# CS-417 INTRODUCTION TO ROBOTICS AND INTELLIGENT SYSTEMS

## Localization

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# Fundamental Problems In Robotics

- How to Go From A to B ? (Path Planning)
- What does the world looks like? (mapping)
  - sense from various positions
  - integrate measurements to produce map
  - assumes perfect knowledge of position
- Where am I in the world? (localization)
  - Sense
  - relate sensor readings to a world model
  - compute location relative to model
  - assumes a perfect world model
- Together, the above two are called SLAM

  (Simultaneous Localization and Mapping)

# Localization

- Tracking: Known initial position
- Global Localization: Unknown initial position
- Re-Localization: Incorrect known position
  - (kidnapped robot problem)

# Uncertainty

- Central to any real system!

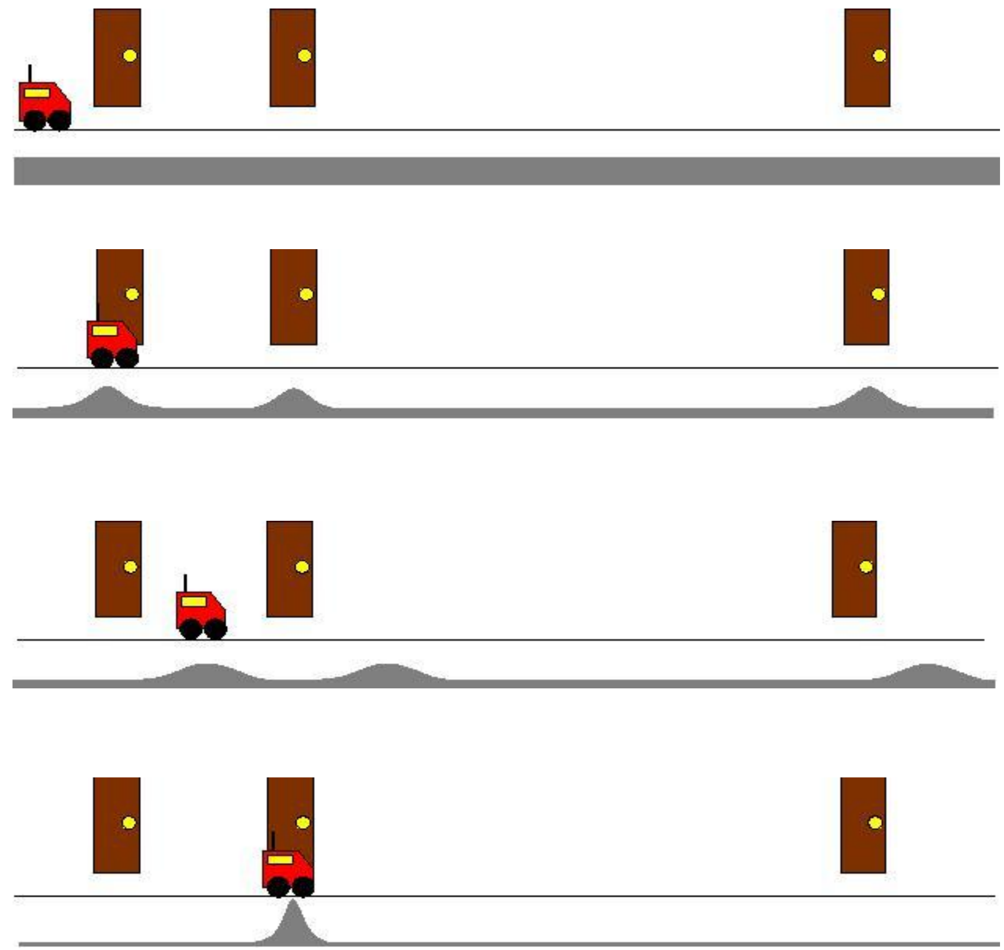
# Localization

Initial state
detects nothing:

Moves and
detects landmark:

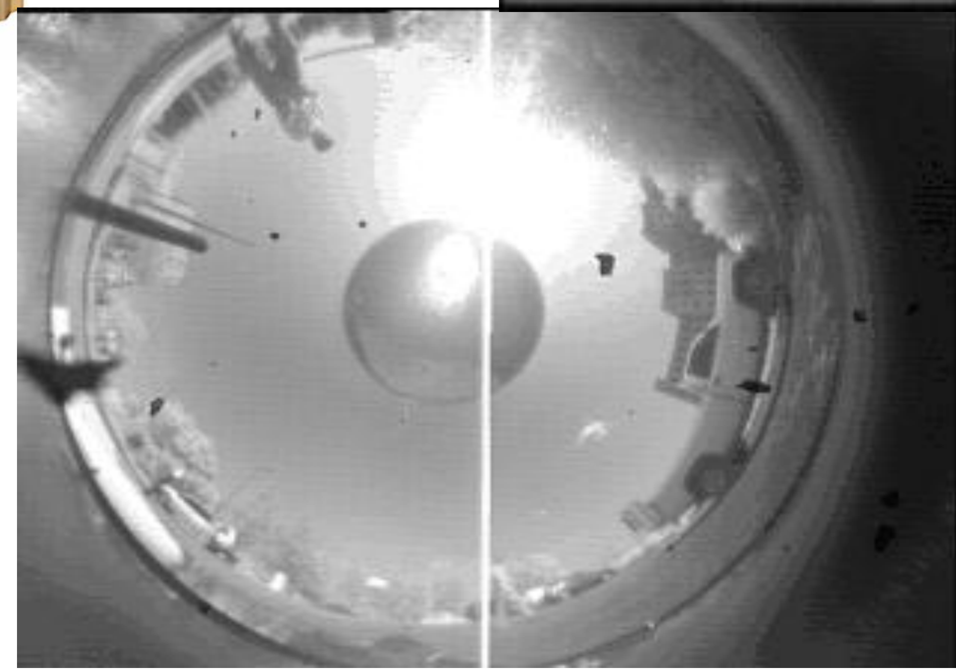
Moves and
detects nothing:

Moves and
detects landmark:



# Sensors

- **Proprioceptive Sensors**
  (monitor state of vehicle-propagate)
  - IMU (accels & gyros)
  - Wheel encoders
  - Doppler radar …
    - **Noise**
- **Exteroceptive Sensors**
  (monitor environment-update)
  - Cameras (single, stereo, omni, FLIR …)
  - Laser scanner
  - MW radar
  - Sonar
  - Tactile…
    - **Uncertainty**

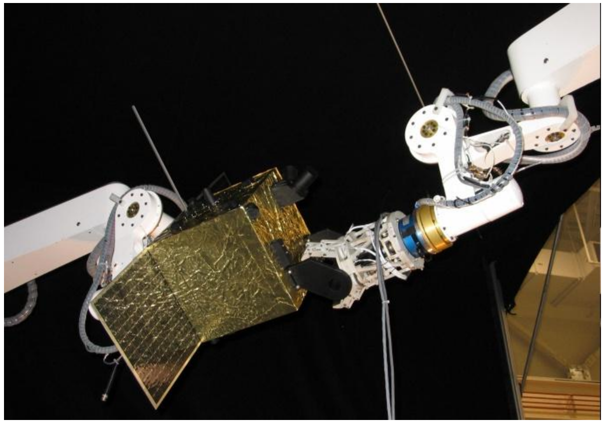



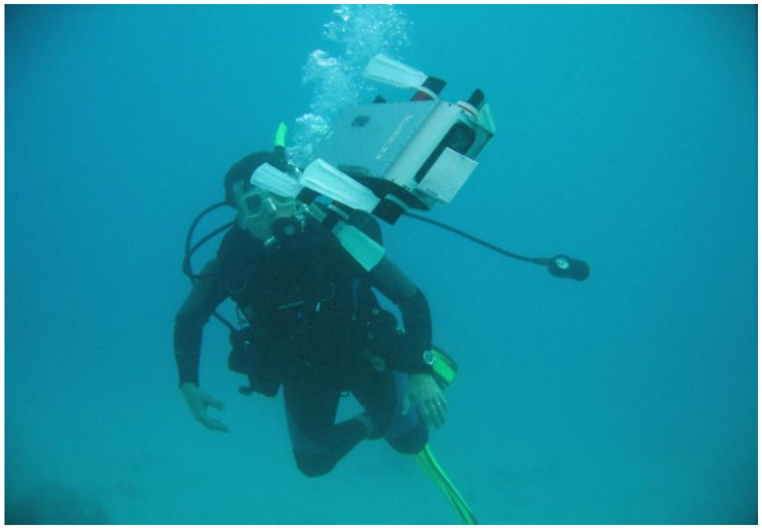
# Bayesian Filter

- "**Filtering**" is a name for combining data.
- Nearly all algorithms that exist for spatial reasoning make use of this approach
  - If you're working in robotics, you'll see it over and over!
- Efficient state estimators
  - Recursively compute the robot's current state based on the previous state of the robot

# State Estimation

- *What is the robot's state?*
- Depends on the robot
  - Indoor mobile robot
    - ***x***=[x, y, θ]
  - 6DOF mobile vehicle
    - ***x***=[x, y, z, φ, ψ, θ]
  - Manipulators
    - ***x***=[$\theta_1$, $\theta_2$, … , $\theta_n$] or
    - ***x***=[x, y, z, φ, ψ, θ] pose of end-effector





# Bayesian Filter

- Estimate state $x$ from data $Z$
  - *What is the probability of the robot being at x?*
- $x$ could be robot location, map information, locations of targets, etc…
- $Z$ could be sensor readings such as range, actions, odometry from encoders, etc…)
- This is a general formalism that does not depend on the particular probability representation
- Bayes filter **recursively** computes the posterior distribution:

$$Bel(x_T) = P(x_T \mid Z_T)$$

# Derivation of the Bayesian Filter

Estimation of the robot's state given the data:

$$Bel(x_t) = p(x_t \mid Z_T)$$

The robot's data, Z, is expanded into two types: observations $o_i$ and actions $a_i$

$$Bel(x_t) = p(x_t \mid o_t, a_{t-1}, o_{t-1}, a_{t-2}, \ldots, o_0)$$

Invoking the Bayesian theorem

$$Bel(x_t) = \frac{p(o_t \mid x_t, a_{t-1}, \ldots, o_0)\, p(x_t \mid a_{t-1}, \ldots, o_0)}{p(o_t \mid a_{t-1}, \ldots, o_0)}$$

# Derivation of the Bayesian Filter

Denominator is constant relative to $x_t$

$$\eta = 1 / p(o_t \mid a_{t-1},\ldots,o_0)$$

$$Bel(x_t) = \eta p(o_t \mid x_t, a_{t-1},\ldots,o_0)\, p(x_t \mid a_{t-1},\ldots,o_0)$$

First-order Markov assumption shortens first term:

$$Bel(x_t) = \eta p(o_t \mid x_t)\, p(x_t \mid a_{t-1},\ldots,o_0)$$

Expanding the last term (theorem of total probability):

$$Bel(x_t) = \eta p(o_t \mid x_t) \int p(x_t \mid x_{t-1}, a_{t-1},\ldots,o_0) p(x_{t-1} \mid a_{t-1},\ldots,o_0) dx_{t-1}$$

# Reminder: Bayes Rule

- Conditional probabilities

$$p(o \wedge S) = p(o \mid S)\, p(S)$$

- Bayes rule relates conditional probabilities

$$p(o \mid S) = \frac{p(S \mid o)\, p(o)}{p(S)}$$

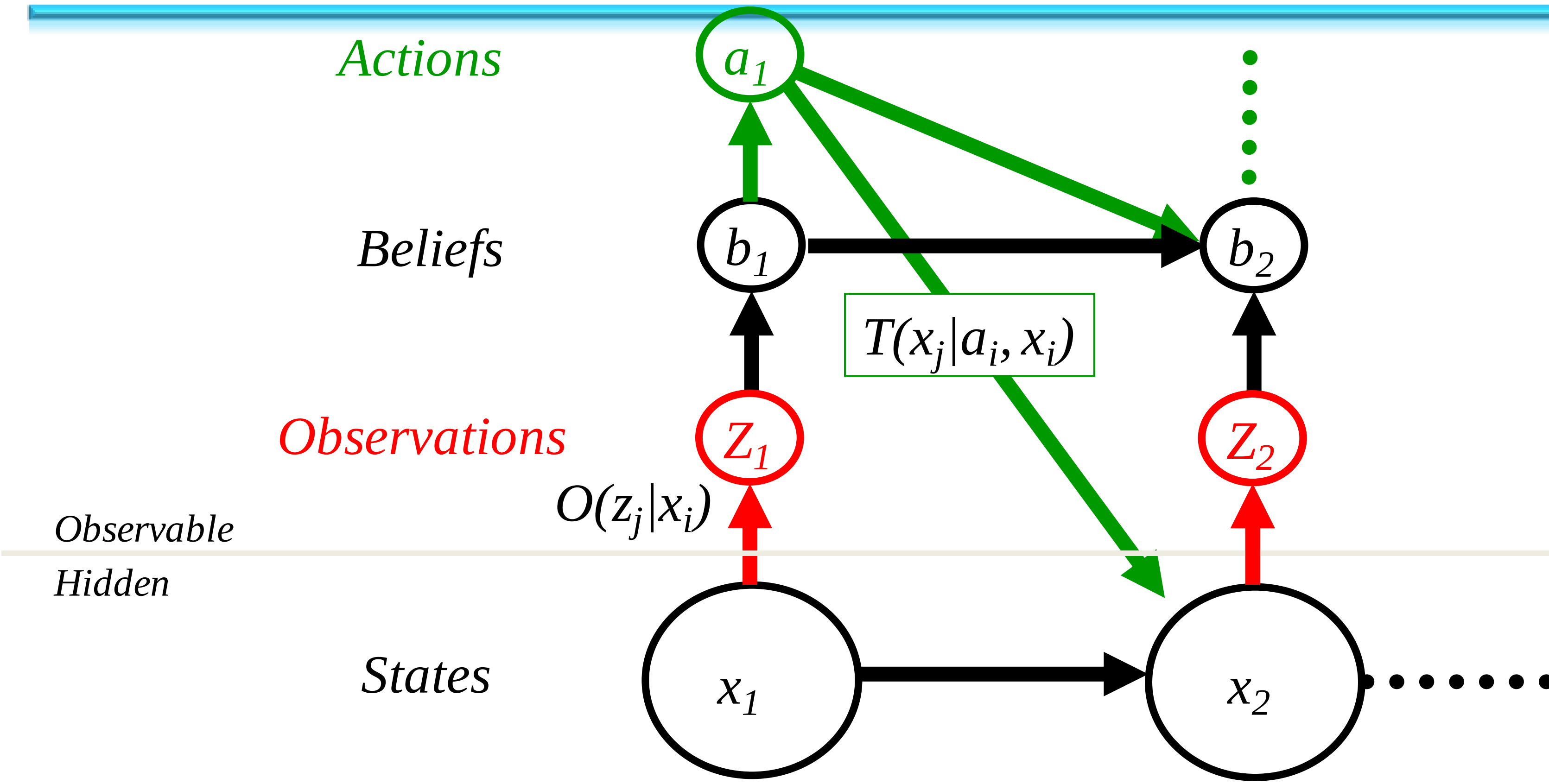
Bayes rule

- So, what does this say about $\text{odds}(o \mid S_2 \wedge S_1)$ ?

Can we update easily ?

$$p(a \mid b, c) = \frac{p(b \mid a, c)\, p(a \mid c)}{p(b \mid c)}$$

# Graphical Models, Bayes' Rule and the Markov Assumption




$$\text{Bayes rule}: p(x \mid y) = \frac{p(y \mid x)\, p(x)}{p(y)}$$

$$\text{Markov}: p(x_t \mid x_{t-1}, a_t, a_0, z_0, a_1, z_1, \ldots, z_{t-1}) = p(x_t \mid x_{t-1}, a_t)$$

# Derivation of the Bayesian Filter

First-order Markov assumption shortens middle term:

$$Bel(x_t) = \eta \boxed{p(o_t \mid x_t)} \int p(x_t \mid x_{t-1}, a_{t-1}) p(x_{t-1} \mid a_{t-1}, \ldots, o_0) dx_{t-1}$$

Finally, substituting the definition of *Bel*($x_{t-1}$):

$$Bel(x_t) = \eta p(o_t \mid x_t) \int p(x_t \mid x_{t-1}, a_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

The above is the probability distribution that must be estimated from the robot's data

# Iterating the Bayesian Filter

- Propagate the motion model:

$$Bel\_(x_t) = \int P(x_t \mid a_{t-1}, x_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

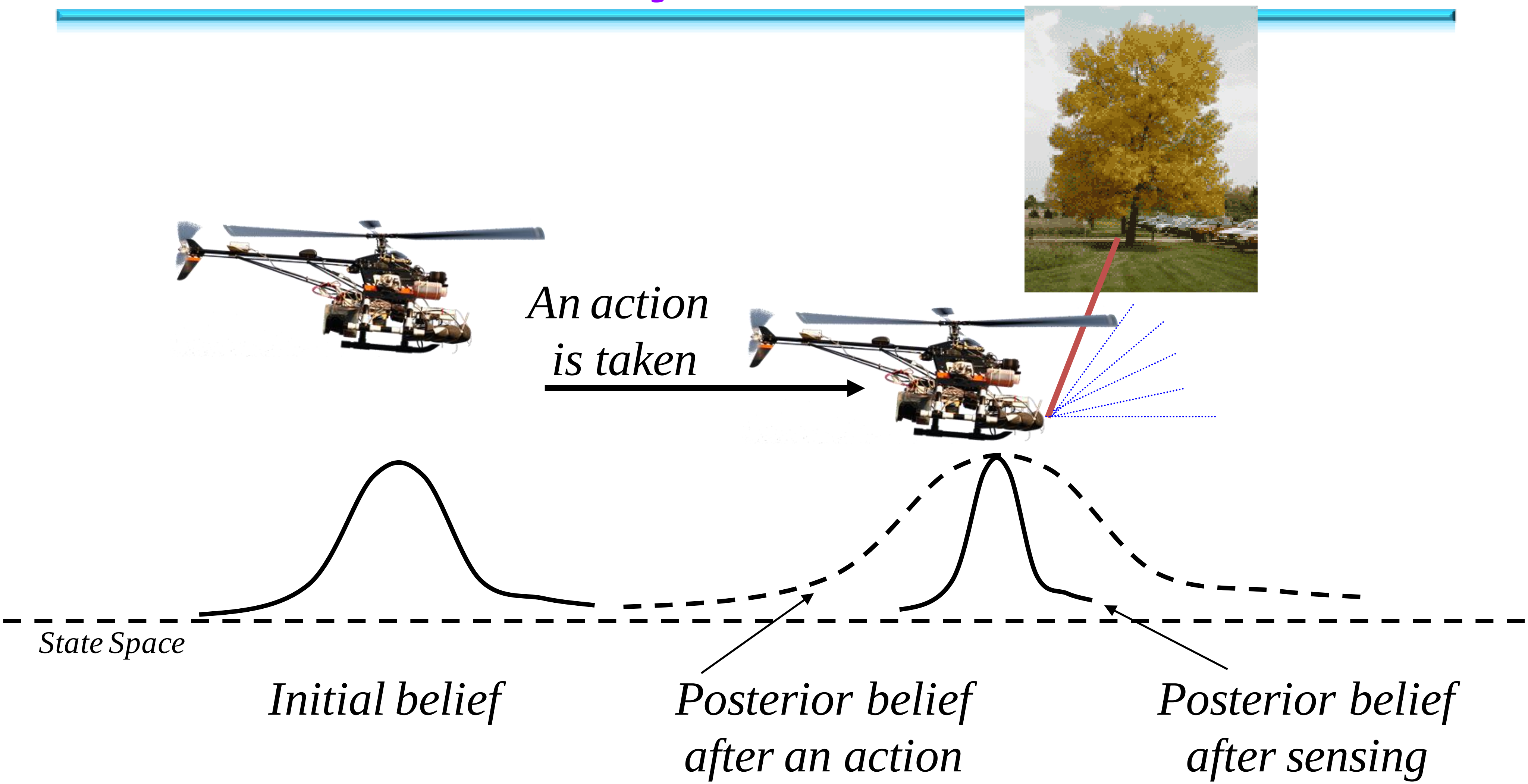
Compute the current state estimate before taking a sensor reading by integrating over all possible previous state estimates and applying the motion model

- Update the sensor model:

$$Bel(x_t) = \eta P(o_t \mid x_t) Bel\_(x_t)$$

Compute the current state estimate by taking a sensor reading and multiplying by the current estimate based on the most recent motion history

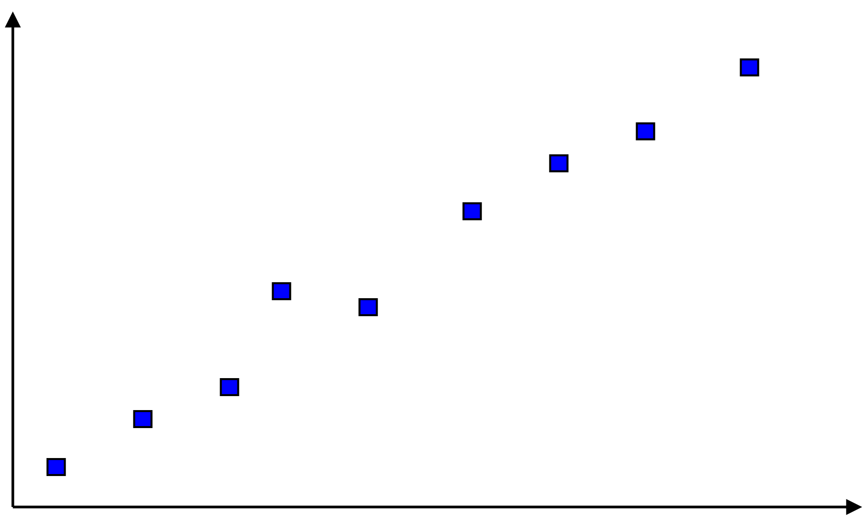
# Bayes Filter



# Representation of the Belief Function

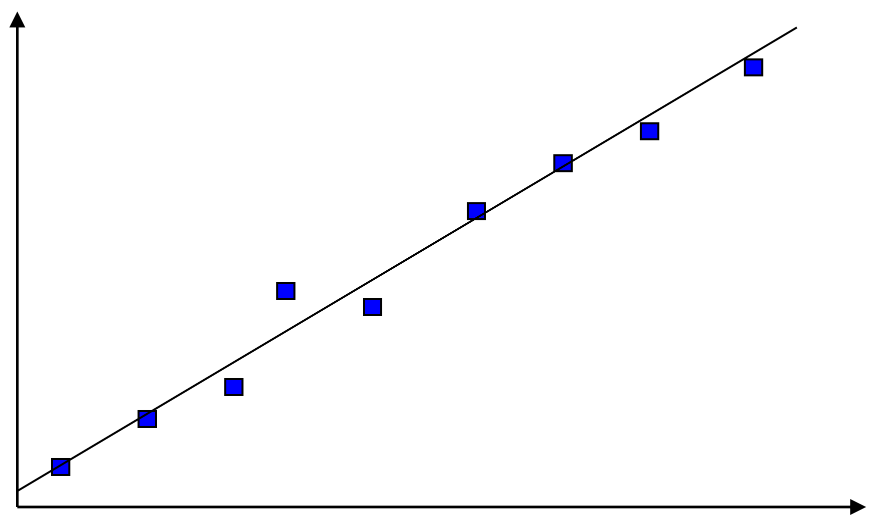
Sample-based representations

*e.g.* Particle filters



$$(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots (x_n, y_n)$$

Parametric representations



$$y = mx + b$$

# Different Approaches

**Kalman filters** (Early-60s?)
- Gaussians
- approximately linear models
- position tracking

**Extended Kalman Filter**

**Information Filter**

**Unscented Kalman Filter**

**Multi-hypothesis** ('00)
- Mixture of Gaussians
- Multiple Kalman filters
- Global localization, recovery

**Discrete approaches** ('95)
- Topological representation ('95)
- Uncertainty handling (POMDPs)
- occas. global localization, recovery
- Grid-based, metric representation ('96)
- global localization, recovery

**Particle filters** ('98)
- Condensation (Isard and Blake '98)
- Sample-based representation
- Global localization, recovery
- Rao-Blackwellized Particle Filter

# Bayesian Filter : Requirements for Implementation

- Representation for the belief function
- Update equations
- Motion model
- Sensor model
- Initial belief state