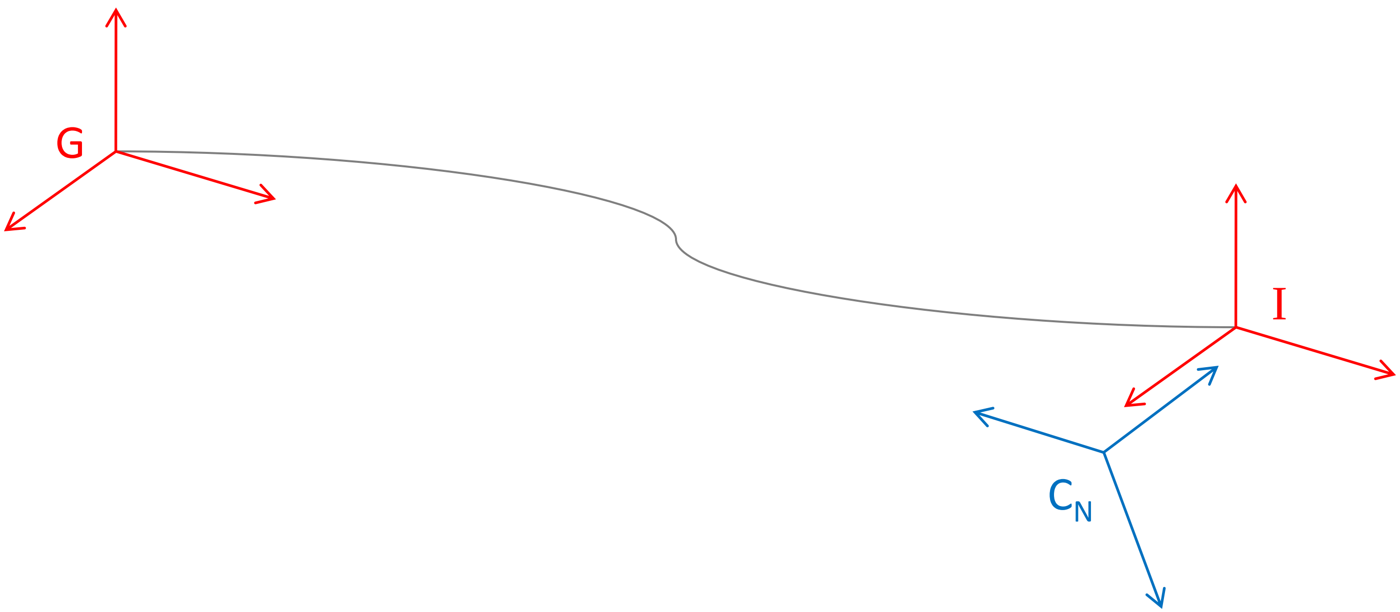
- Because errors will also be integrated.

Back to the algorithm

- Main idea: assuming feature-rich scenes we can use visual motion estimation to correct IMU drift.



The state vector



$$\mathbf{x} = [\underbrace{{}^{I}_{G}\mathbf{q}\ \ \mathbf{b}_g\ \ {}^{G}\mathbf{v}_I\ \ \mathbf{b}_a\ \ {}^{G}\mathbf{p}_I}_{\text{IMU state}}\ \ \underbrace{{}^{C_1}_{G}\mathbf{q}\ \ {}^{G}\mathbf{p}_{C_1}}_{\text{1}^{\text{st}}\text{ camera frame}}\ \cdots\ \underbrace{{}^{C_N}_{G}\mathbf{q}\ \ {}^{G}\mathbf{p}_{C_N}}_{\text{N}^{\text{th}}\text{ camera frame}}\ ]$$

The state vector

- ${}^{I}_{G}\mathbf{q} = [\mathbf{u}\sin(\theta/2) \;\; \cos(\theta/2)]$ is a quaternion that represents the rotation from the global frame G to the current IMU frame.

- ${}^{G}\mathbf{p}_{I}$ is the origin of the IMU frame in global coordinates.

- ${}^{G}\mathbf{v}_{I}$ is the velocity of the IMU frame in global coordinates.

- ${}^{C_i}_{G}\mathbf{q}$ is the rotation from the global frame to the ith camera frame.

- ${}^{G}\mathbf{p}_{C_i}$ is the origin of the ith camera frame in global coordinates.

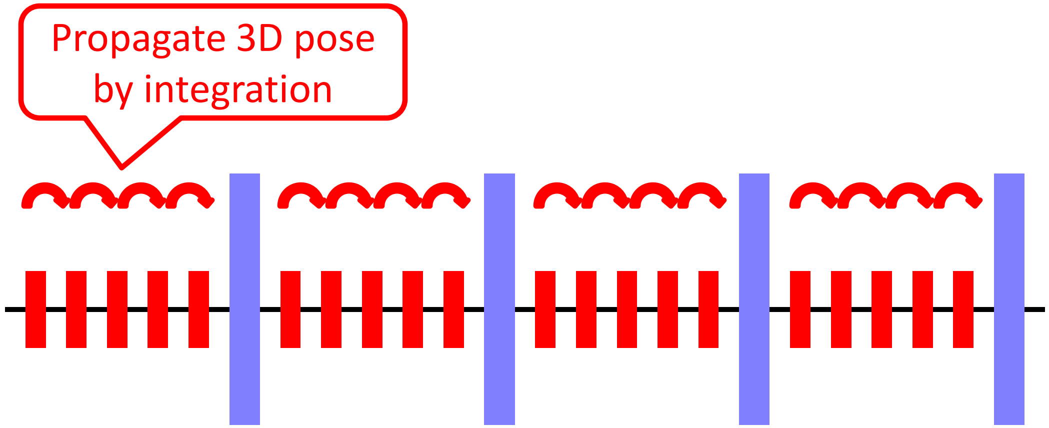
The covariance matrix

$$\mathbf{P} = \mathrm{cov}(\tilde{\mathbf{x}}) = \mathrm{cov}(\mathbf{x} - \hat{\mathbf{x}}) = \begin{bmatrix} \mathbf{P}_{II} & \mathbf{P}_{IC} \\ \mathbf{P}_{IC}^T & \mathbf{P}_{CC} \end{bmatrix}$$

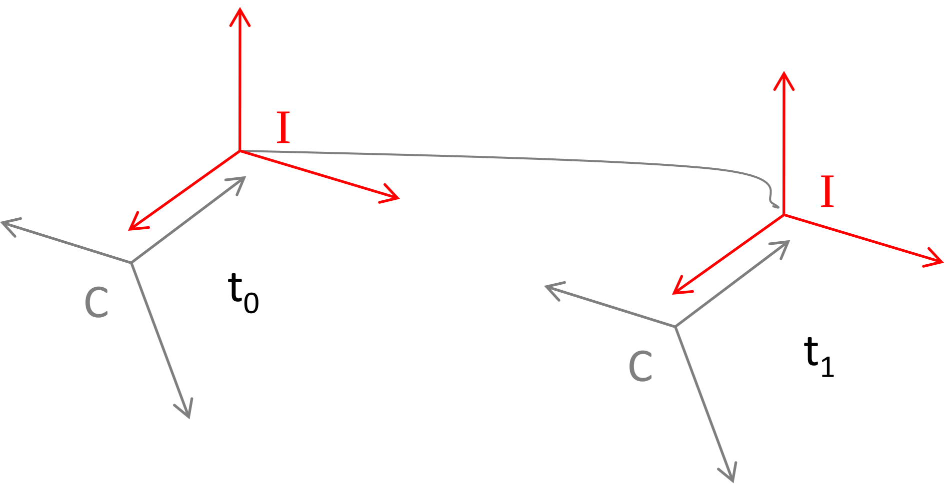
- At t=0 $\mathbf{P} = \mathbf{P}_{II} = 0$ and $\hat{\mathbf{x}} = \hat{\mathbf{x}}_{IMU}$ because no images have been recorded yet.

EKF Propagation

- Is done every time we receive an IMU measurement.



Propagation (state)



$$ {}^{G}\mathbf{p}_I(t_1) = {}^{G}\mathbf{p}_I(t_0) + {}^{G}\mathbf{v}_I(t_0)\,dt $$

$$ {}^{G}\mathbf{v}_I(t_1) = {}^{G}\mathbf{v}_I(t_0) + ({}^{G}_{I}\mathbf{R}(t_0)\,{}^{I}\hat{\mathbf{a}} + {}^{G}\mathbf{g})\,dt $$

$$ \mathbf{b}_g(t_1) = \mathbf{b}_g(t_0) $$

$$ \mathbf{b}_a(t_1) = \mathbf{b}_a(t_0) $$

$$ {}^{I}\hat{\mathbf{a}} = \mathbf{a}_m - \mathbf{b}_a(t_0) \qquad {}^{I}\hat{\boldsymbol{\omega}} = \boldsymbol{\omega}_m - \mathbf{b}_g(t_0) \quad \Rightarrow $$

$$ {}^{I}_{G}\mathbf{q}(t_1) = \begin{bmatrix} \dfrac{{}^{I}\hat{\boldsymbol{\omega}}}{\left\| {}^{I}\hat{\boldsymbol{\omega}} \right\|} \sin\!\left(\dfrac{\left\| {}^{I}\hat{\boldsymbol{\omega}} \right\|}{2} dt\right) \\ \cos\!\left(\dfrac{\left\| {}^{I}\hat{\boldsymbol{\omega}} \right\|}{2} dt\right) \end{bmatrix} \otimes {}^{I}_{G}\mathbf{q}(t_0) $$

Note: the camera frames of the state are not propagated

Propagation (covariance)

- How do we propagate $\mathbf{P} = \mathrm{cov}(\tilde{\mathbf{x}}) = \mathrm{cov}(\mathbf{x} - \hat{\mathbf{x}}) = \begin{bmatrix} \mathbf{P}_{II} & \mathbf{P}_{IC} \\ \mathbf{P}_{IC}^{T} & \mathbf{P}_{CC} \end{bmatrix}$?

- Since we didn't modify the camera frames: $\begin{bmatrix} \mathbf{P}_{II} & \mathbf{P}_{IC} \\ \mathbf{P}_{IC}^{T} & \mathbf{P}_{CC} \end{bmatrix}$

- We can show that IMU propagation errors increase according to:

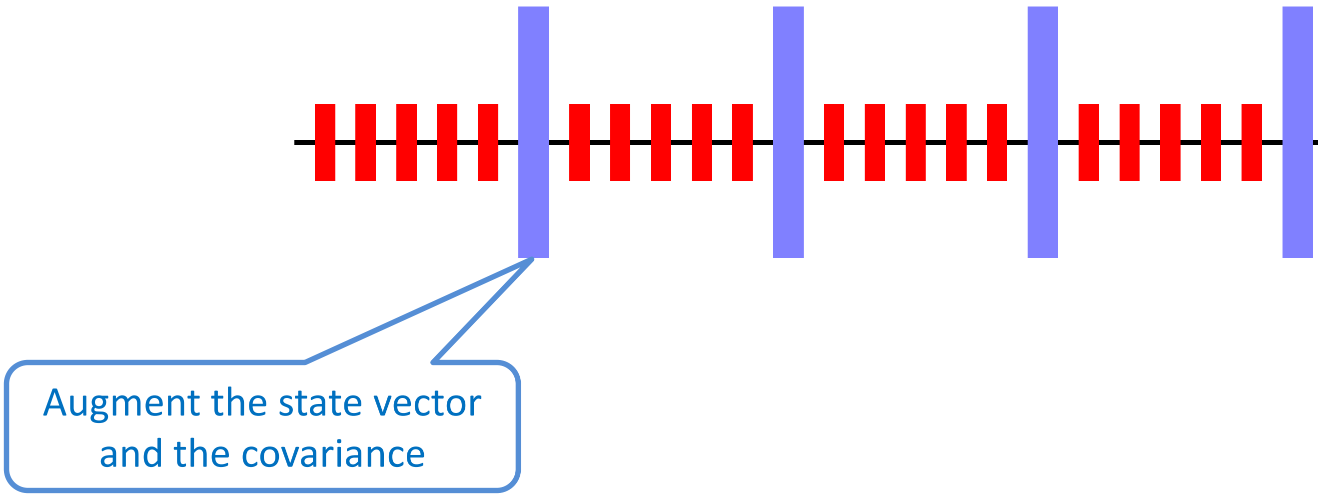
$$\frac{d}{dt}\tilde{\mathbf{x}}_{IMU}(t) = \mathbf{F}\tilde{\mathbf{x}}_{IMU}(t) + \mathbf{G}[\mathbf{n}_g \;\; \dot{\mathbf{b}}_g \;\; \mathbf{n}_a \;\; \dot{\mathbf{b}}_a]^T$$

where $\mathbf{F}$ depends on $\hat{\boldsymbol{\omega}}$, $\hat{\mathbf{a}}$, ${}^{\mathrm{I}}_{\mathrm{G}}\hat{\mathbf{q}}$ and $\mathbf{G}$ depends on ${}^{\mathrm{I}}_{\mathrm{G}}\hat{\mathbf{q}}$.

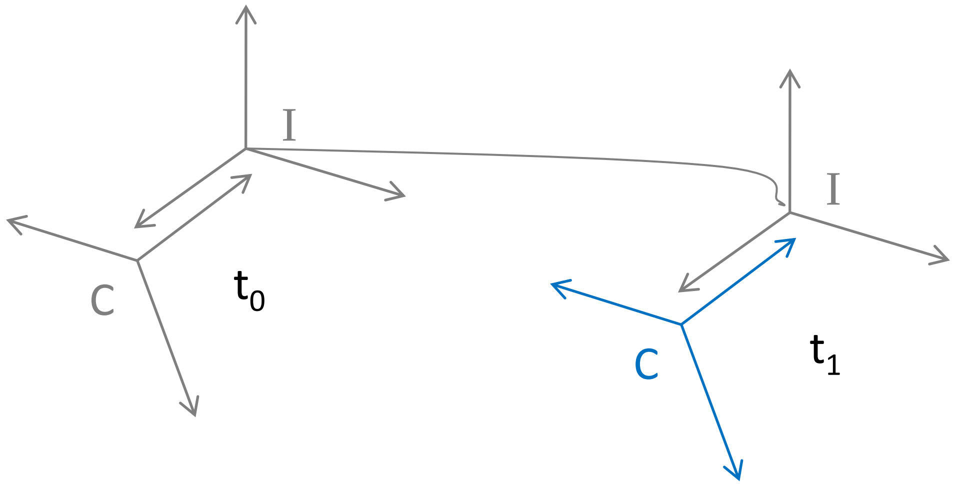
- We can get the propagated covariance by integration.

State augmentation

- Is done every time an image is recorded



State augmentation



$$\mathbf{x} = [{}^{I}_{G}\mathbf{q}\ \ \mathbf{b}_g\ \ {}^{G}\mathbf{v}_I\ \ \mathbf{b}_a\ \ {}^{G}\mathbf{p}_I]$$

$$\Downarrow$$

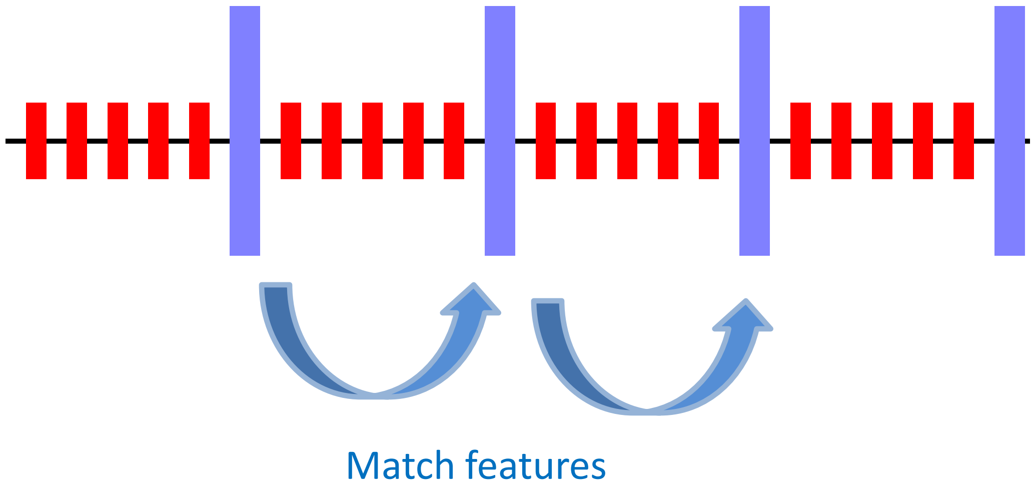
$$\mathbf{x} = [{}^{I}_{G}\mathbf{q}\ \ \mathbf{b}_g\ \ {}^{G}\mathbf{v}_I\ \ \mathbf{b}_a\ \ {}^{G}\mathbf{p}_I\ \ \boxed{{}^{C_1}_{G}\mathbf{q}\ \ {}^{G}\mathbf{p}_{C_1}}]$$

Transformation from camera to global coordinates

- The covariance is also augmented.

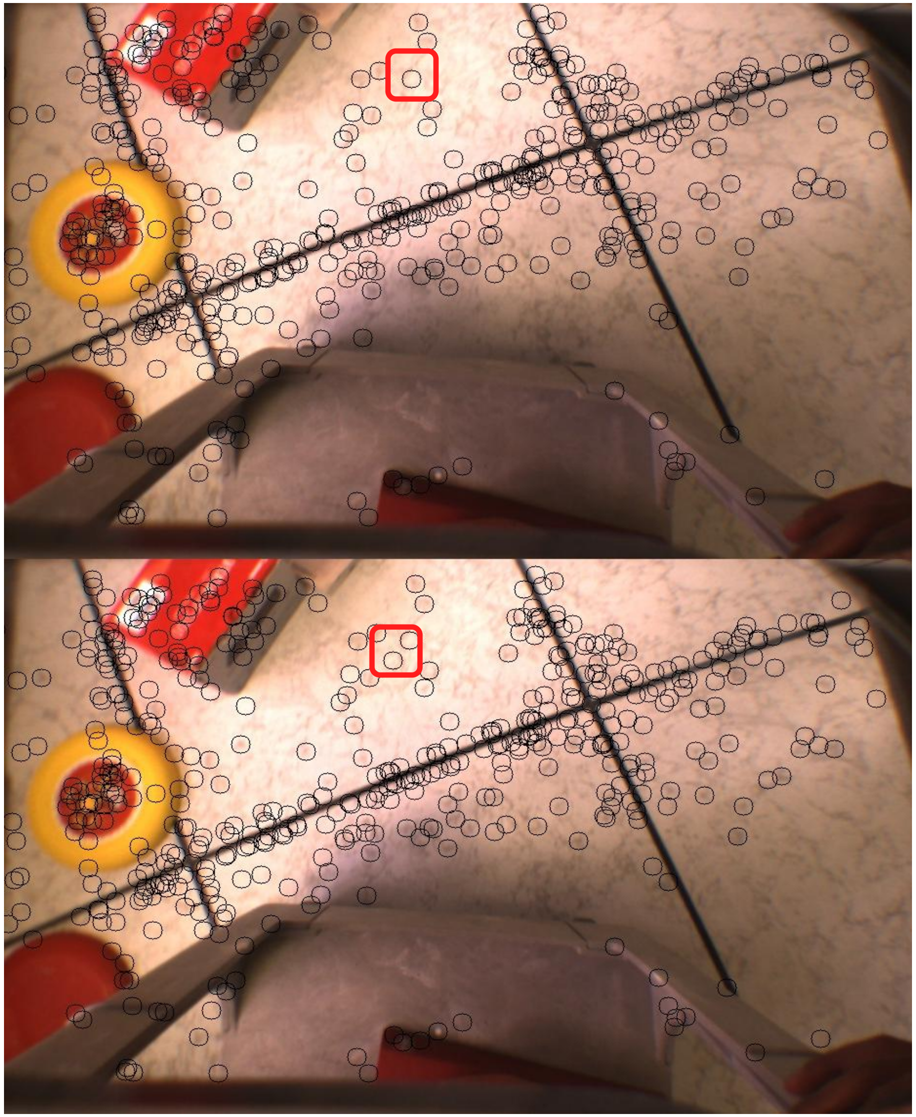
Feature matching

- Is done between every pair of consecutive images.

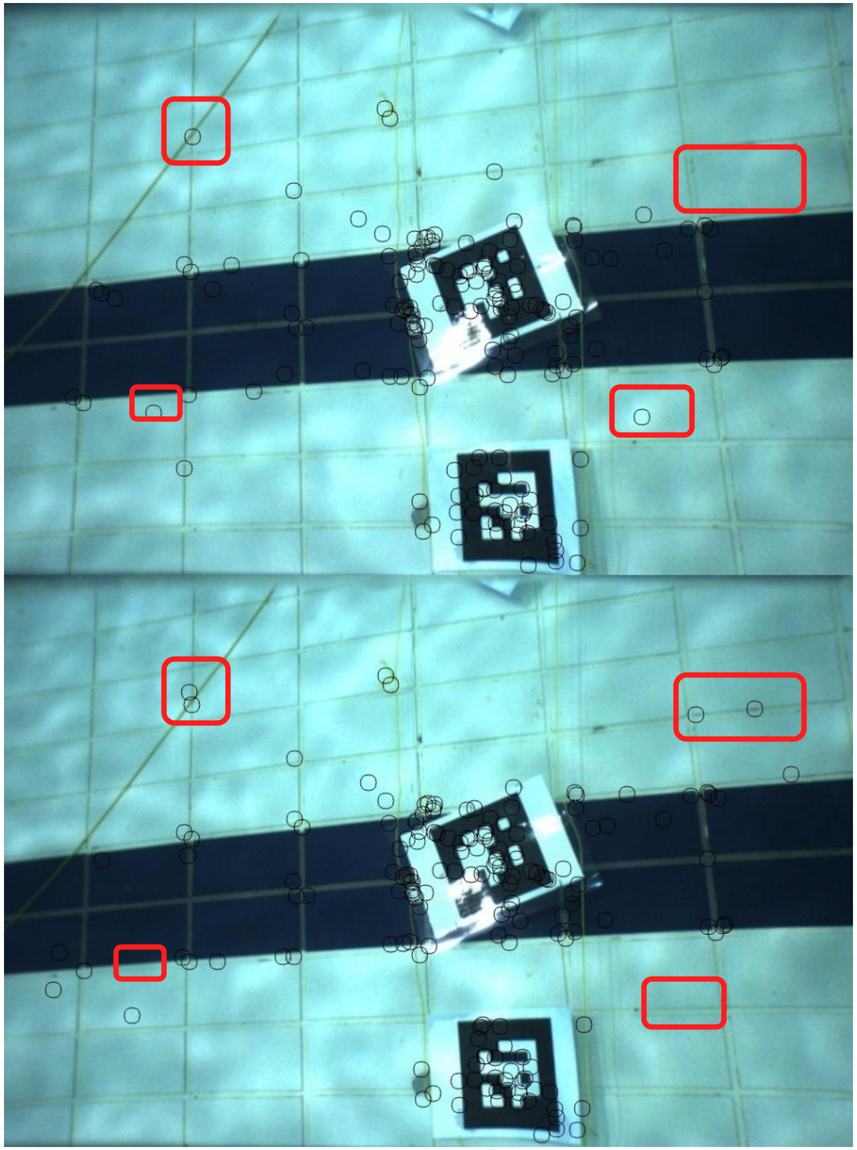


- SURF-64 features are matched using the Fast Library for Approximate Nearest Neighbors, by Muja & Lowe at UBC.
- Approximately 1000 features per frame.
- Average feature tracking length is 4 frames.

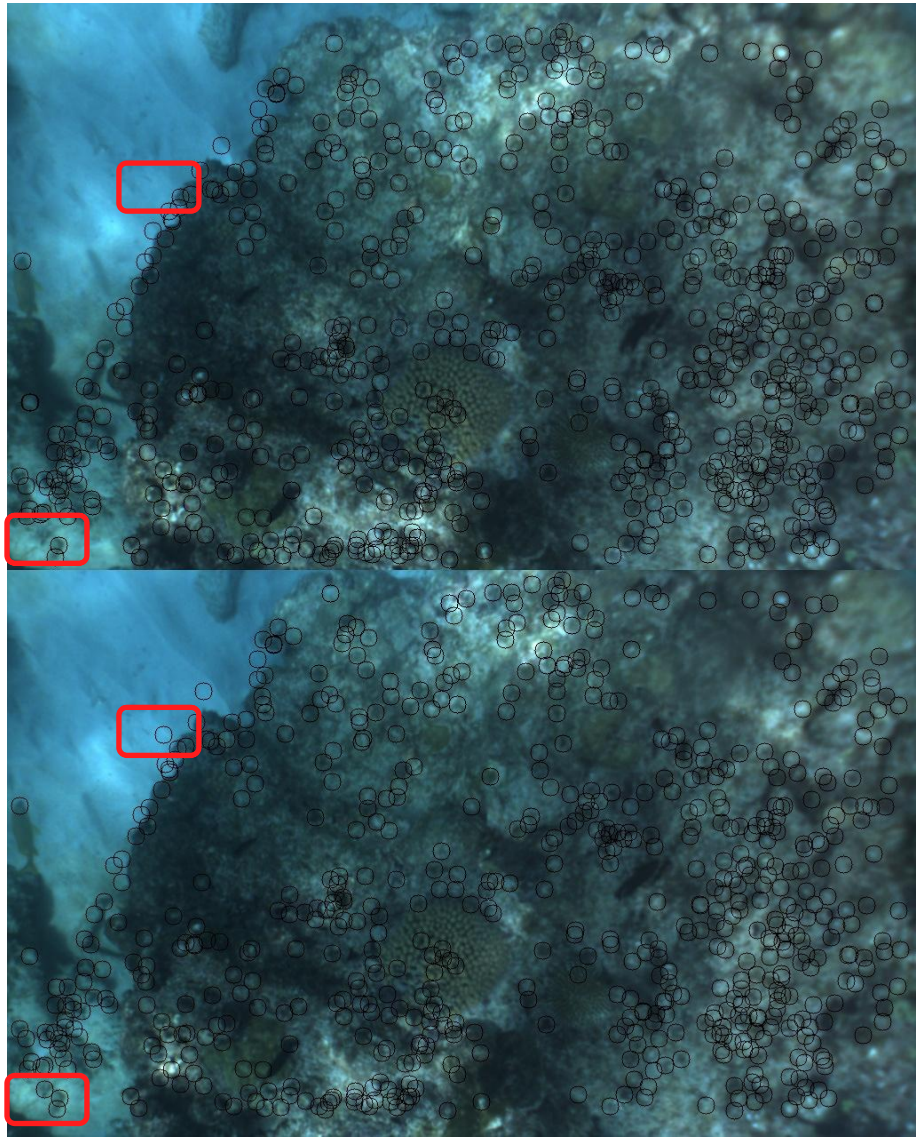
Feature matching (in the lab)



Feature matching (in the pool)

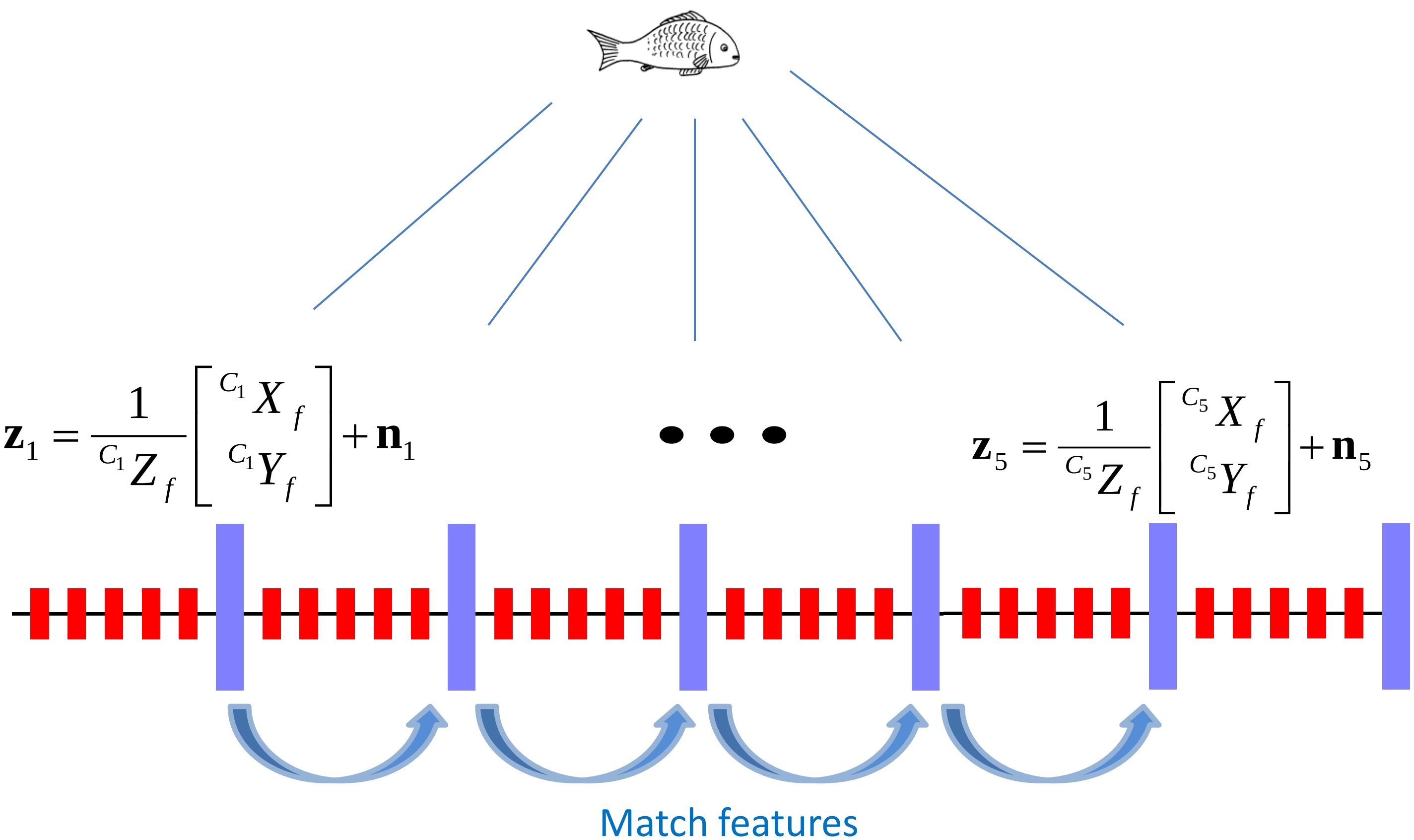


Feature matching (in the ocean)



Feature matching

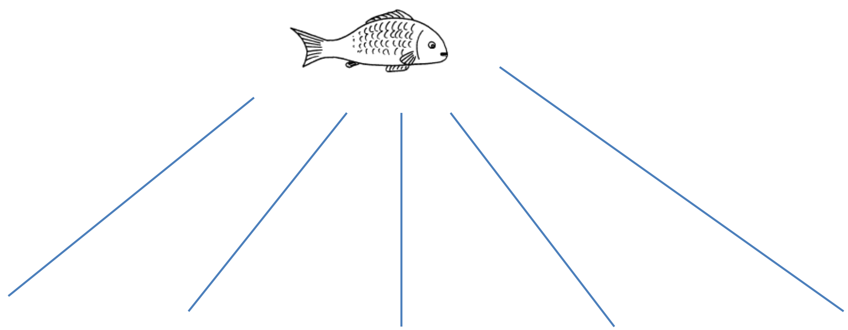
- Not very reliable, so outlier detection is necessary.
- If done carefully, it allows us to estimate the 3D position of a feature:



$$\mathbf{z}_1 = \frac{1}{{}^{C_1}Z_f}\begin{bmatrix} {}^{C_1}X_f \\ {}^{C_1}Y_f \end{bmatrix} + \mathbf{n}_1 \quad \bullet\bullet\bullet \quad \mathbf{z}_5 = \frac{1}{{}^{C_5}Z_f}\begin{bmatrix} {}^{C_5}X_f \\ {}^{C_5}Y_f \end{bmatrix} + \mathbf{n}_5$$

Structure Estimation

- We are searching for the 3D position of the fish in global coordinates, ${}^{G}\hat{\mathbf{p}}_f$

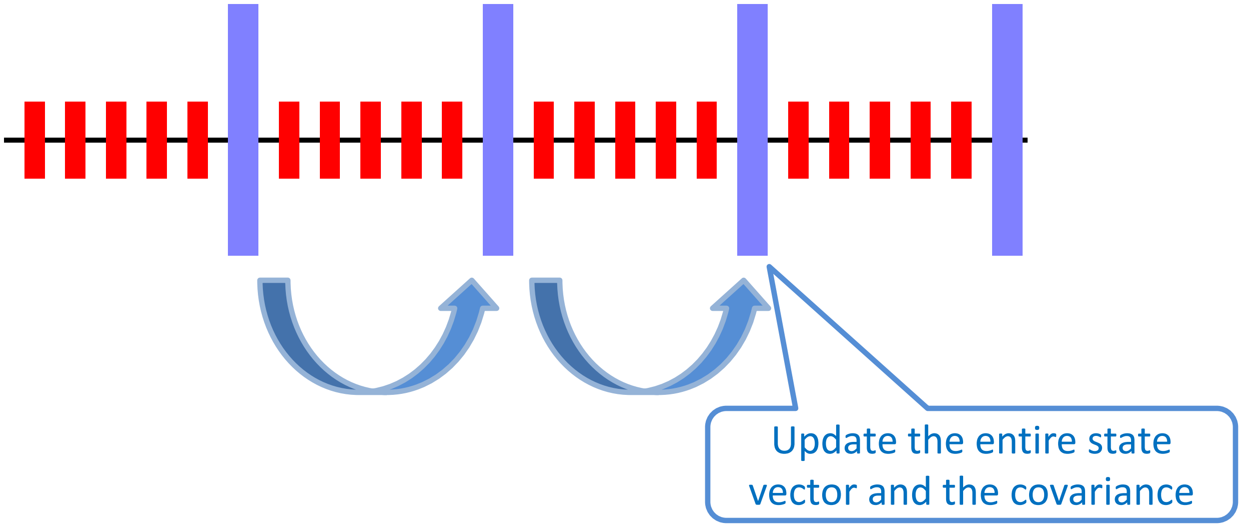


$$\mathbf{z}_1 = \frac{1}{{}^{C_1}Z_f}\begin{bmatrix} {}^{C_1}X_f \\ {}^{C_1}Y_f \end{bmatrix} + \mathbf{n}_1 \quad \bullet\bullet\bullet \quad \mathbf{z}_5 = \frac{1}{{}^{C_5}Z_f}\begin{bmatrix} {}^{C_5}X_f \\ {}^{C_5}Y_f \end{bmatrix} + \mathbf{n}_5$$

- If we work in global coordinates we have 3 unknowns ${}^{G}X_f, {}^{G}Y_f, {}^{G}Z_f$ and 5 measurements. We can solve the nonlinear problem using iterative minimization methods (e.g. Levenberg-Marquardt).

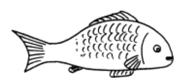
EKF Update

- Is done every time a feature stops being tracked.



EKF Update (the residual)

- The only source of correction we have is the camera.



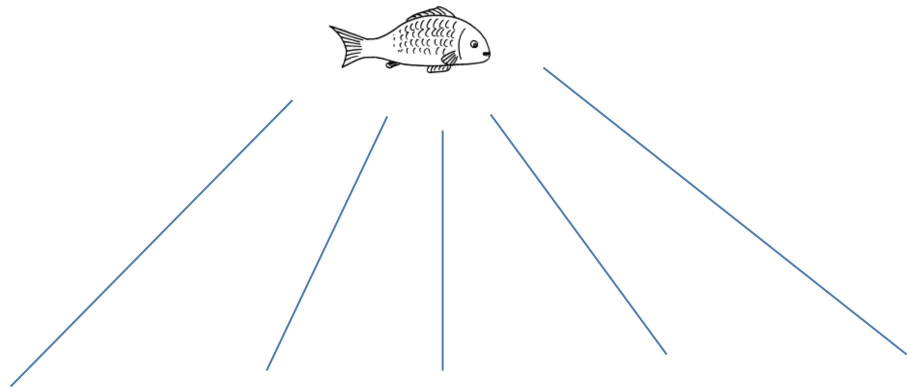
What we measured

$$\mathbf{z}_1 = \frac{1}{{}^{C_1}Z_f}\begin{bmatrix} {}^{C_1}X_f \\ {}^{C_1}Y_f \end{bmatrix} + \mathbf{n}_1 \quad \bullet\bullet\bullet \quad \mathbf{z}_5 = \frac{1}{{}^{C_5}Z_f}\begin{bmatrix} {}^{C_5}X_f \\ {}^{C_5}Y_f \end{bmatrix} + \mathbf{n}_5$$

$$\hat{\mathbf{z}}_1 = \frac{1}{{}^{C_1}\hat{Z}_f}\begin{bmatrix} {}^{C_1}\hat{X}_f \\ {}^{C_1}\hat{Y}_f \end{bmatrix} \quad \bullet\bullet\bullet \quad \hat{\mathbf{z}}_5 = \frac{1}{{}^{C_5}\hat{Z}_f}\begin{bmatrix} {}^{C_5}\hat{X}_f \\ {}^{C_5}\hat{Y}_f \end{bmatrix}$$

What we expected to measure after we estimated the 3D position of the fish

EKF Update (the residual)



$$\mathbf{r}_1 = \mathbf{z}_1 - \hat{\mathbf{z}}_1 \qquad \bullet\bullet\bullet \qquad \mathbf{r}_5 = \mathbf{z}_5 - \hat{\mathbf{z}}_5$$

$$\mathbf{r}_i = \mathbf{z}_i - \frac{1}{{}^{C_i}\hat{Z}_f} \begin{bmatrix} {}^{C_i}\hat{X}_f \\ {}^{C_i}\hat{Y}_f \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} {}^{C_i}\hat{X}_f \\ {}^{C_i}\hat{Y}_f \\ {}^{C_i}\hat{Z}_f \end{bmatrix} = {}^{C_i}_{G}\hat{\mathbf{R}}({}^{G}\hat{\mathbf{p}}_f - {}^{G}\hat{\mathbf{p}}_{C_i})$$

- Each residual is a nonlinear function of $({}^{C_i}_{G}\mathbf{R},\ {}^{G}\mathbf{p}_{C_i}, {}^{G}\mathbf{p}_f)$

EKF Update

- We linearize the residual $\mathbf{r}_i \approx \mathbf{H}_i \tilde{\mathbf{x}} + \mathbf{n}_i$ for each frame.
- We stack the residuals for all camera frames and for all features into one vector $\mathbf{r} \approx \mathbf{H}\tilde{\mathbf{x}} + \mathbf{n}$
- We apply the usual update equations of the Extended Kalman Filter:

$$\mathbf{K} = \mathbf{P}\mathbf{H}^T(\mathbf{H}\mathbf{P}\mathbf{H}^T + \mathrm{cov}(\mathbf{n}))^{-1}$$

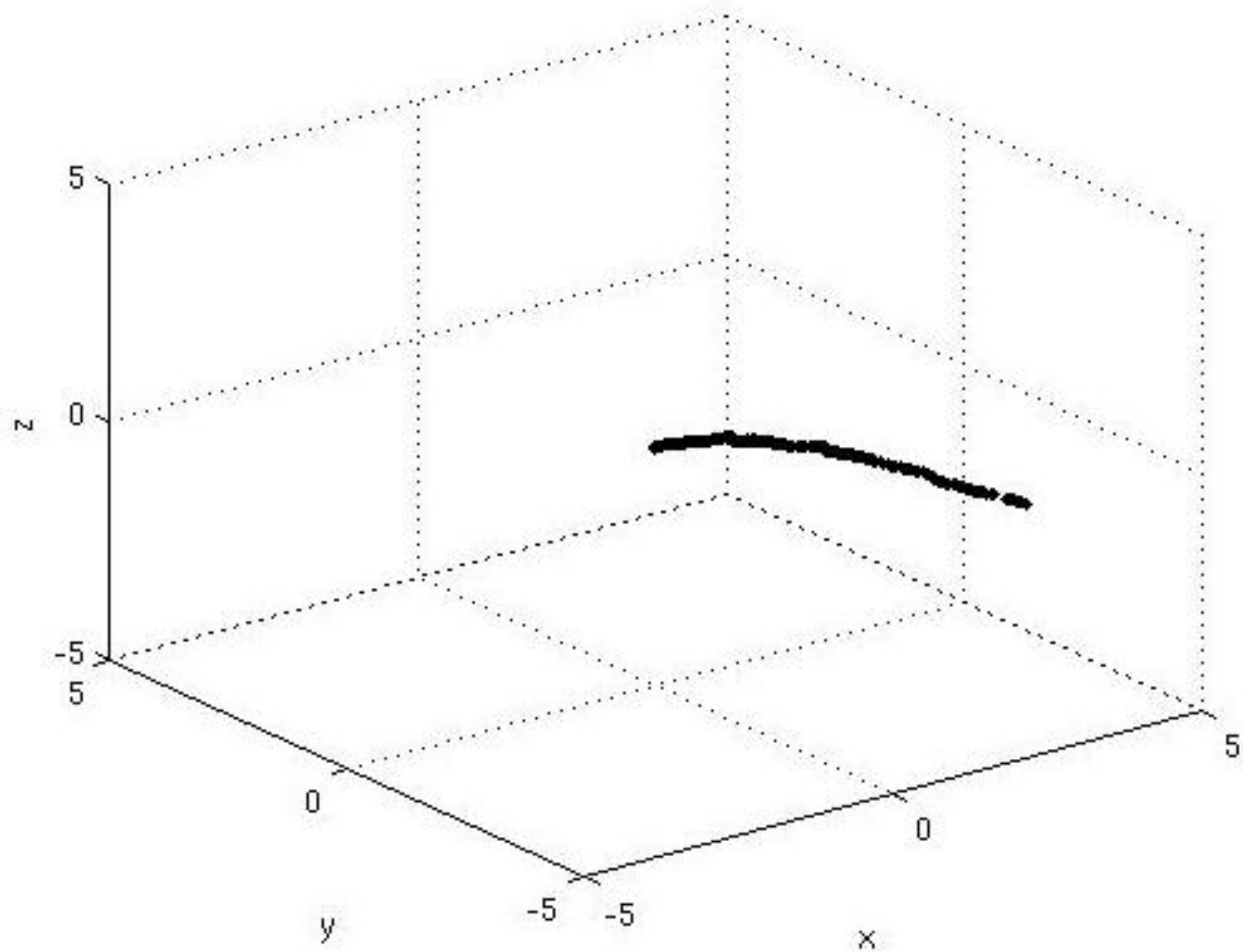
$$\Delta\mathbf{x} = \mathbf{K}\mathbf{r}$$

$$\mathbf{x}_{t+1|t+1} = \mathbf{x}_{t+1|t} + \Delta\mathbf{x}$$

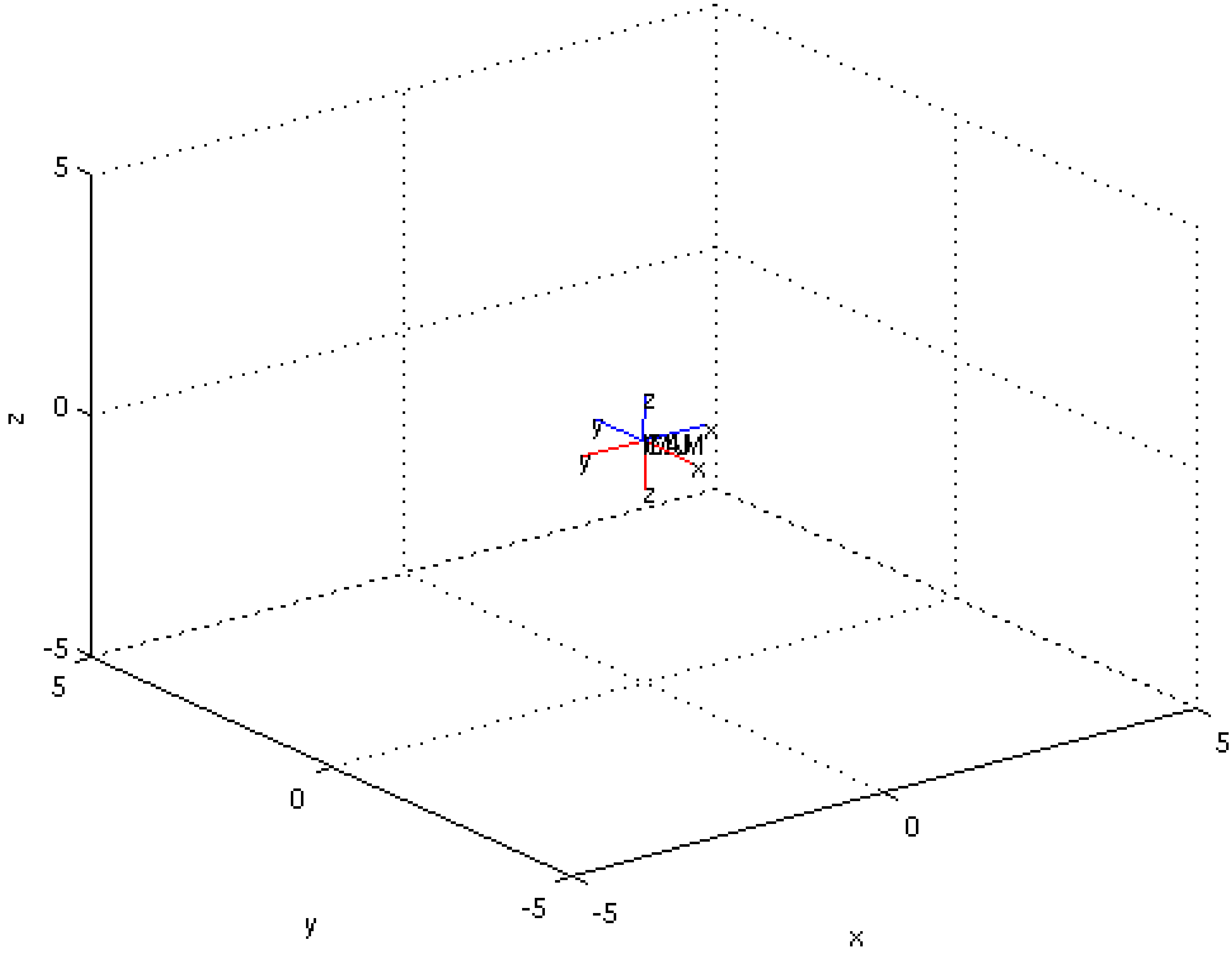
$$\mathbf{P}_{t+1|t+1} = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}_{t+1|t}$$

Lab experiment

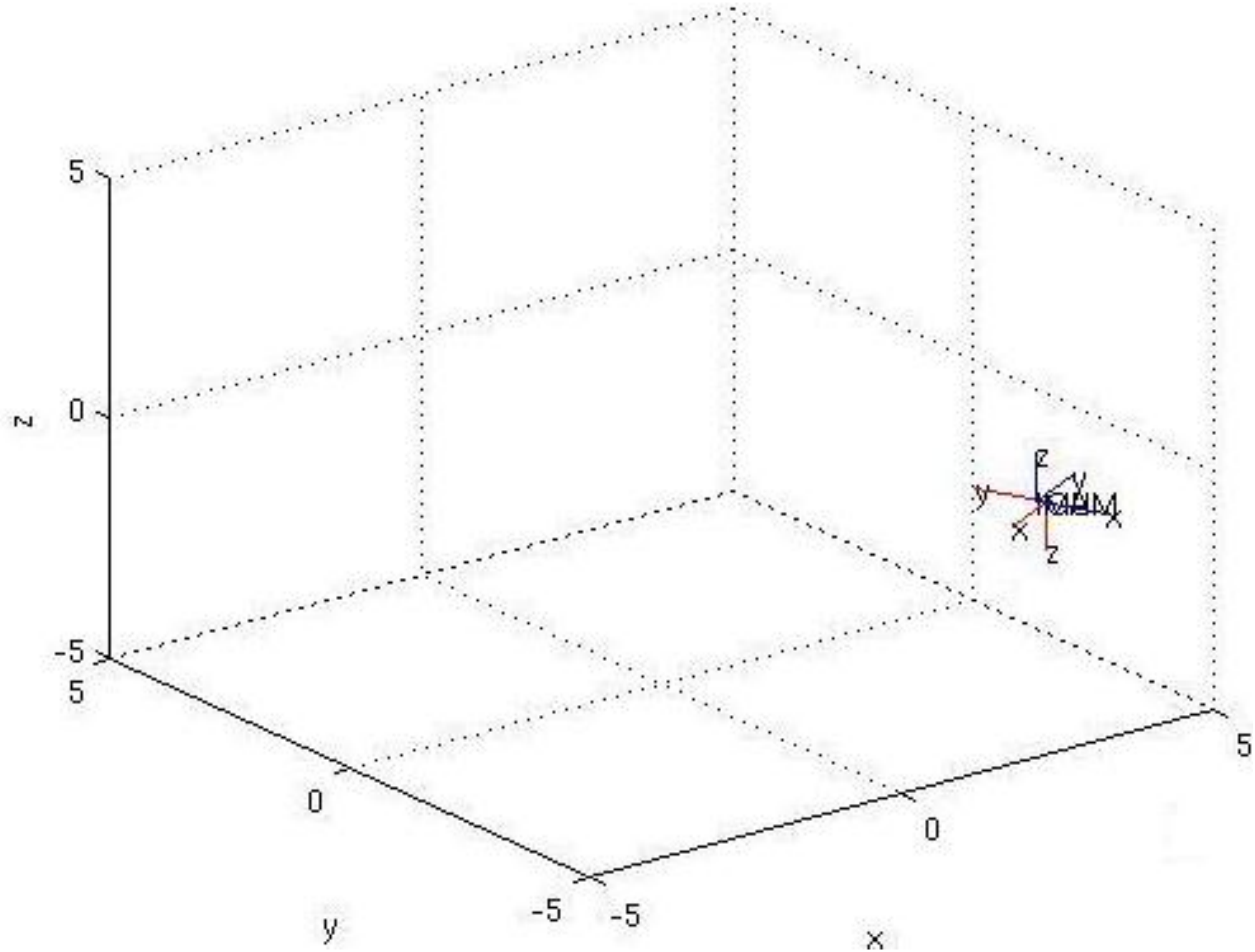
Estimated trajectory



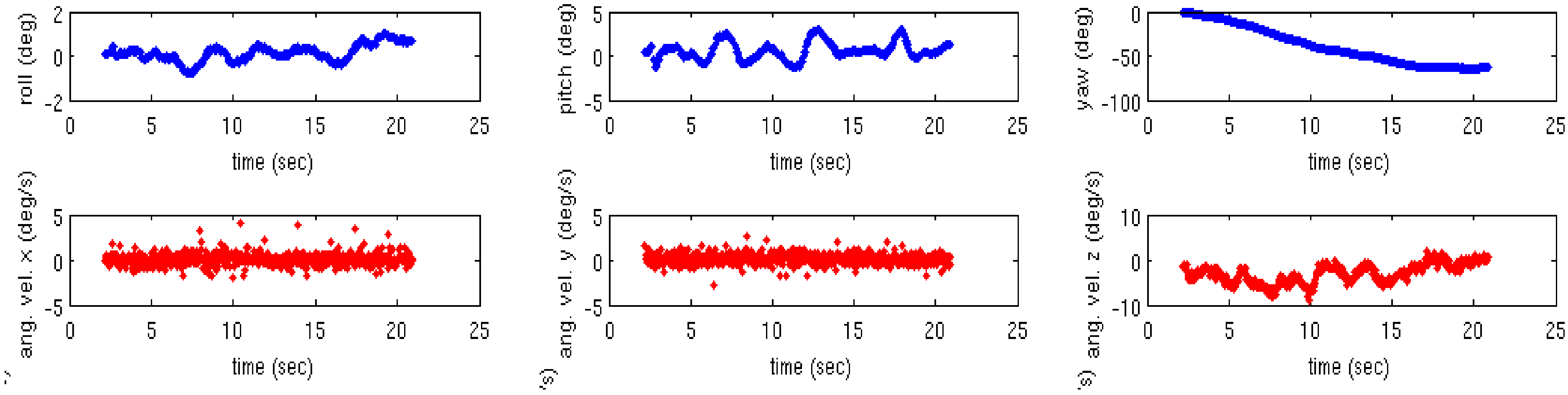
Initial pose



Estimated final pose



Orientation estimates



$\tilde{r} \approx 0^0$

$r \approx 0^0$

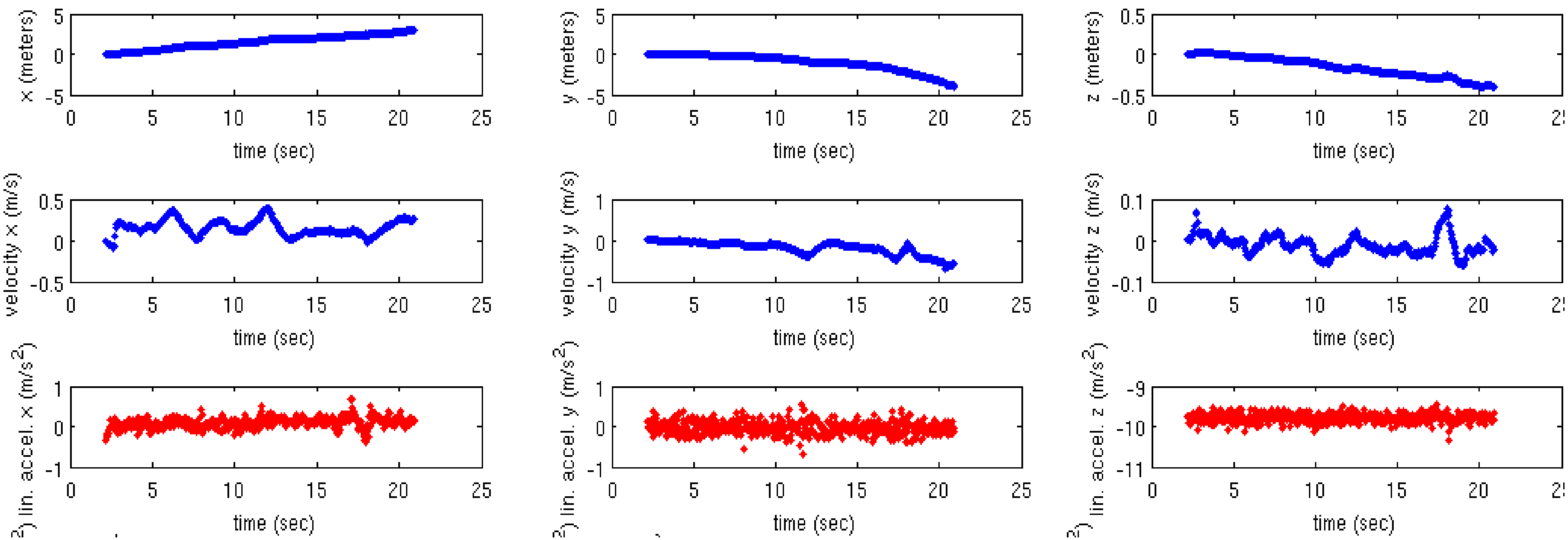
$\tilde{p} \approx 2^0$

$p \approx 0^0$

$\tilde{y} \approx 5^0$

$y \approx -70^0$

Velocity and position estimates



$\tilde{x} \approx 0.3m$

$x \approx 2.2m$

$\tilde{y} \approx 1m$

$y \approx 2m$

$\tilde{z} \approx 0.3m$

$z \approx 0m$

Concluding remarks

- The algorithm is very sensitive to the visual motion estimation.

- Current work:
- Figure out how to better estimate 3D locations of features to improve the position and velocity estimates.
- Implement the algorithm so that it runs in real time.

- Future work:
- Integrate it with the robot controller to enable the robot to perform requested trajectories
- Examine if bundle adjustment methods will make the estimate more robust .

Thank you

- Questions, feedback, and criticisms would be appreciated!

Optional: Propagation (covariance)

- From $\frac{d}{dt}\tilde{\mathbf{x}}_{IMU}(t) = \mathbf{F}\tilde{\mathbf{x}}_{IMU}(t) + \mathbf{G}[\mathbf{n}_g \;\; \dot{\mathbf{b}}_g \;\; \mathbf{n}_a \;\; \dot{\mathbf{b}}_a]^T$ we get

$$\frac{d}{dt}\mathbf{P}_{II}(t) = \mathbf{F}\mathbf{P}_{II}(t) + \mathbf{P}_{II}(t)\mathbf{F}^T + \mathbf{G}\,\mathrm{cov}([\mathbf{n}_g \;\; \dot{\mathbf{b}}_g \;\; \mathbf{n}_a \;\; \dot{\mathbf{b}}_a])\mathbf{G}^T$$

- We can numerically integrate $\frac{d}{dt}\mathbf{P}_{II}(t)$ in the propagation interval to obtain $\mathbf{P}_{II}$.

- And then compute the IMU-camera covariance:

$$\begin{aligned}
\mathbf{P}_{IC}(t+1\,|\,t) &= \mathrm{cov}(\tilde{\mathbf{x}}_{IMU}(t+1), \tilde{\mathbf{x}}_{CAM}(t)) \\
&= \mathrm{cov}(e^{\mathbf{F}dt}\tilde{\mathbf{x}}_{IMU}(t), \tilde{\mathbf{x}}_{CAM}(t)) \\
&= e^{\mathbf{F}dt}\mathbf{P}_{IC}(t\,|\,t)
\end{aligned}$$