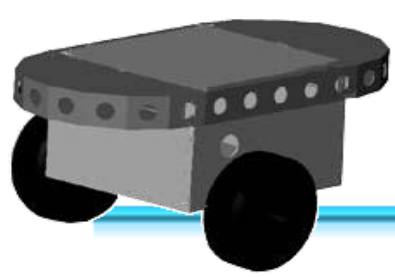


# CS-417 INTRODUCTION TO ROBOTICS AND INTELLIGENT SYSTEMS

## Coverage



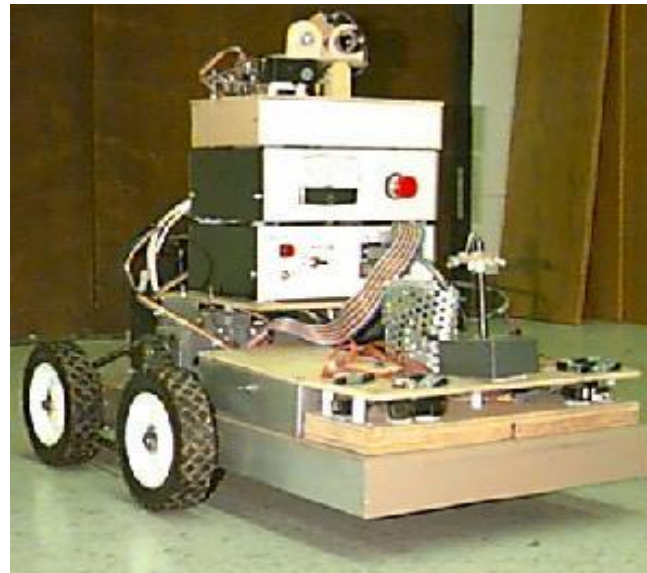
# Motivation

## Humanitarian Demining



# Motivation

## Lawn Mowing



# Motivation

## Vacuum Cleaning



# Robotic Coverage

- More than 2 million Roombas sold!
- Automated Car Painting



# Roomba Costumes



# Coverage

---

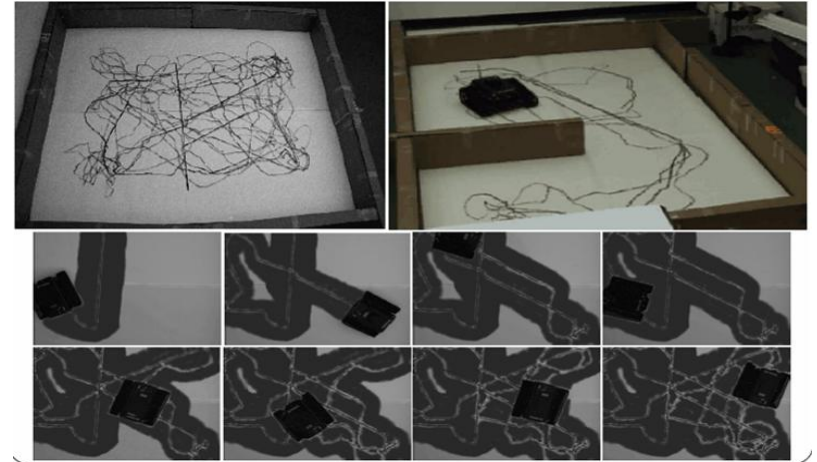
- First Distinction
  - Deterministic **Demining**
  - Random **Vacuum Cleaning**
- Second Distinction
  - Complete
  - No Guarantee
- Third Distinction
  - Known Environment
  - Unknown Environment



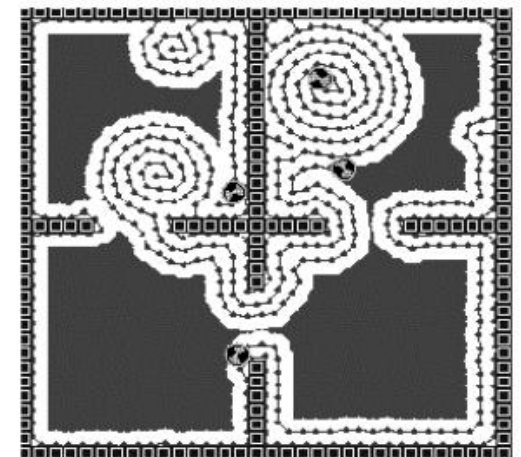
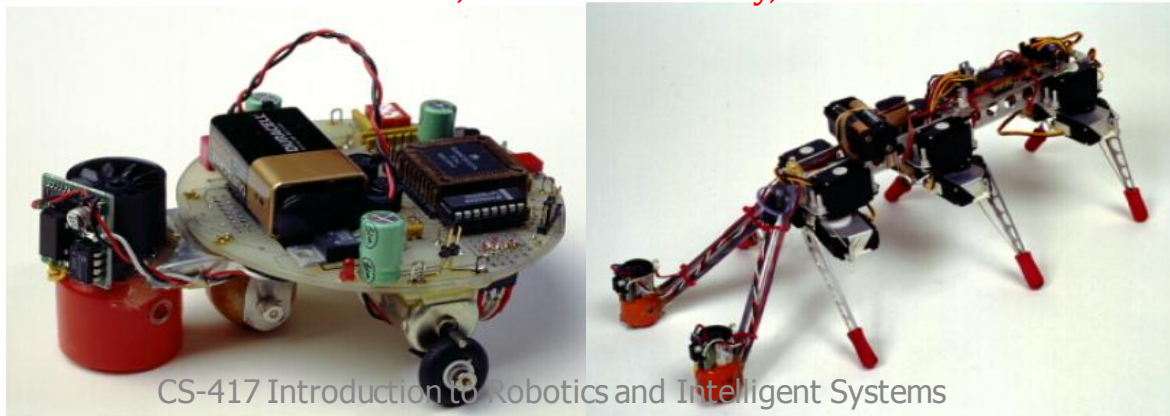
# Non-Deterministic Coverage

- Complete Random Walk
- Ant Robotics
  - Leave trail
  - Bias the behavior towards or away from the trails

S. Koenig Ant Robotics, terrain coverage



Andrew Russell, Monash University, Australia



Ant Robotics: I. Wagner, IBM & Technion





# Deterministic Coverage

---

- Complete Algorithm
- Guarantees Complete Coverage



# Cell-Decomposition Methods

---

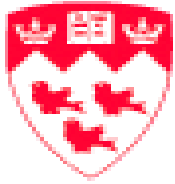
Two families of methods:

- **Exact cell decomposition**

The free space  $F$  is represented by a collection of non-overlapping cells whose union is exactly  $F$

**Examples:** trapezoidal and cylindrical decompositions



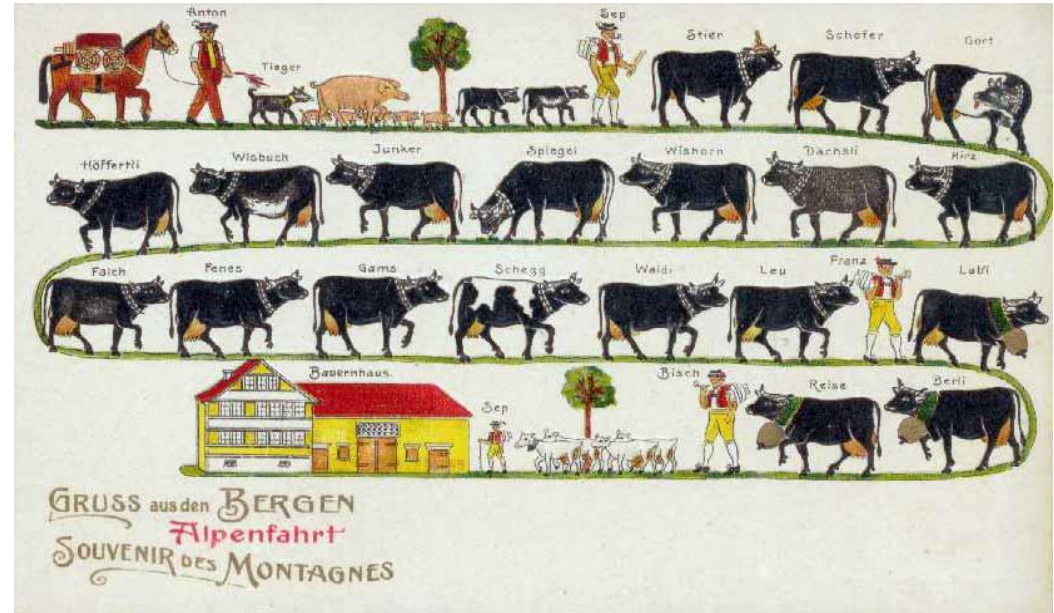


# **BOUSTROPHEDON CELLULAR DECOMPOSITION**

**The way of the Ox!**

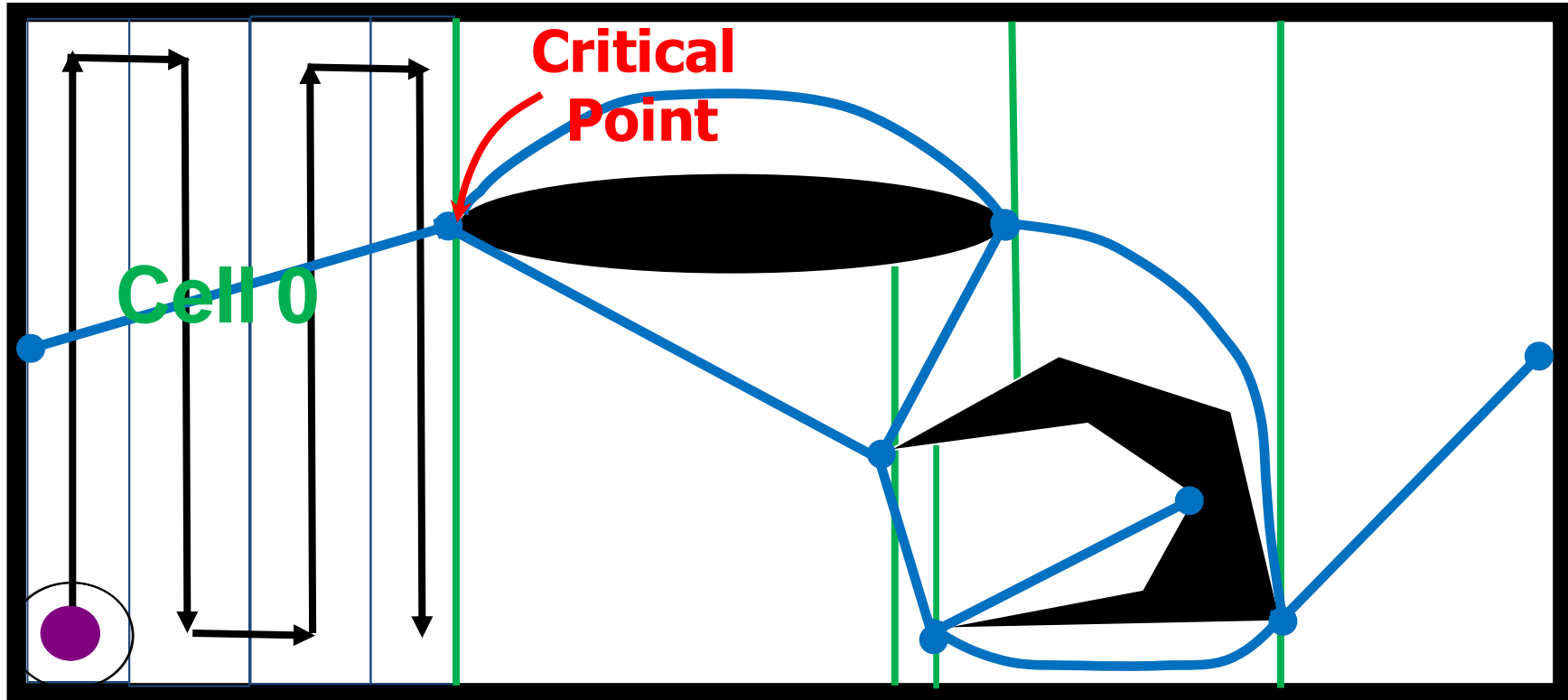
# Single Robot Coverage

- Deterministic algorithm
- Guarantee of completeness
- Sensor based
- Unknown Environment



- Seed spreader algorithm: Lumelsky et al, “Dynamic path planning in sensor-based terrain acquisition”, IEEE Transactions on Robotics and Automation, August 1990.
- Boustrophedon algorithm: Choset and Pignon, “Coverage path planning: The boustrophedon cellular decomposition”, International Conference on Field and Service Robotics, 1997.

# Single Robot Coverage



→  
Direction of Coverage

—  
Cellular Decomposition

—•—•—  
Reeb graph  
Vertices: Critical Points  
Edges: Cells



# Critical Points

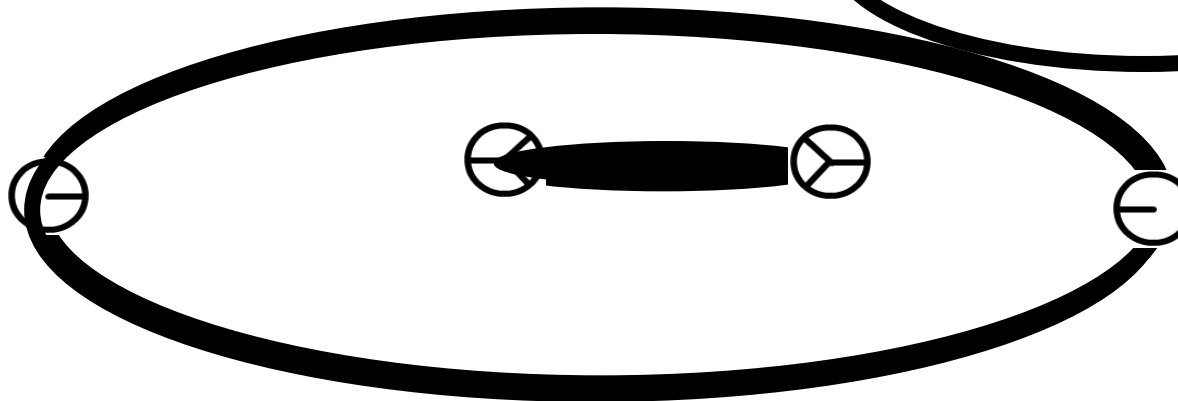
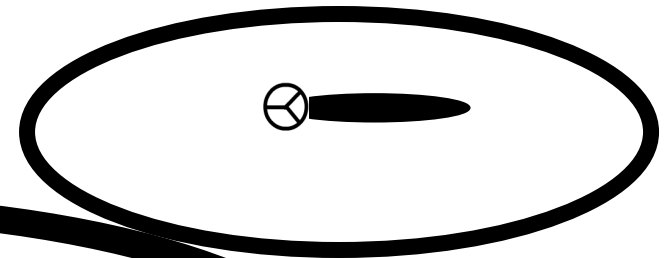
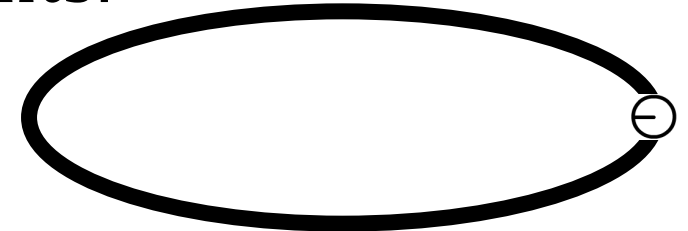
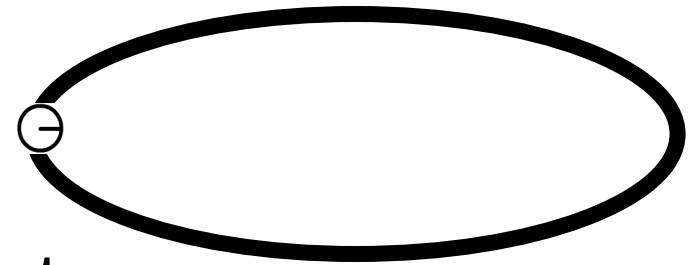
- There are four types of critical points:

⊖ Forward Concave critical point

⊖ Reverse Concave critical point

⊕ Reverse Convex critical point

⊕ Forward Convex critical point



  
Direction of Coverage

# Optimal Coverage

---

- Find an order for traversing the Reeb graph such that the robot would not go through a cell more times than necessary

## Solution

- Use the Chinese Postman Problem



# Chinese Postman Problem

---

- The Chinese postman problem (CPP), is to find a shortest closed path that visits every edge of a (connected) undirected graph. When the graph has an Eulerian circuit (a closed walk that covers every edge once), that circuit is an optimal solution.

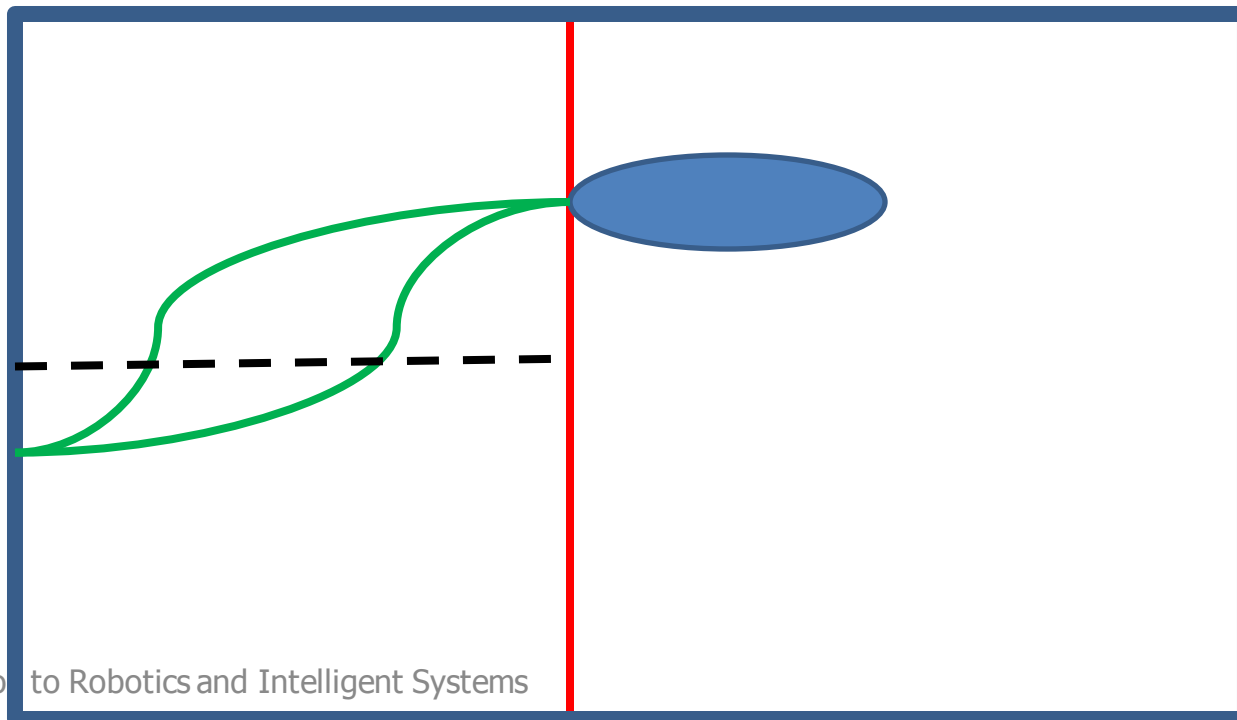
**See:** J. Edmonds and E.L. Johnson, Matching Euler tours and the Chinese postman problem, Math. Program. (1973).





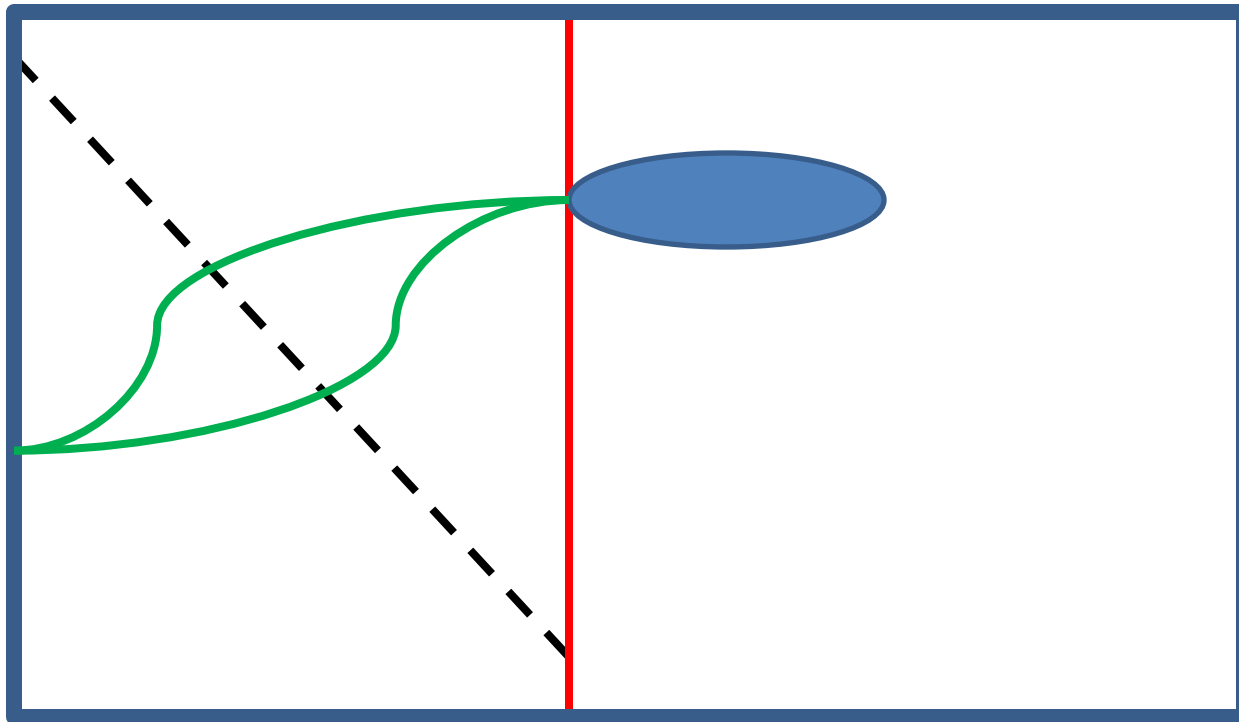
# Chinese Postman Problem

- The solution of the CPP guarantees that no edge is doubled more than once
- That means some cells have to be traversed twice
- Cells that have to be traversed/covered are divided in half



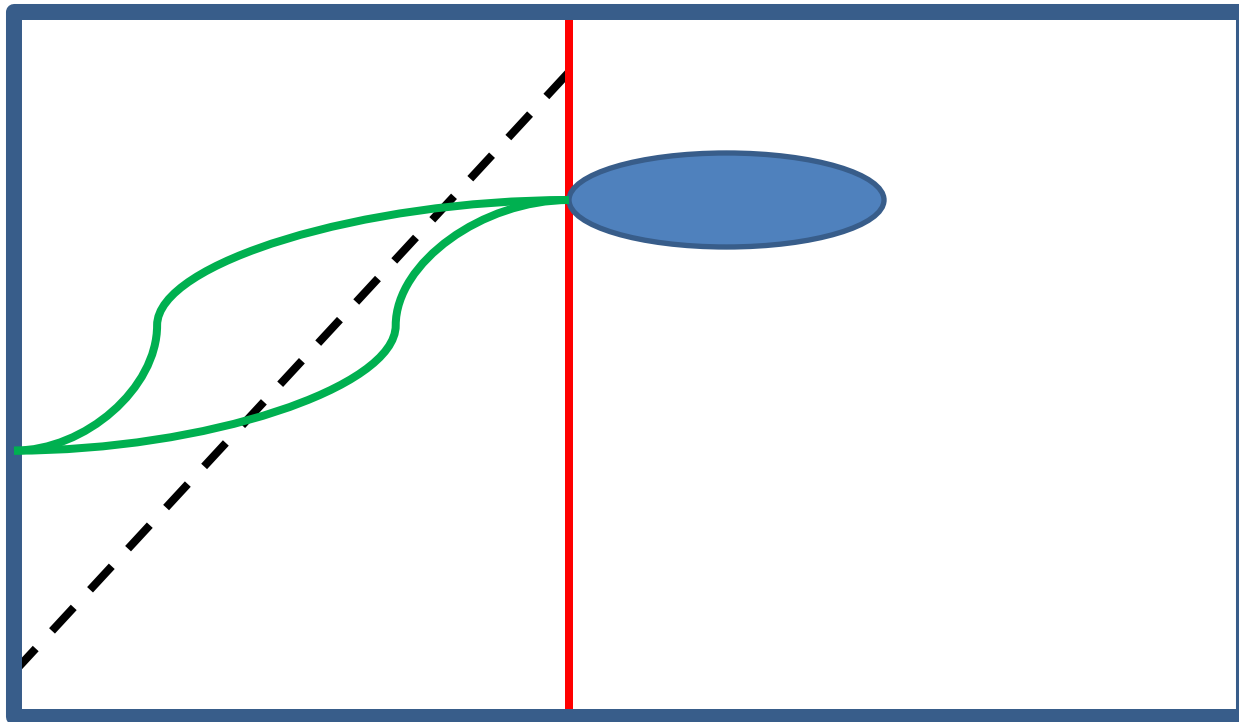
# Double Coverage of a Single Cell

- By dividing the cell diagonally we control the beginning and end of the coverage



# Double Coverage of a Single Cell

- By dividing the cell diagonally we control the beginning and end of the coverage



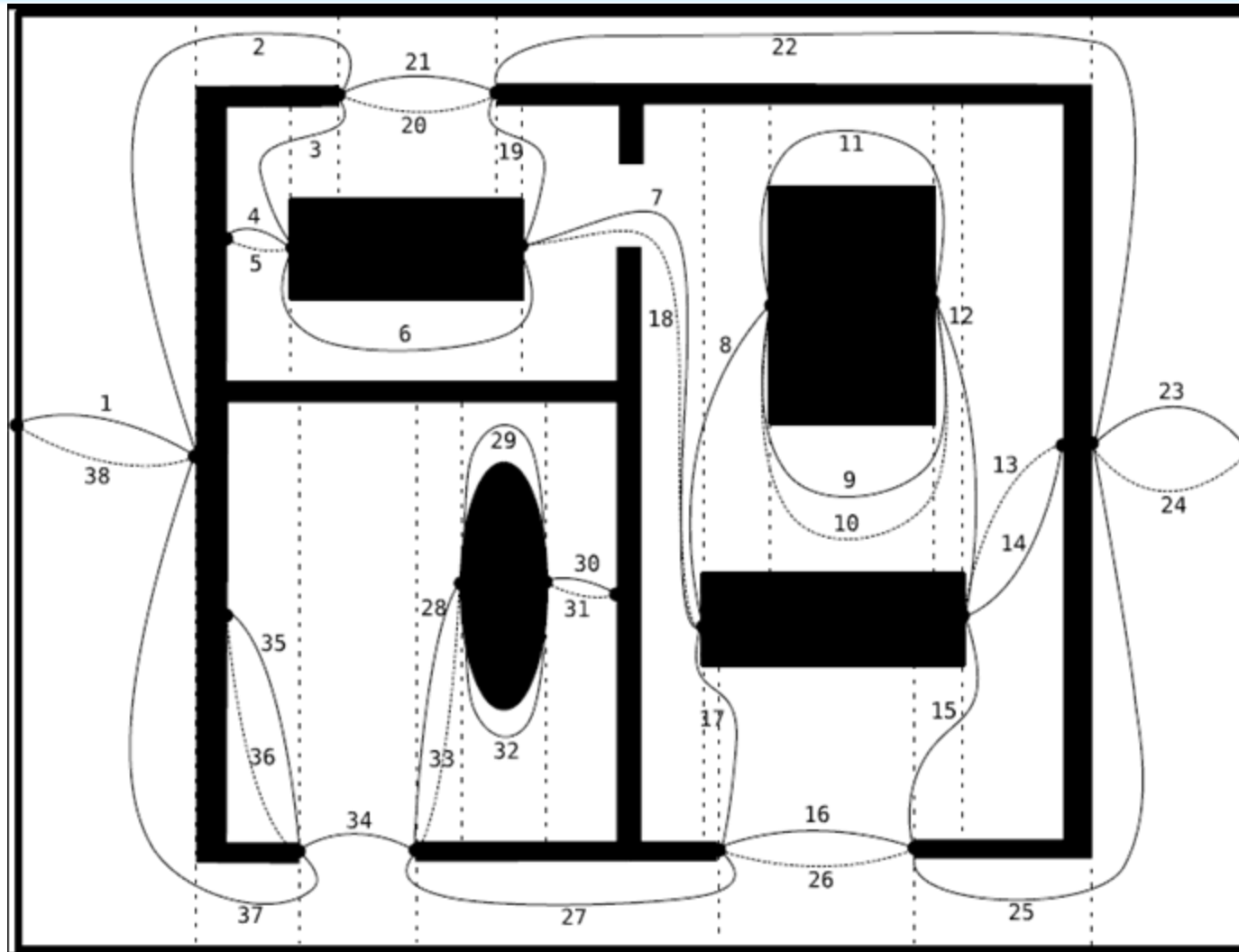
# Optimal Coverage Algorithm

---

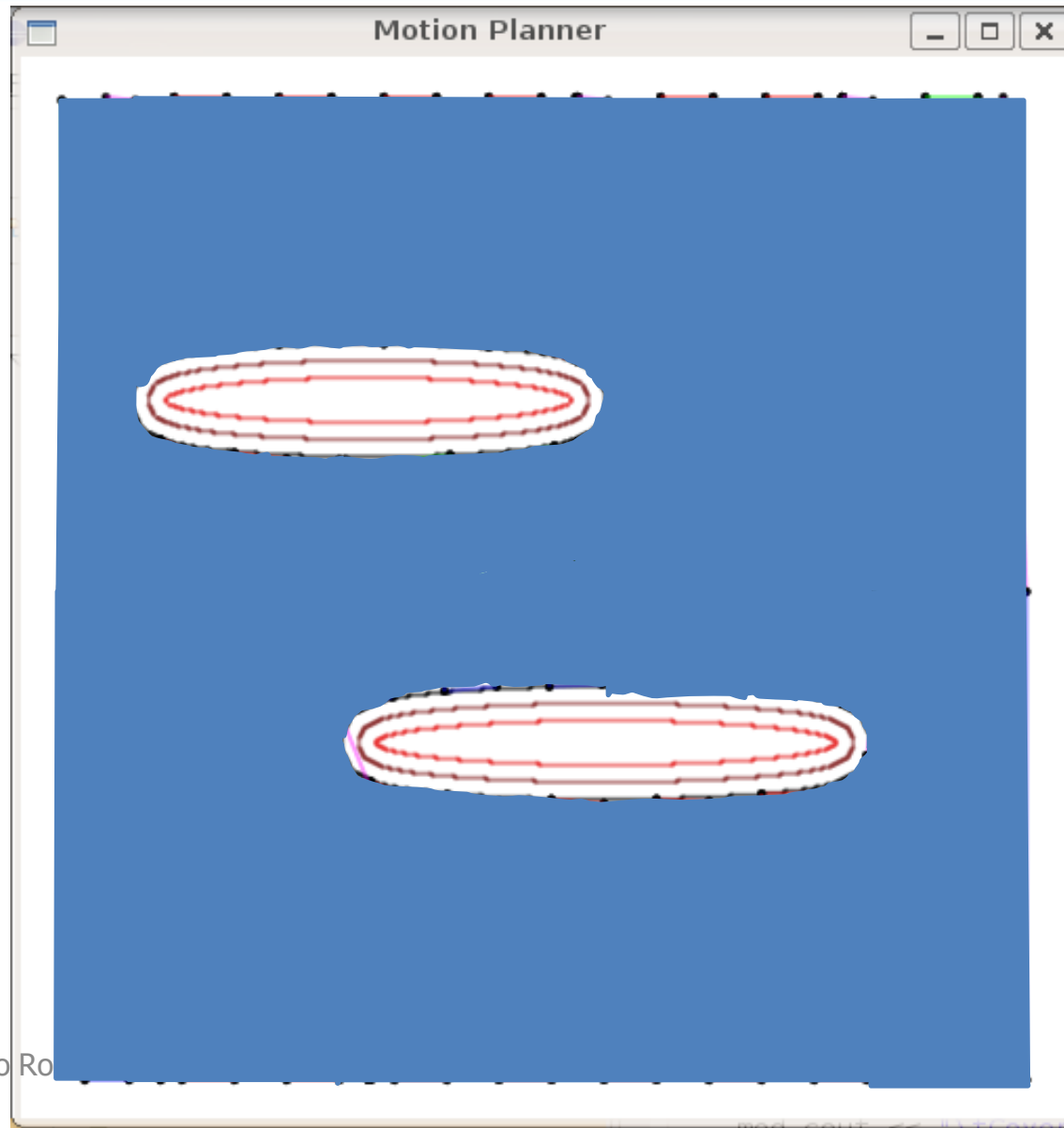
- Given a known environment:
  - Calculate the Boustrophedon decomposition
  - Construct the Reeb graph
  - Use the Reeb graph as input to the Chinese Postman Problem (CPP)
  - Use the solution of the CPP to find a minimum cost cycle traversing every edge of the Reeb graph
  - For every doubled edge divide the corresponding cell in half
  - Traverse the Reeb graph by covering each cell in order



# Traversal order of the Reeb graph

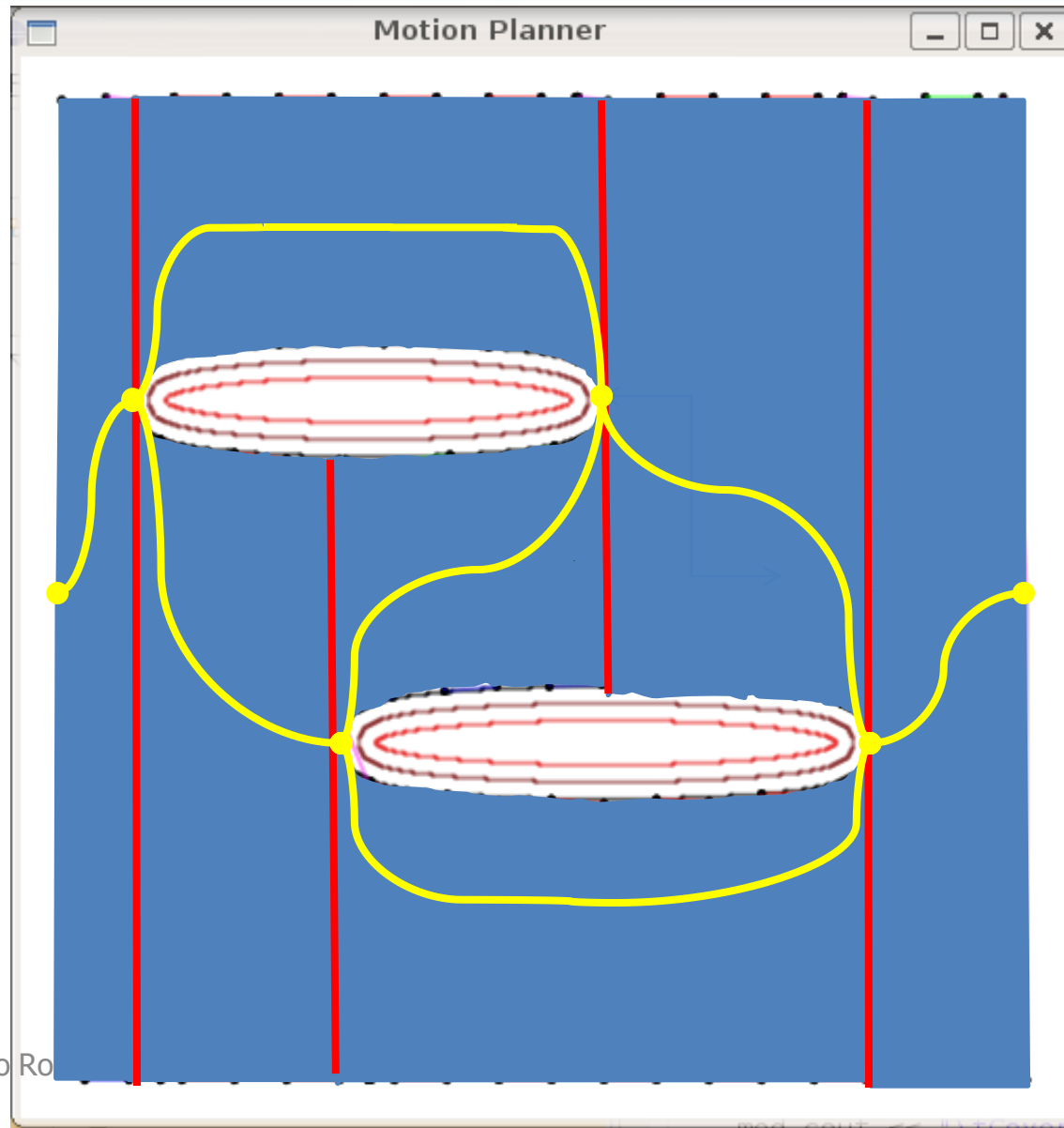


# Example



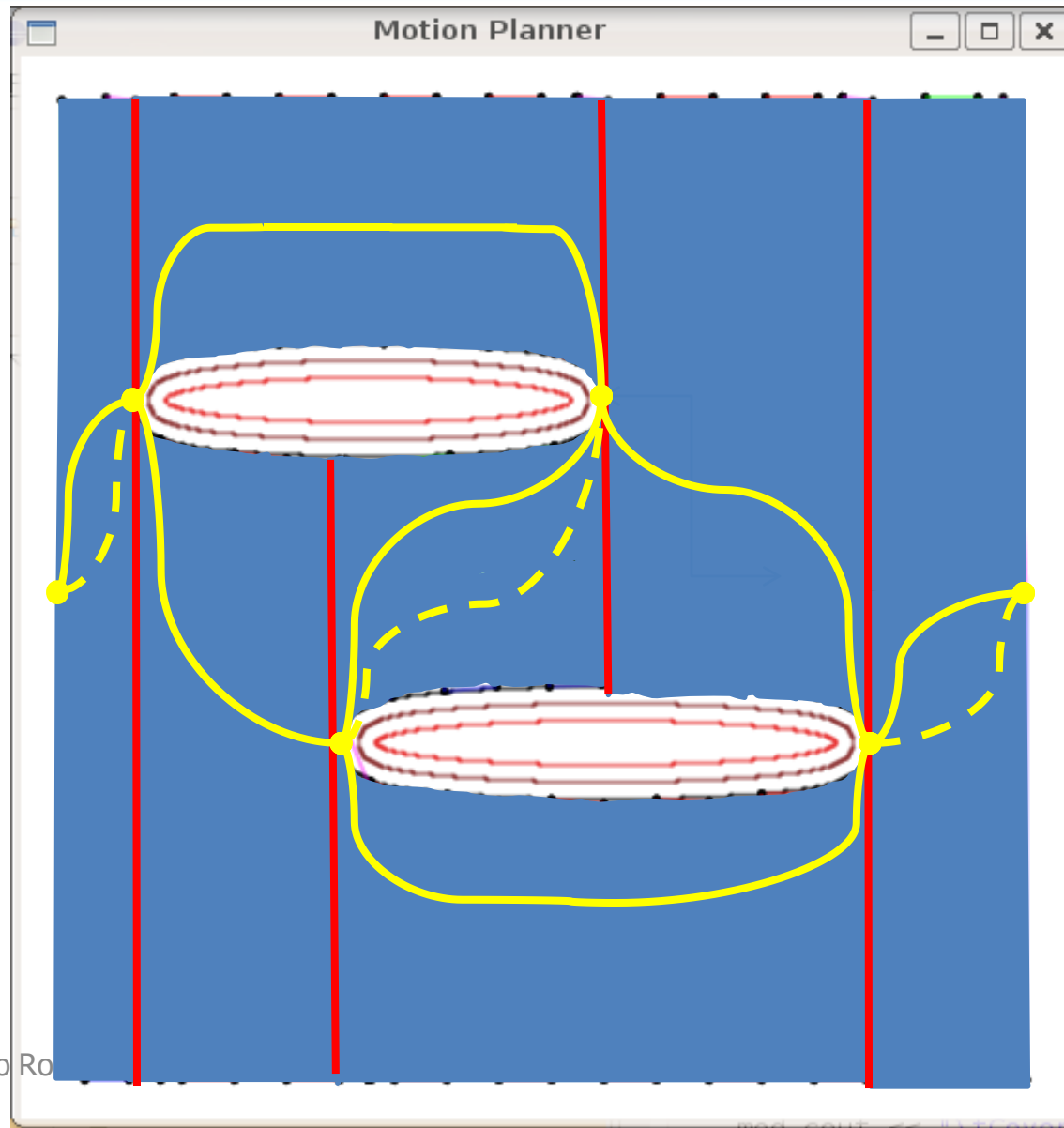


# Example: Reeb Graph

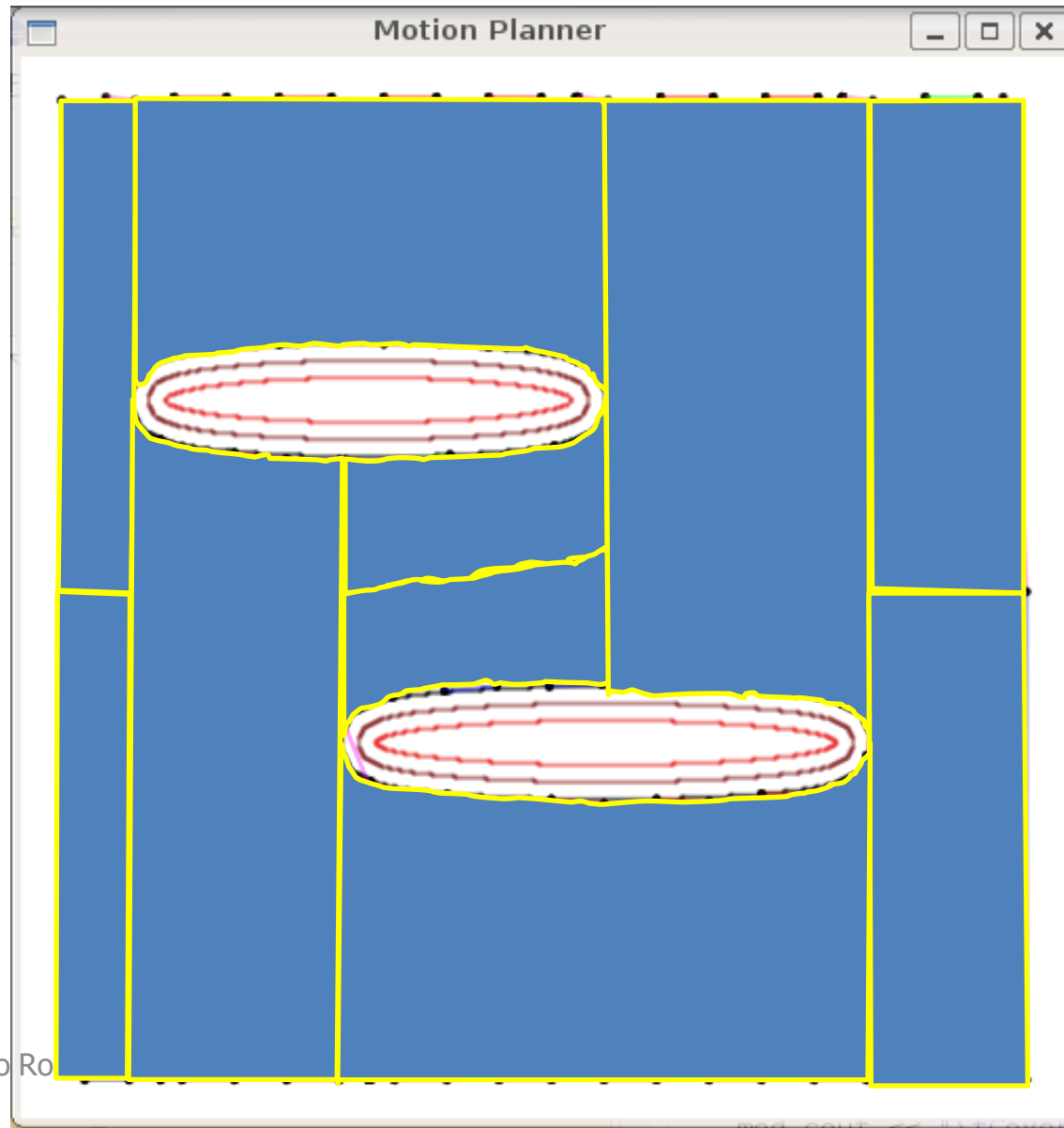




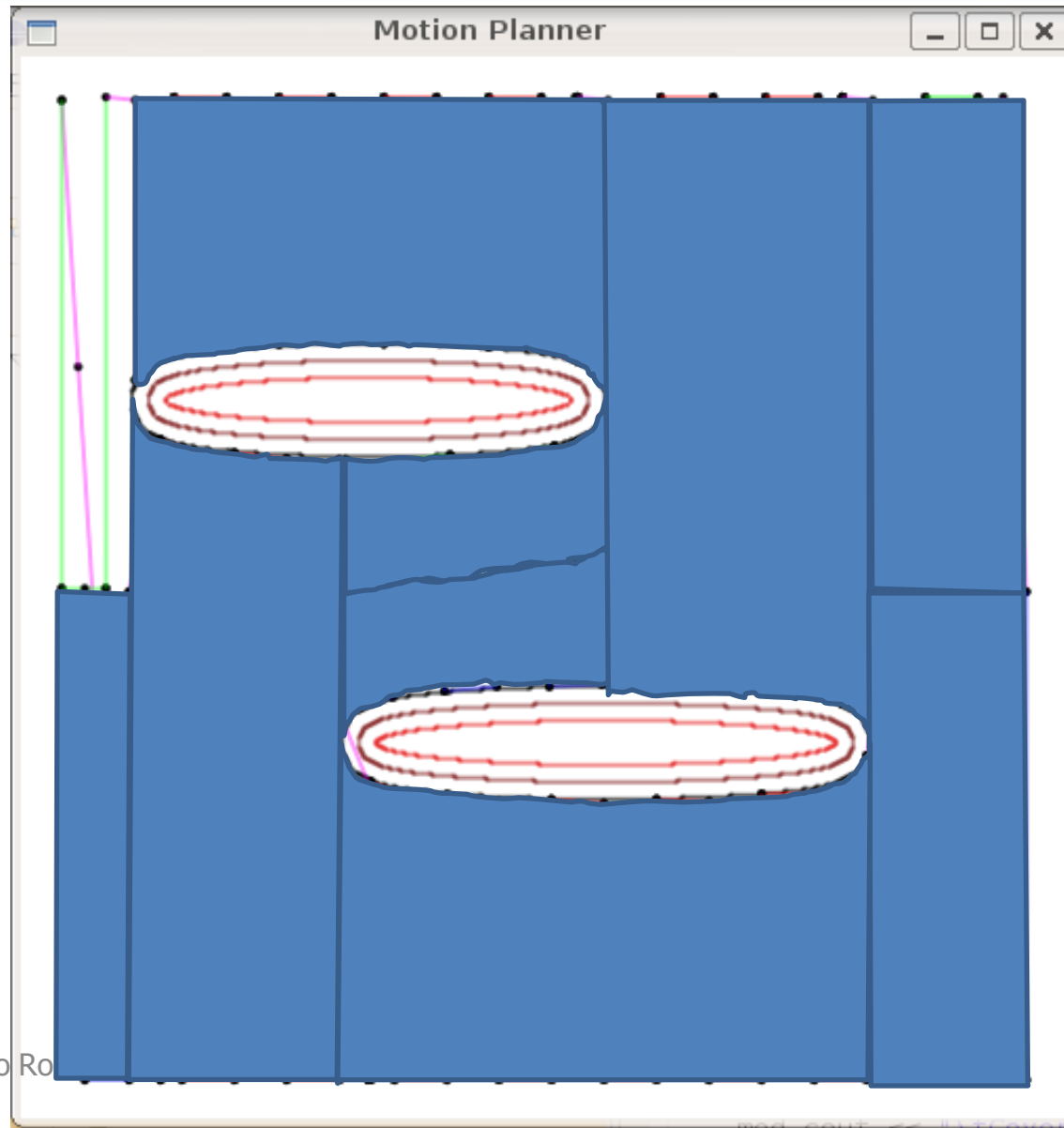
# Example: CPP solution



# Example

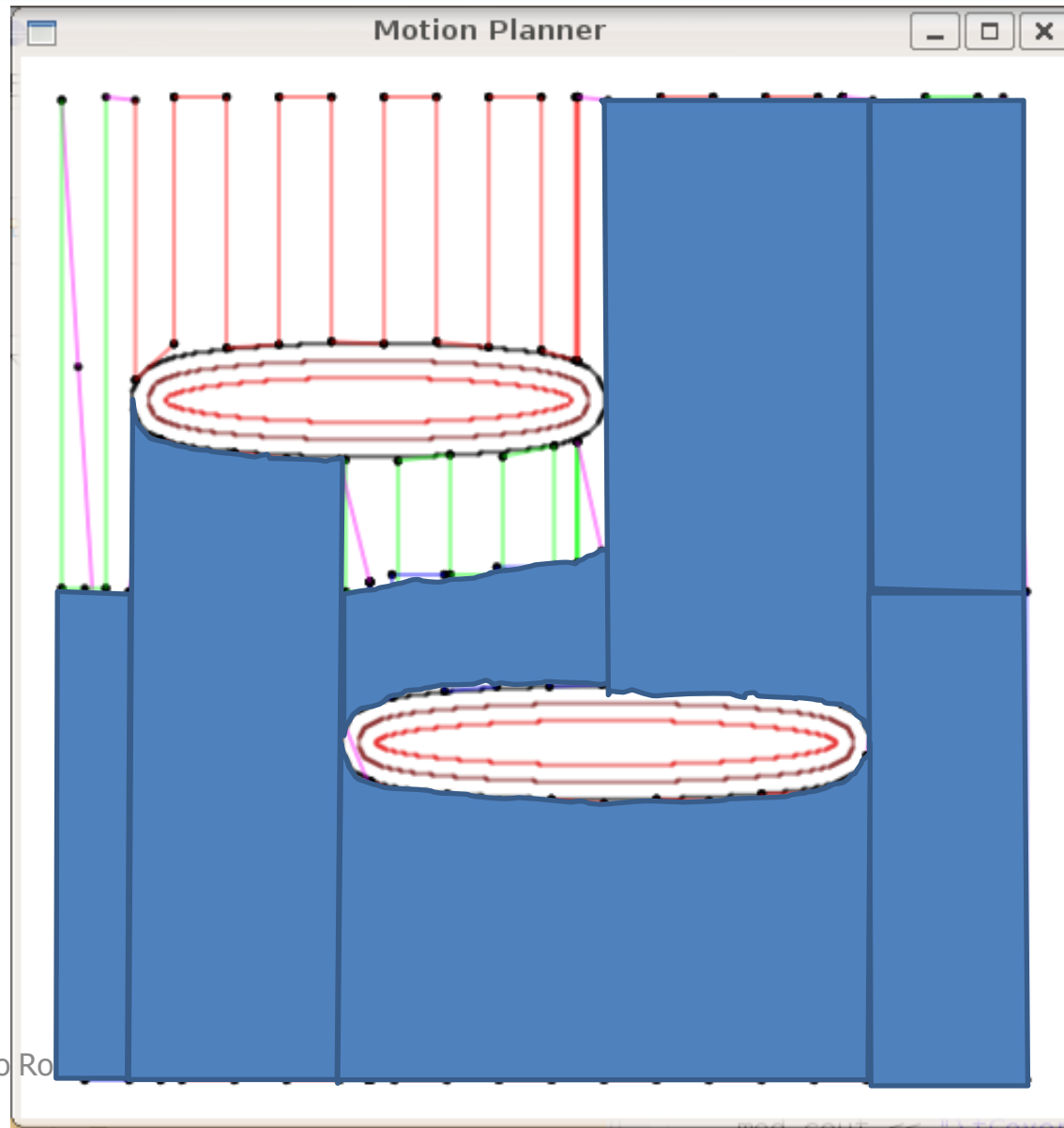


# Example





# Example





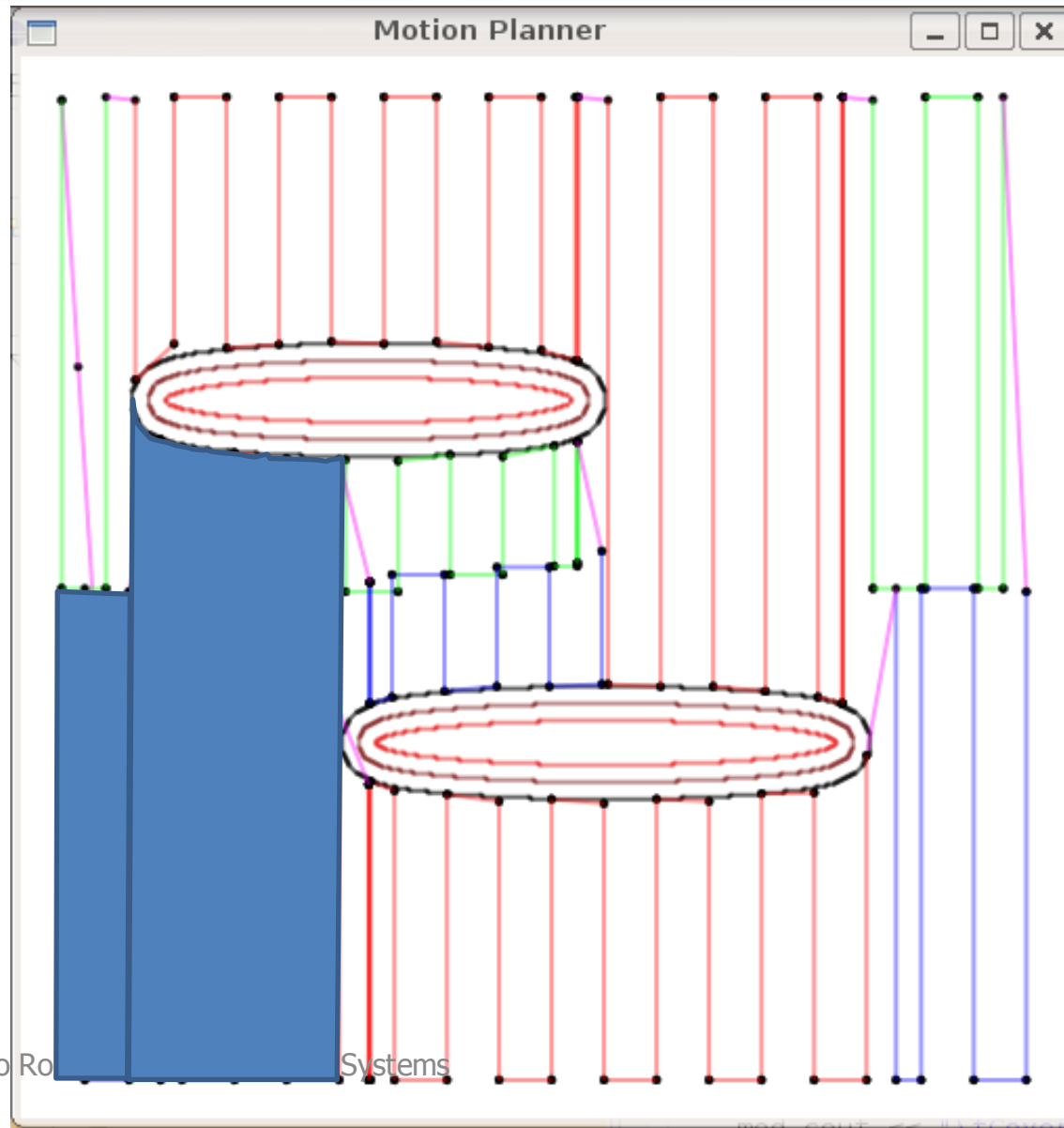




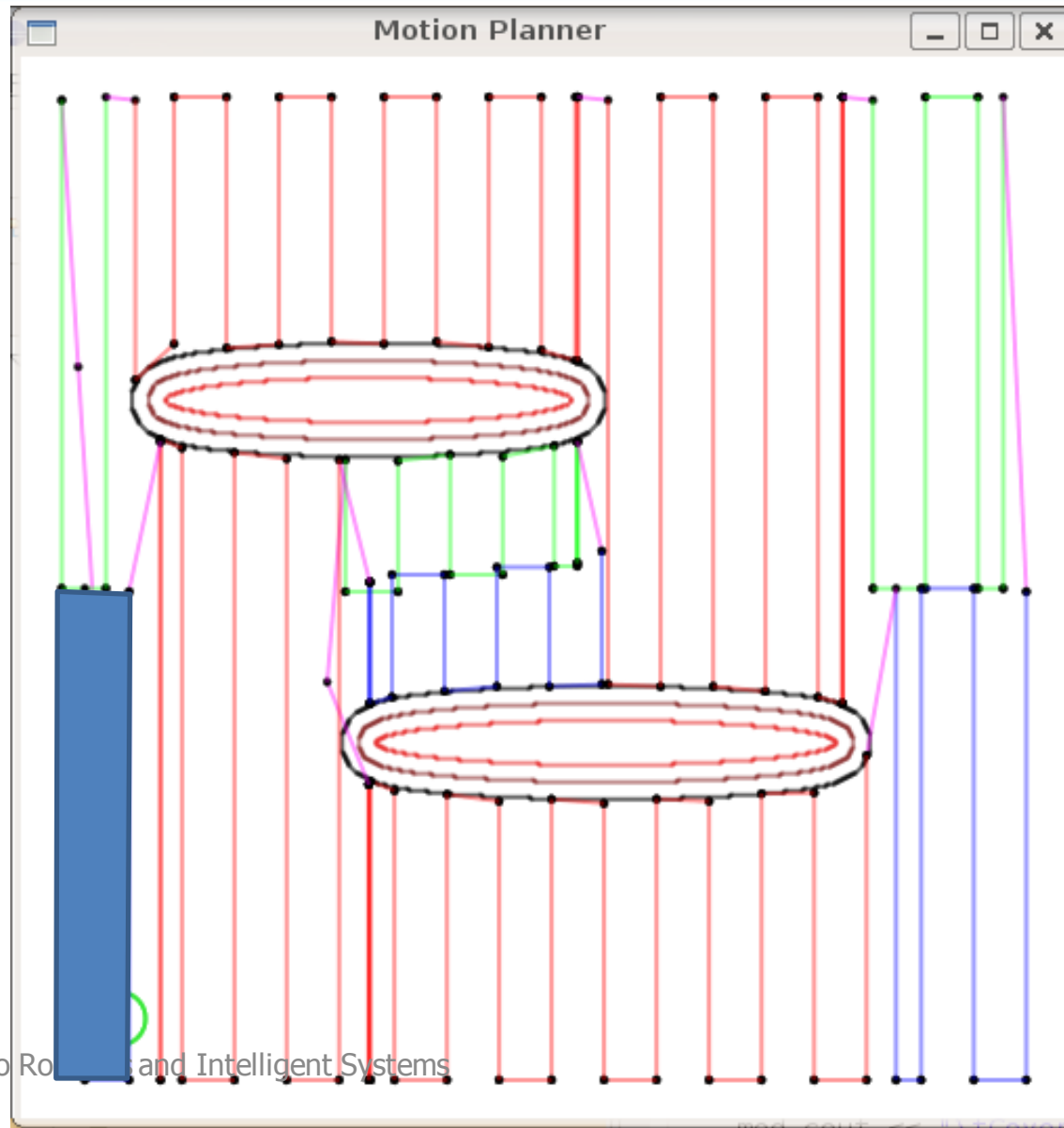




# Example

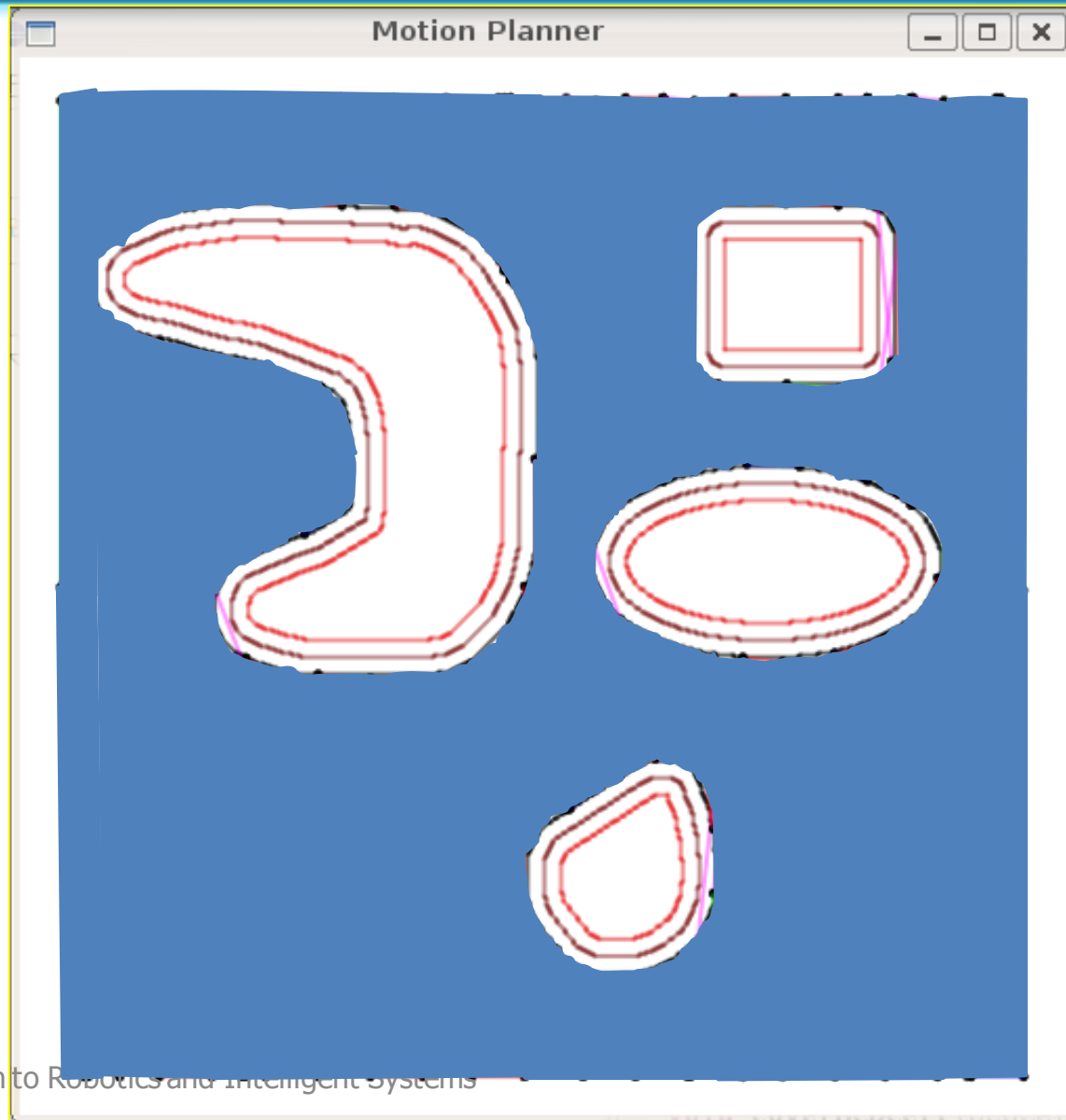


# Example

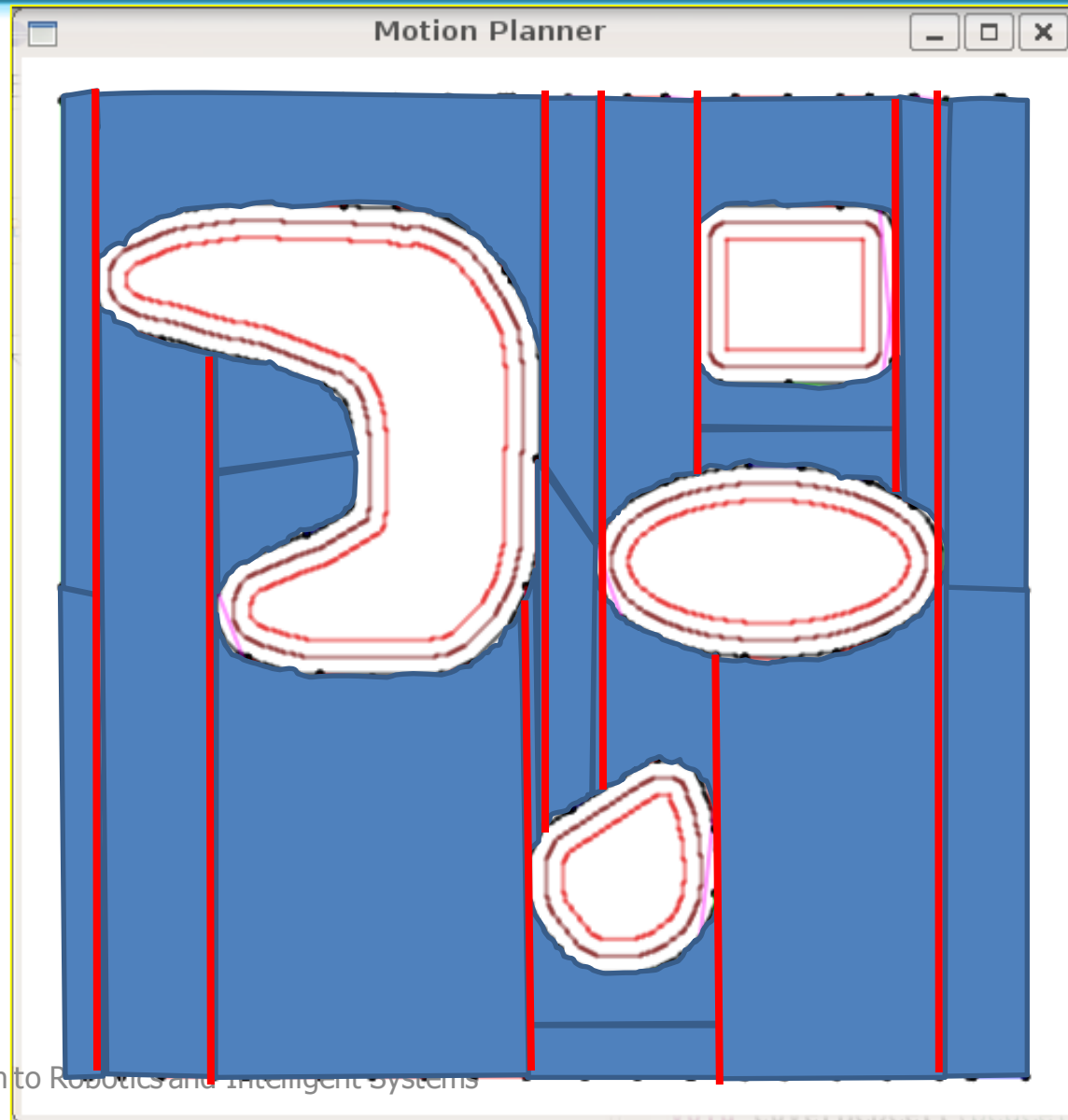




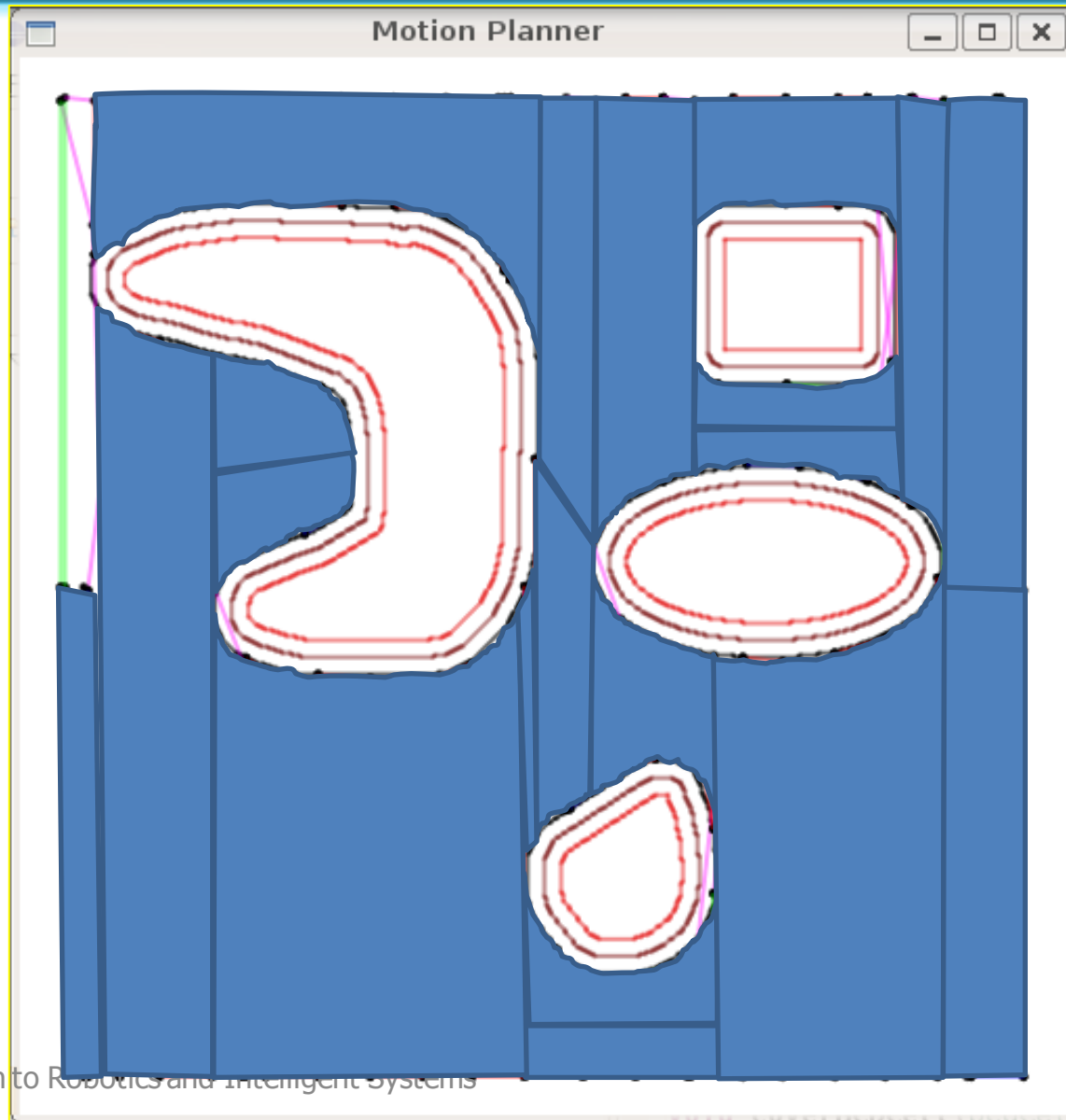
# Example 2



# Example 2 Boustrophedon Decomp.



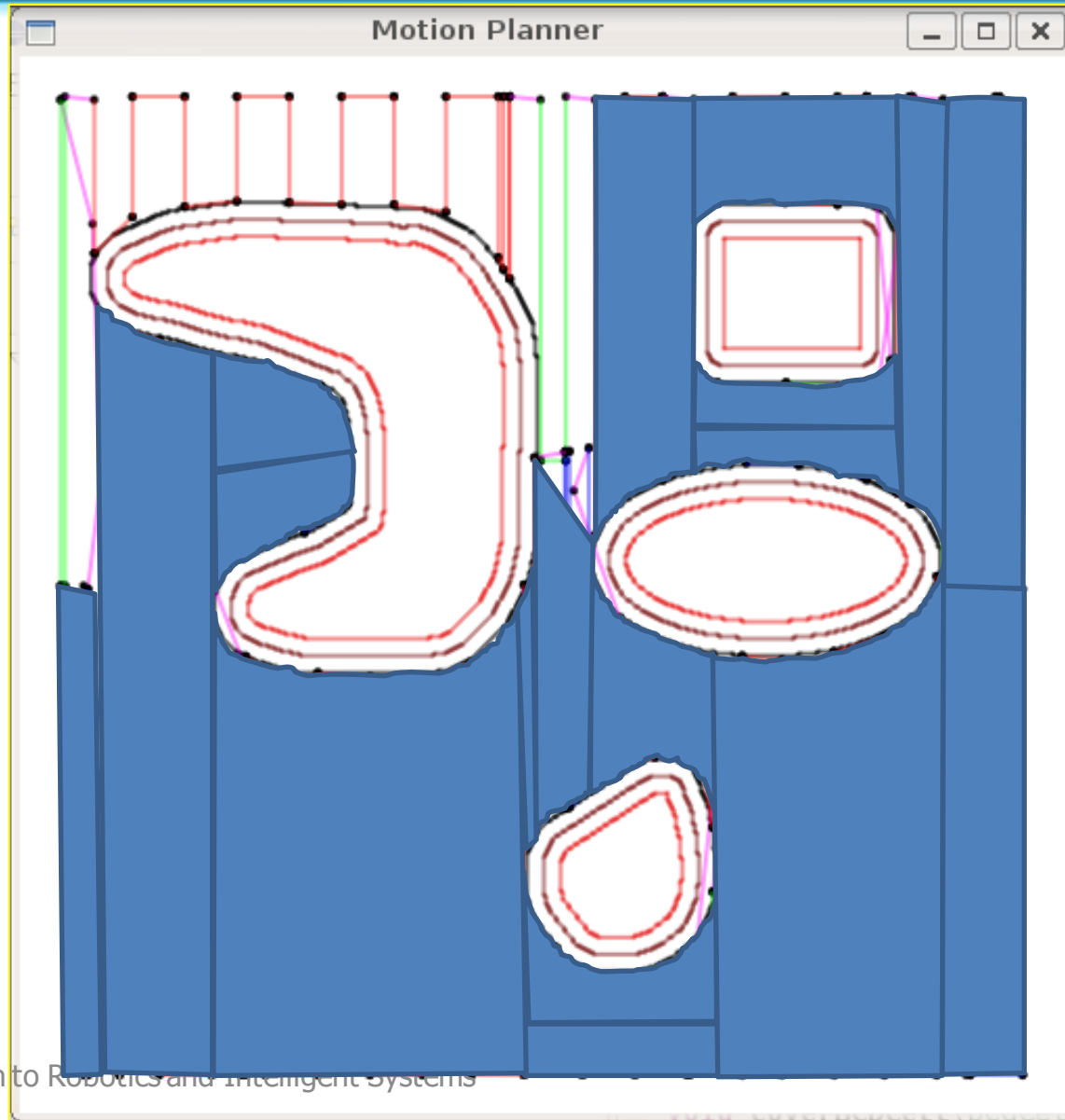
# Example 2



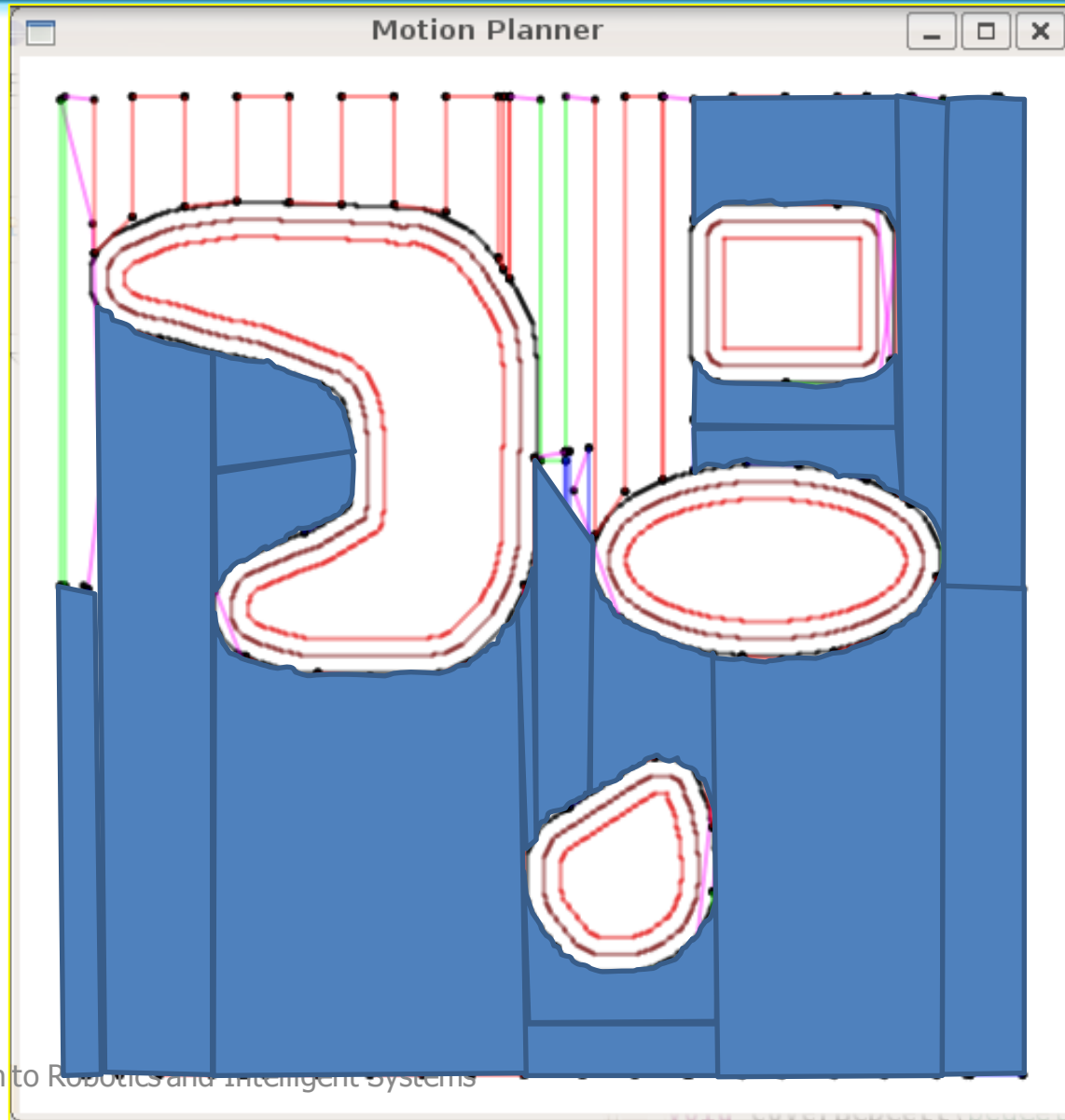




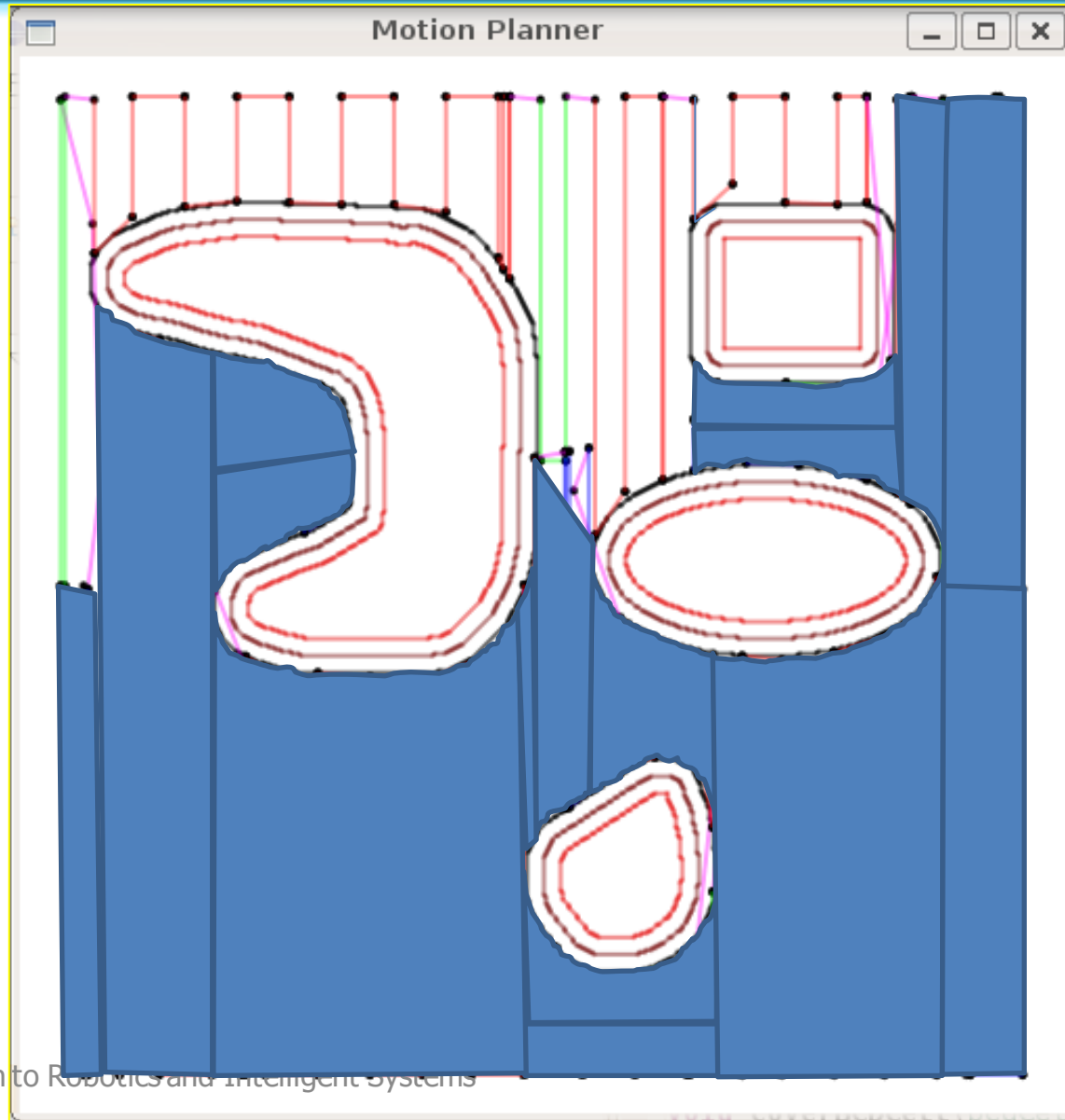
# Example 2



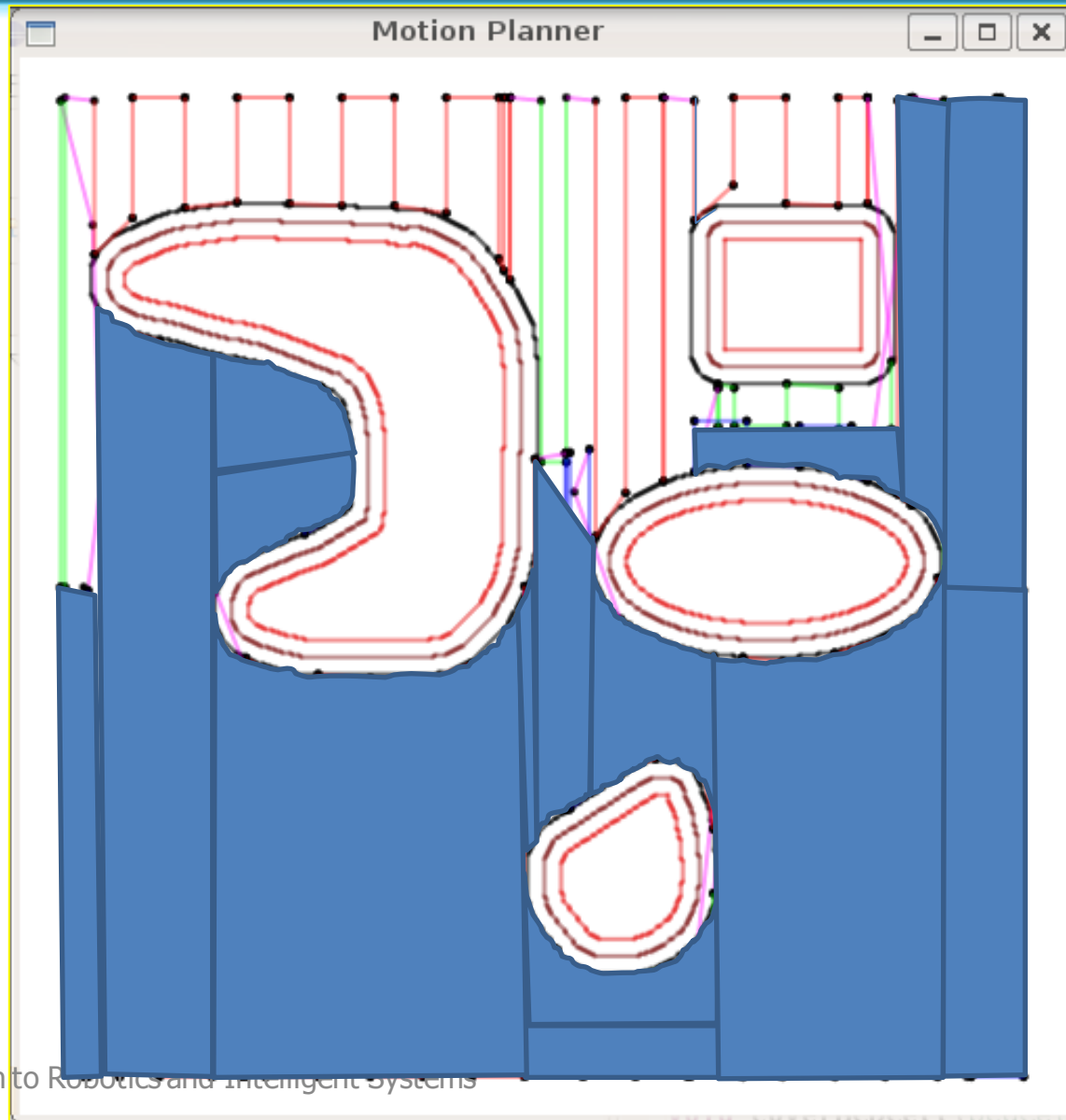
# Example 2



# Example 2

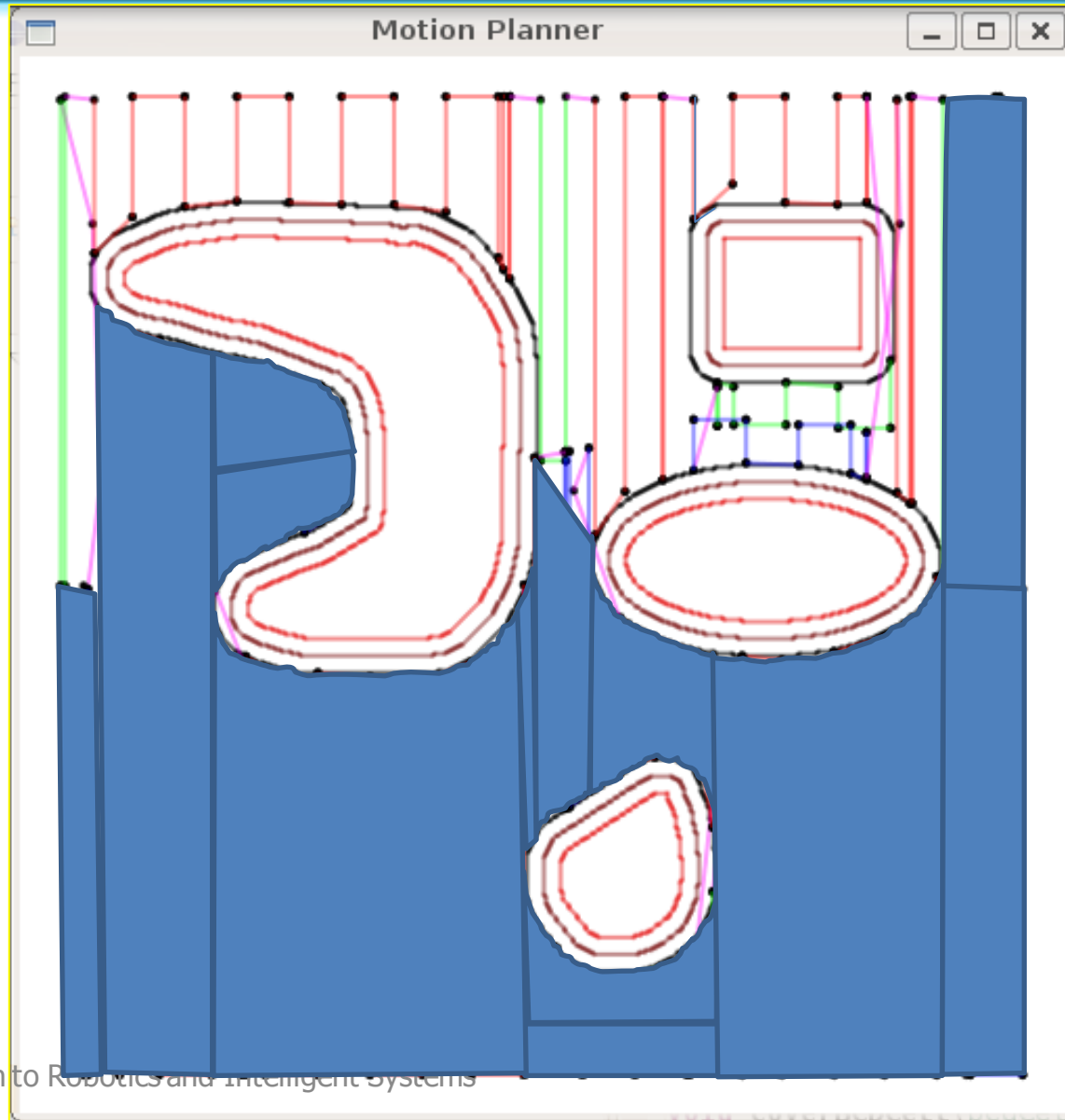


# Example 2

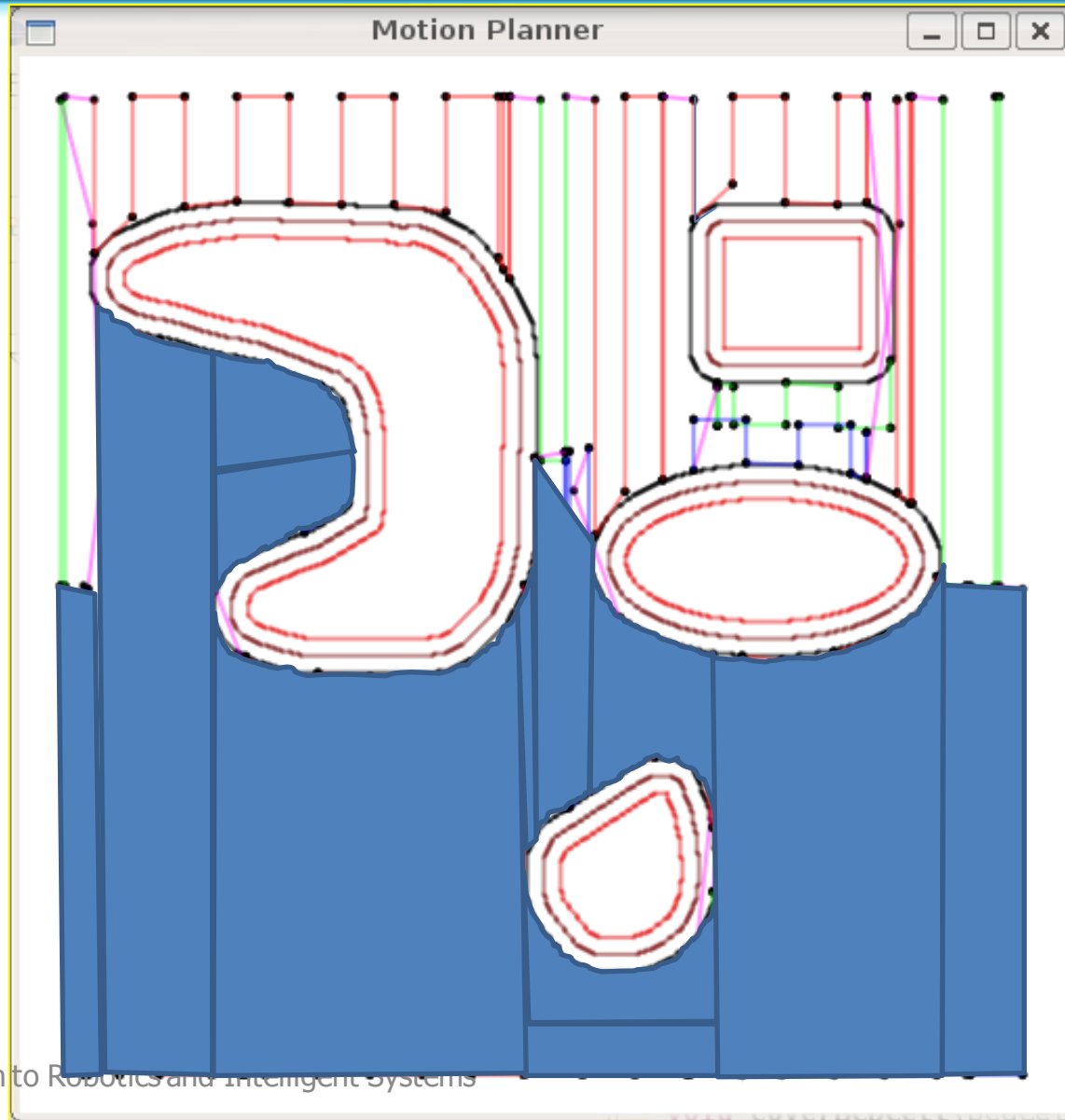




# Example 2



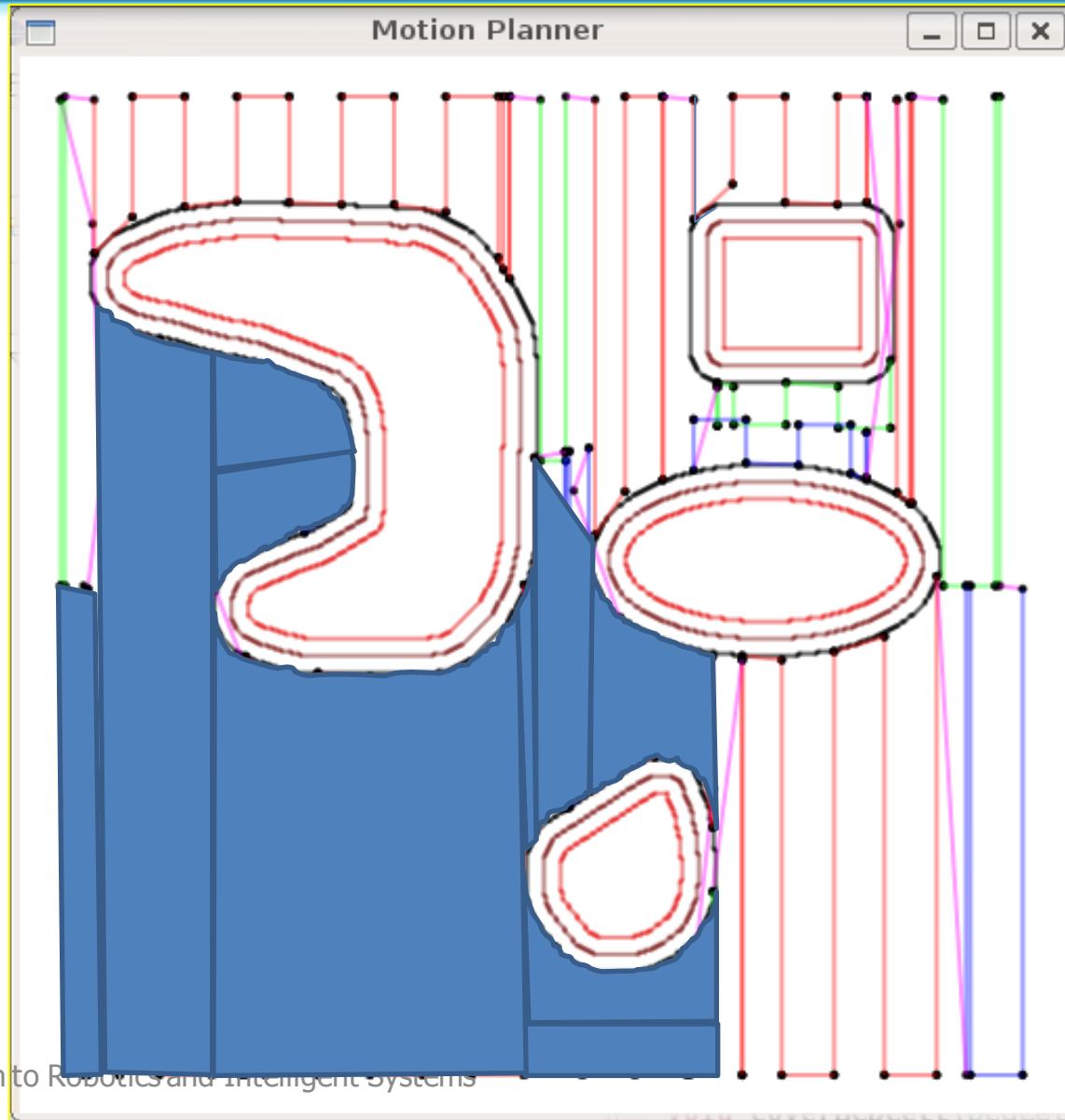
# Example 2



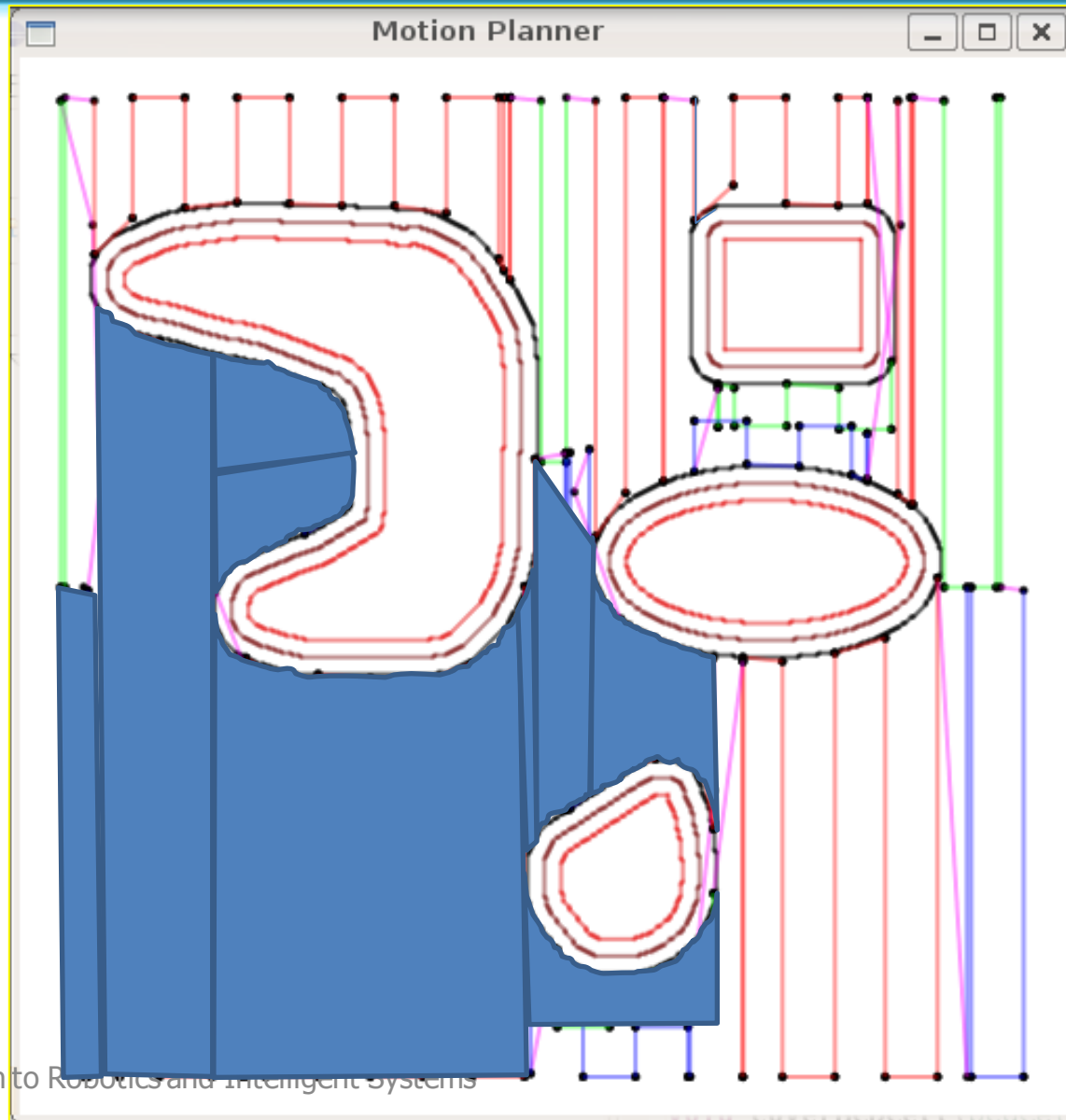




# Example 2

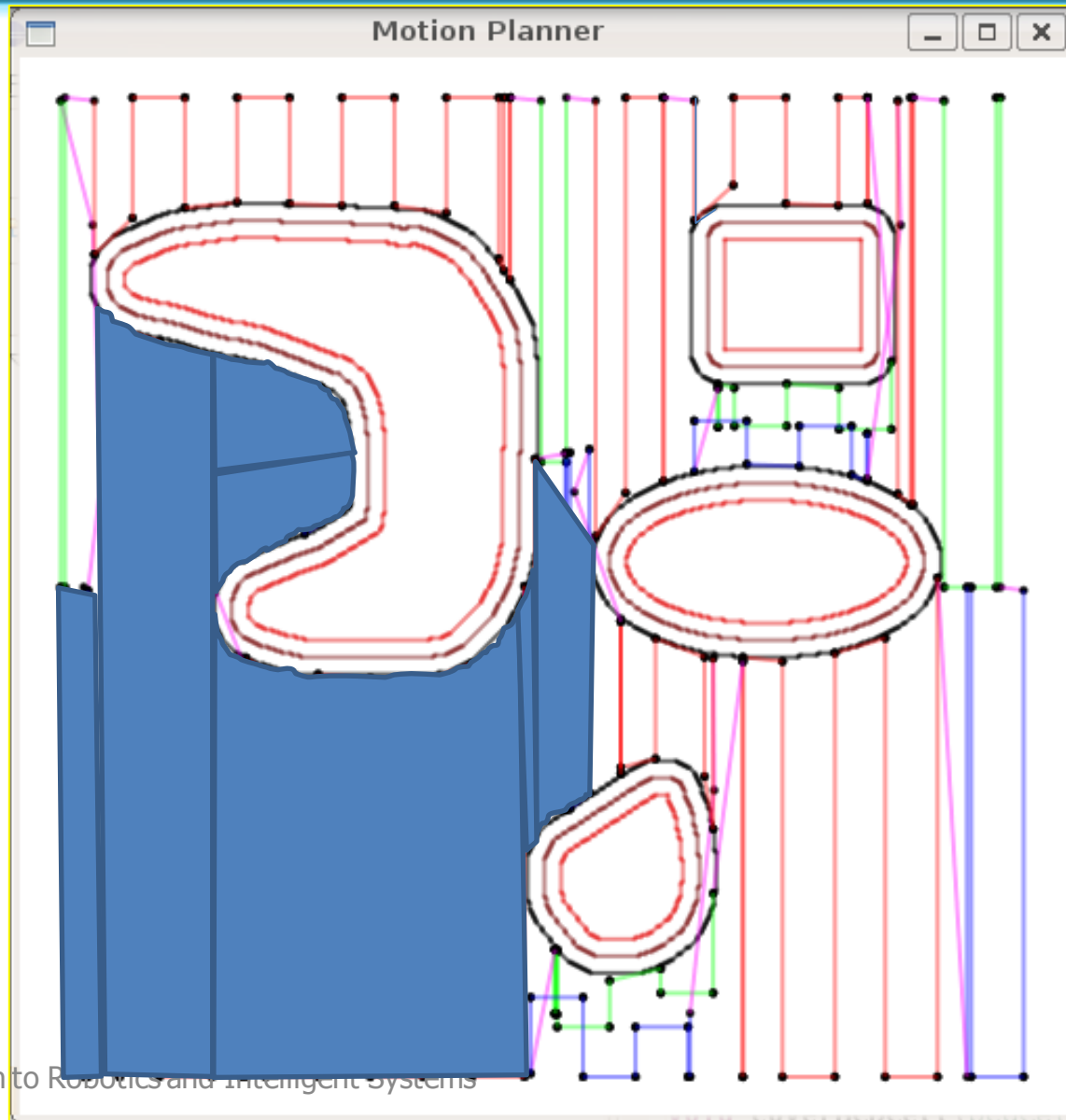


# Example 2





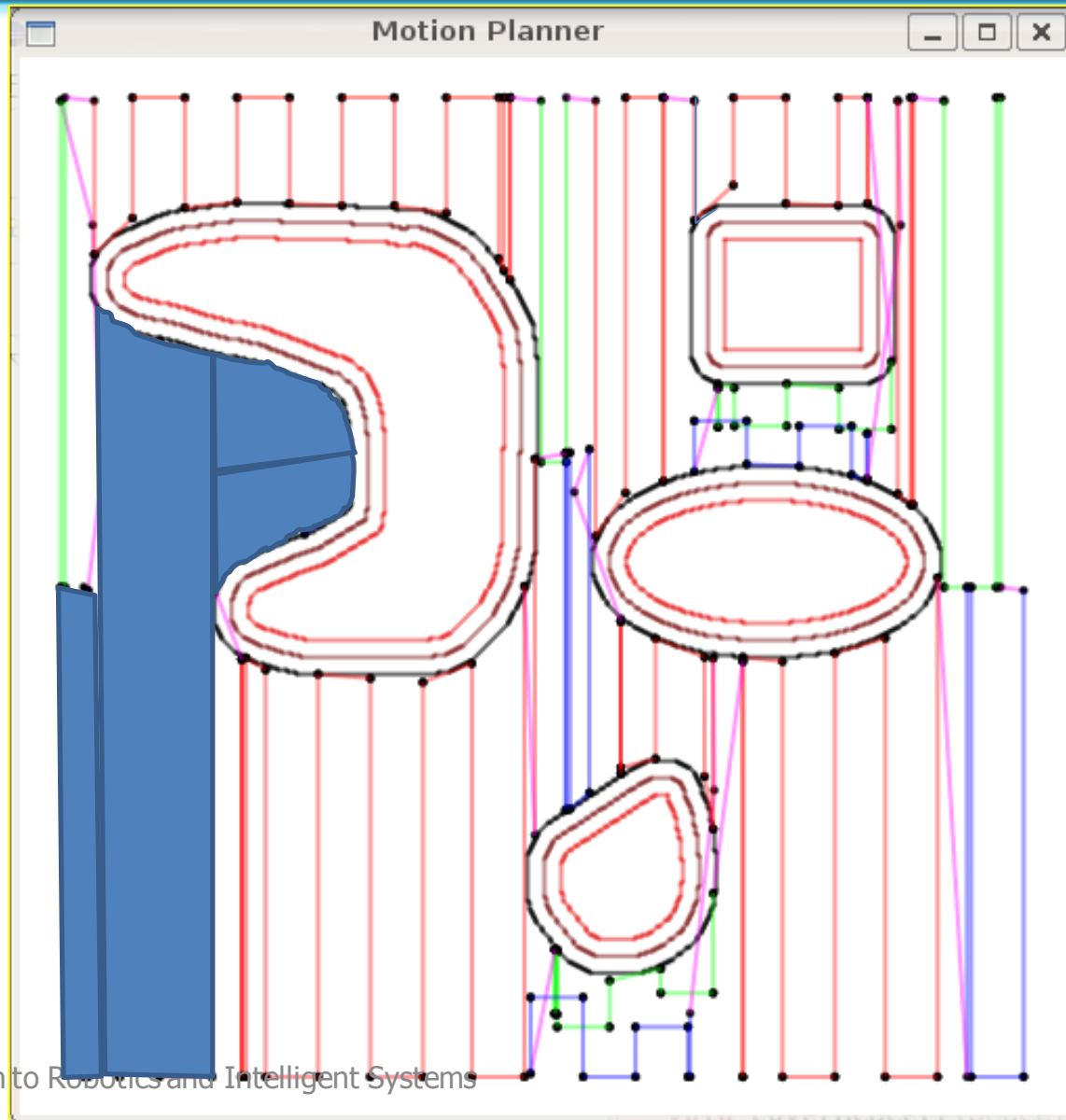
# Example 2







# Example 2

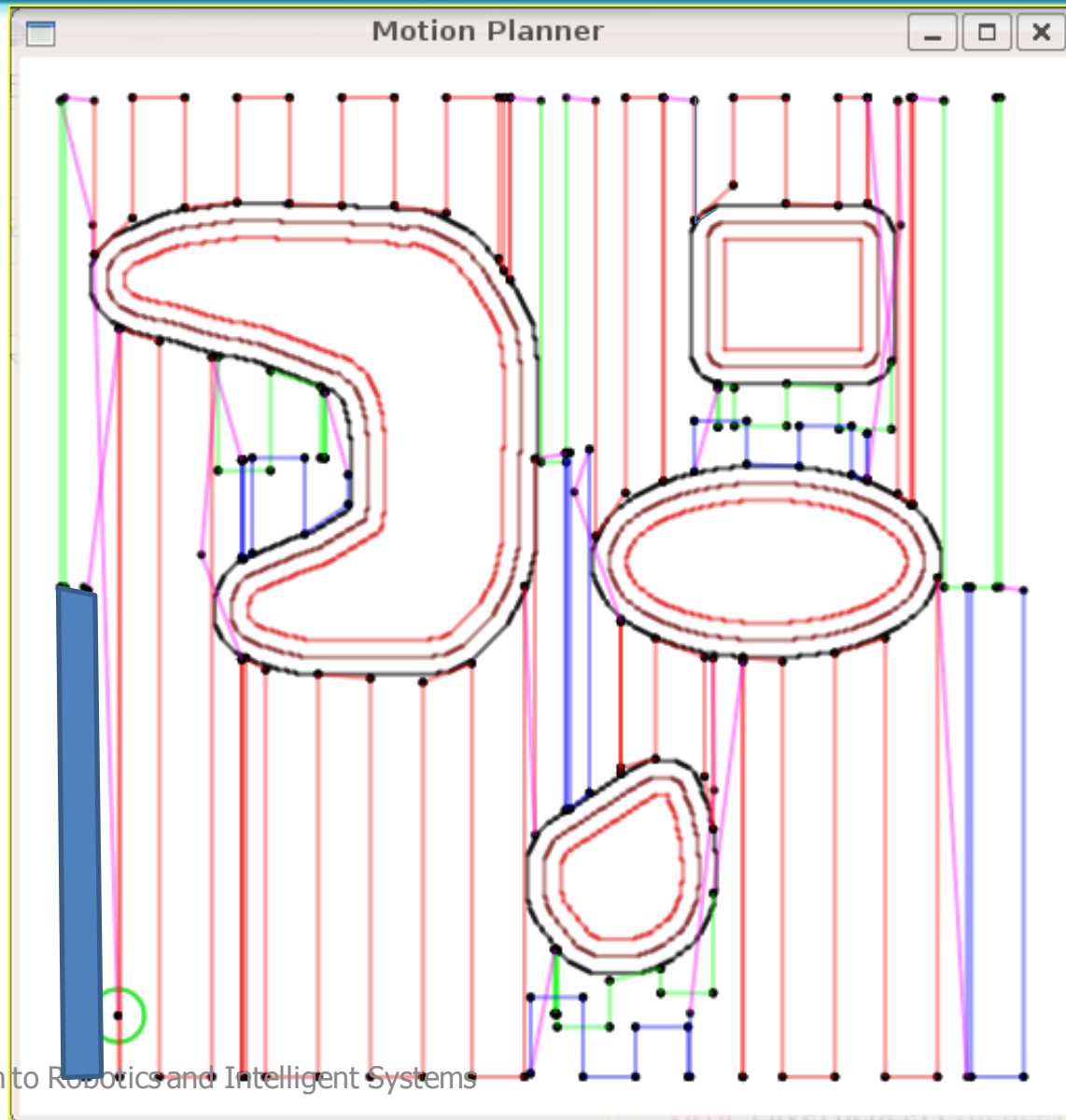








# Example 2





# UAV-Optimal Coverage

---



# UAV-Optimal Coverage

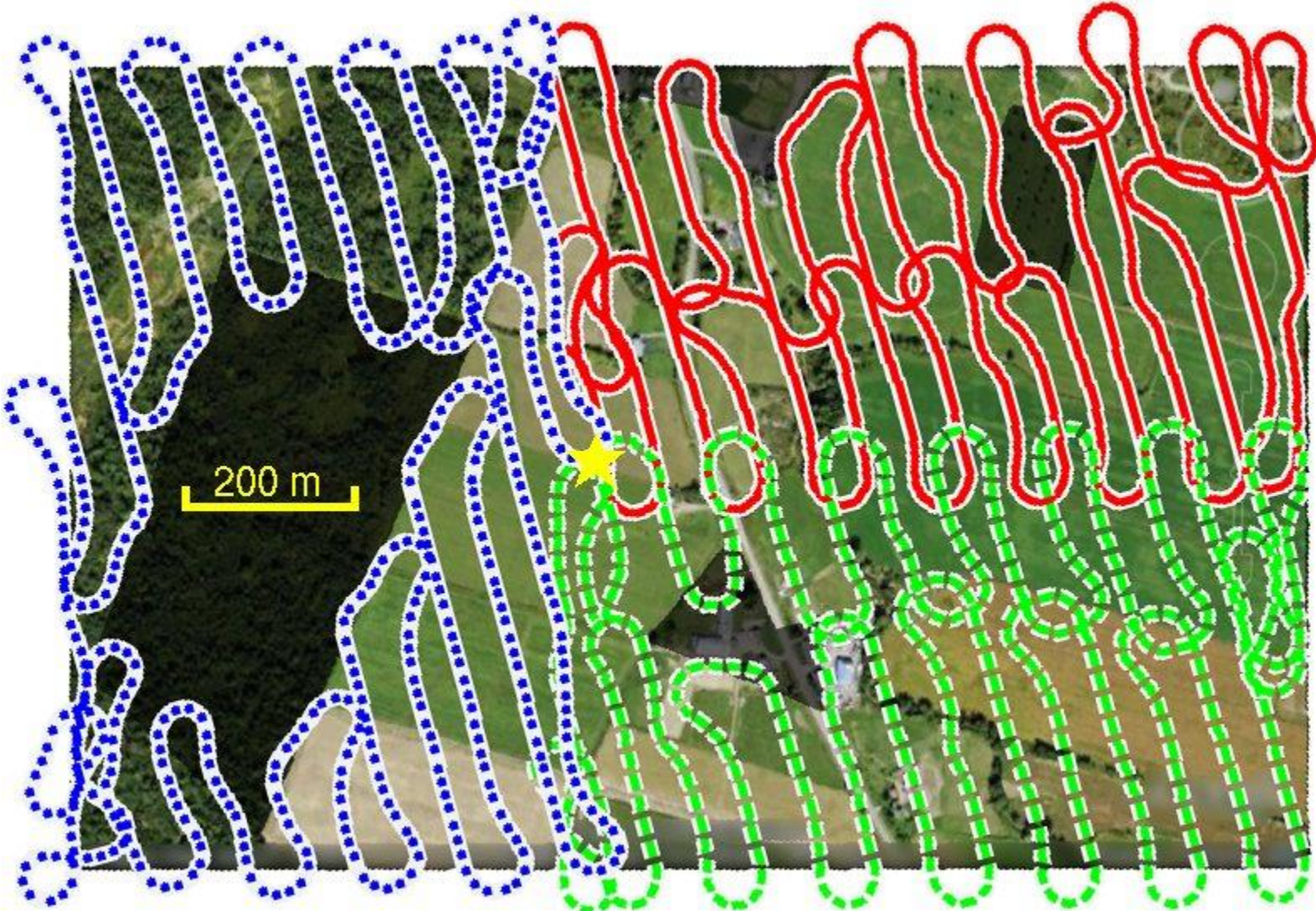
- UAVs non-holonomic constraints require special trajectory planning
- 120 Km of flight during coverage



# Image Mosaic

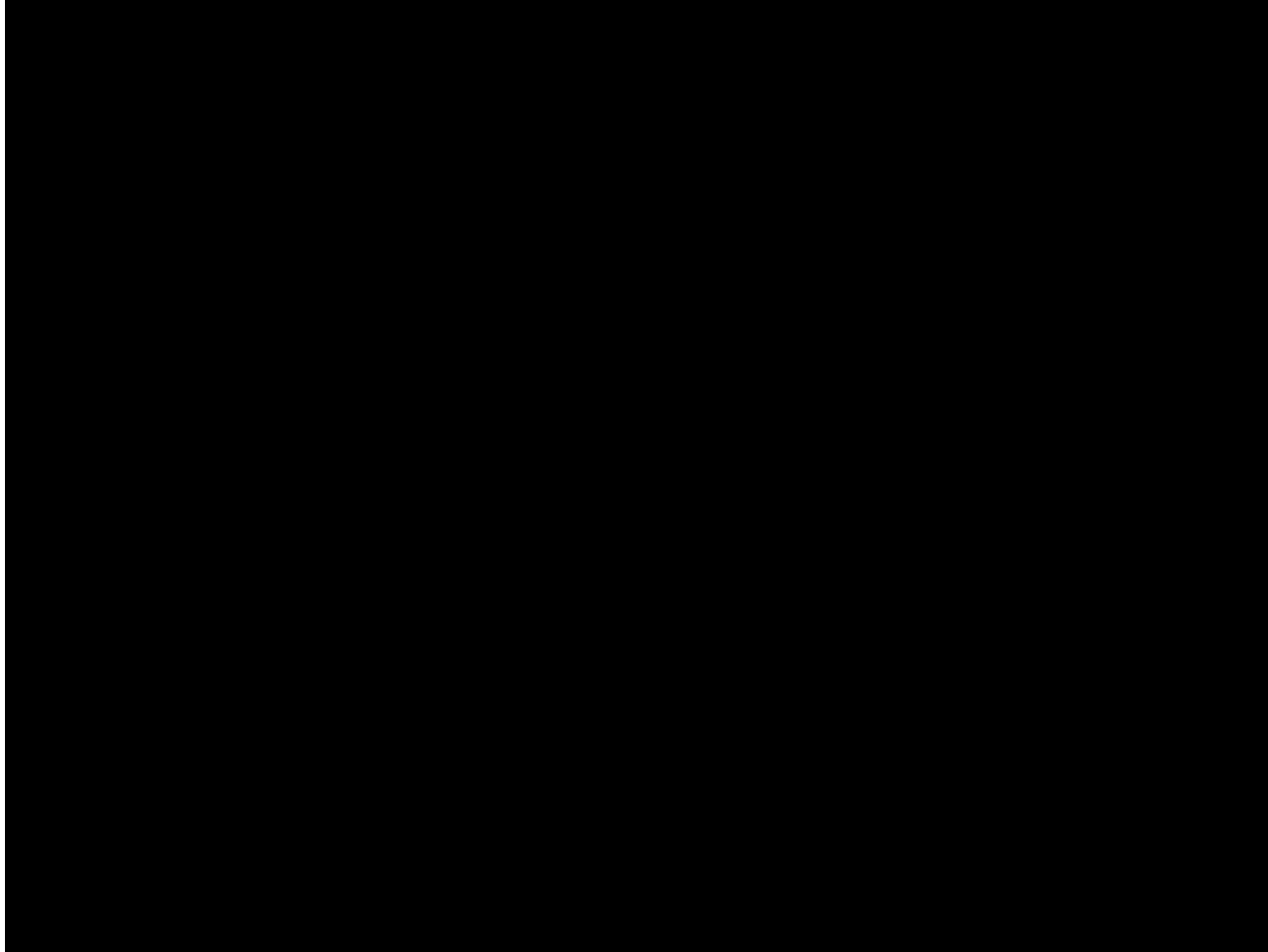


# Multi-UAV



# Just submitted

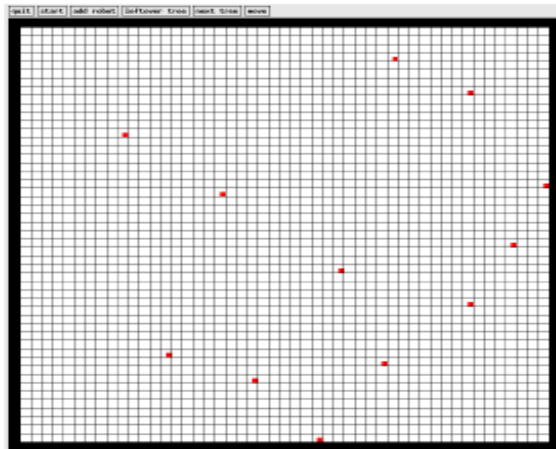
---



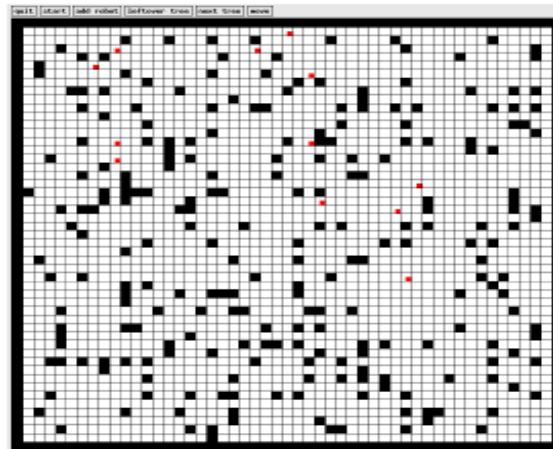


# Coverage of Known Worlds

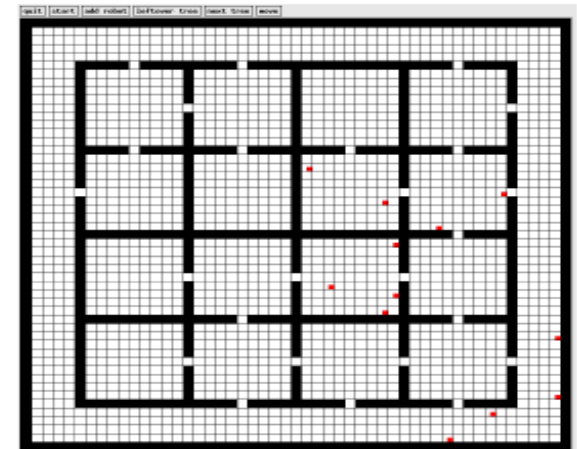
Empty Terrain



Outdoor-Like Terrain



Indoor-Like Terrain

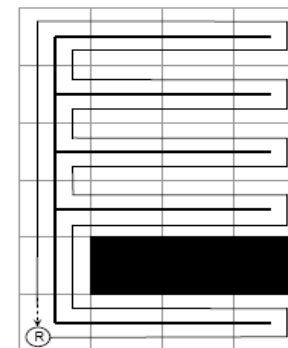


From: X. Zheng and S. Koenig. Robot Coverage of Terrain with Non-Uniform Traversability. In Proc. of the IEEE Int. Conf. on Intelligent Robots and Systems (IROS), pg. 3757-3764, 2007

STC

cover time = 682

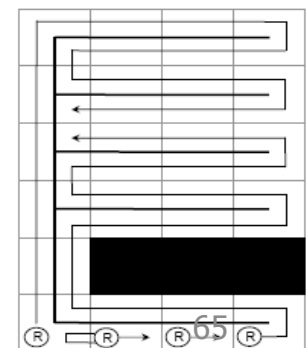
cover and return time = 688



MSTC

cover time = 332

cover and return time = 394



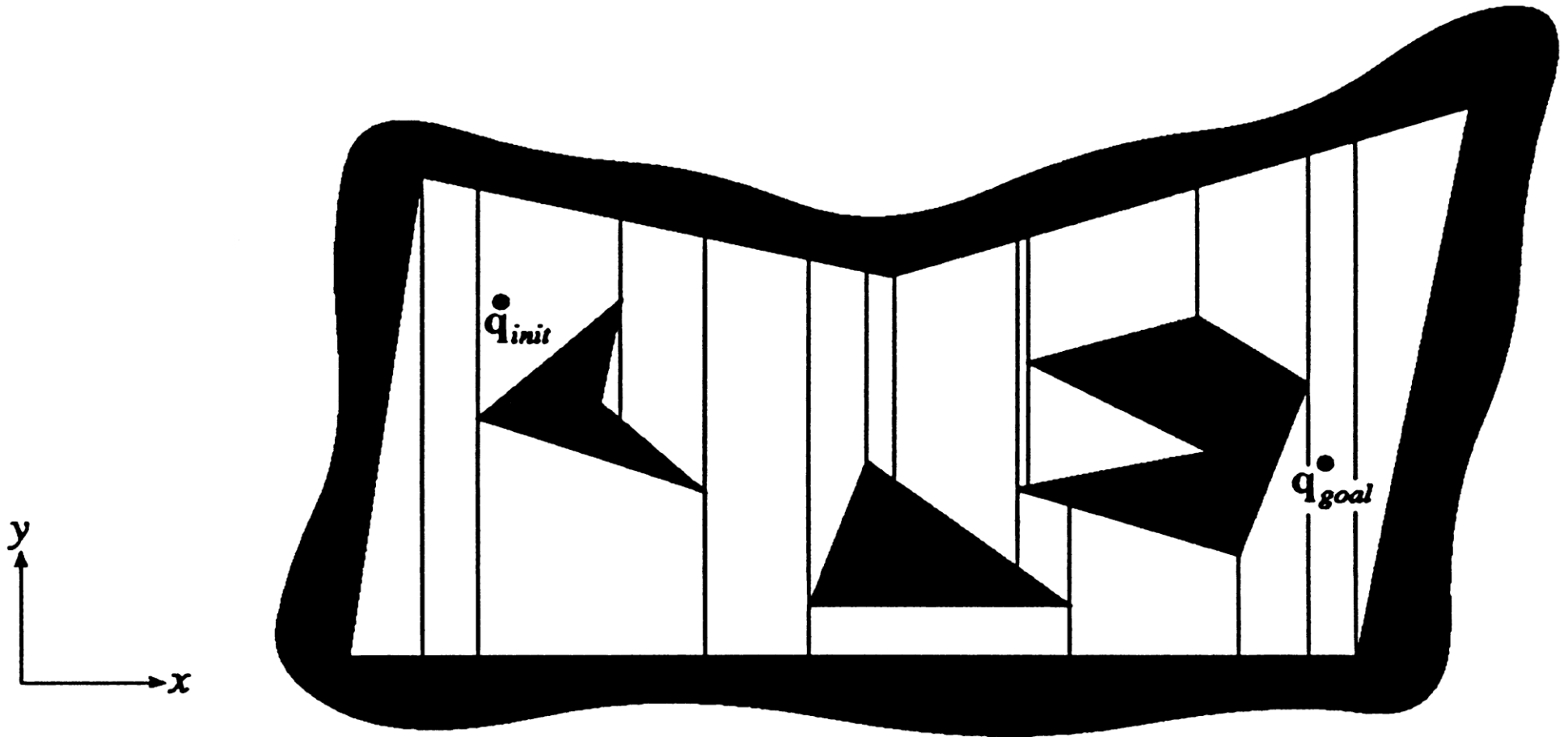
# Cell decomposition for Path Planning

---

- Decompose the free space into **simple** cells and represent the connectivity of the free space by the adjacency graph of these cells



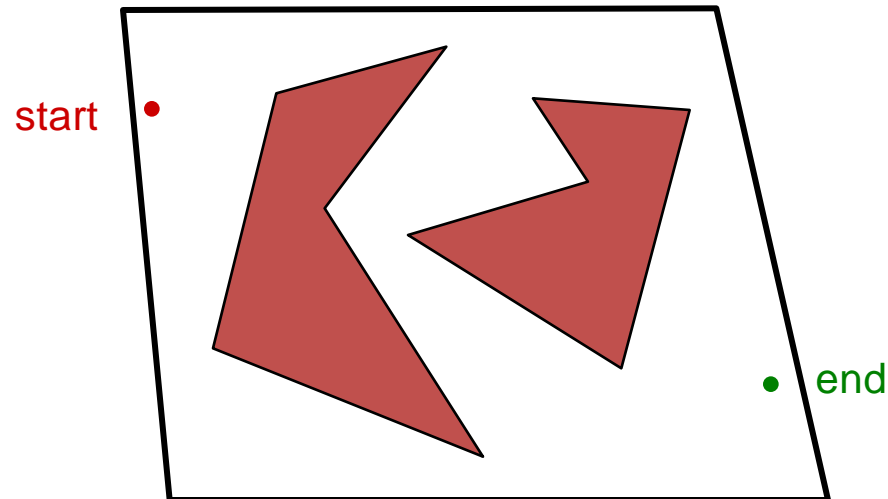
# Trapezoidal decomposition



# Spatial decompositions

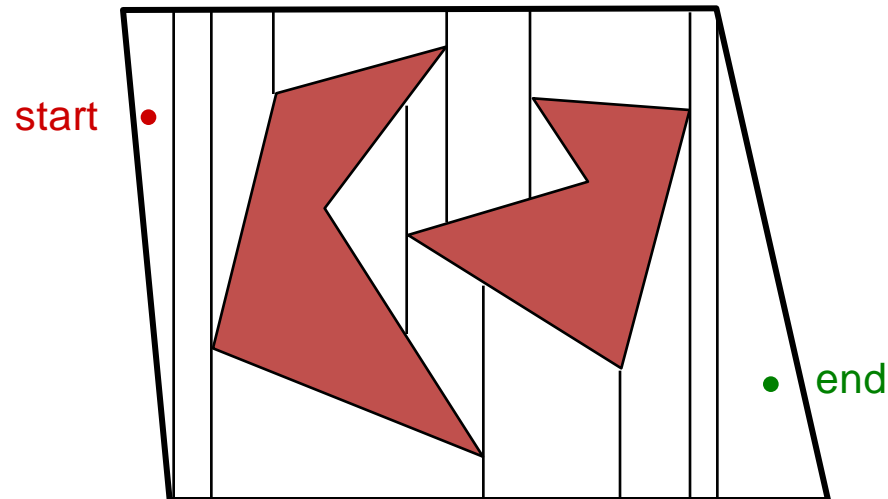
---

- Dividing free space into pieces and using those...



# Spatial decompositions

- Dividing free space into pieces and using those...



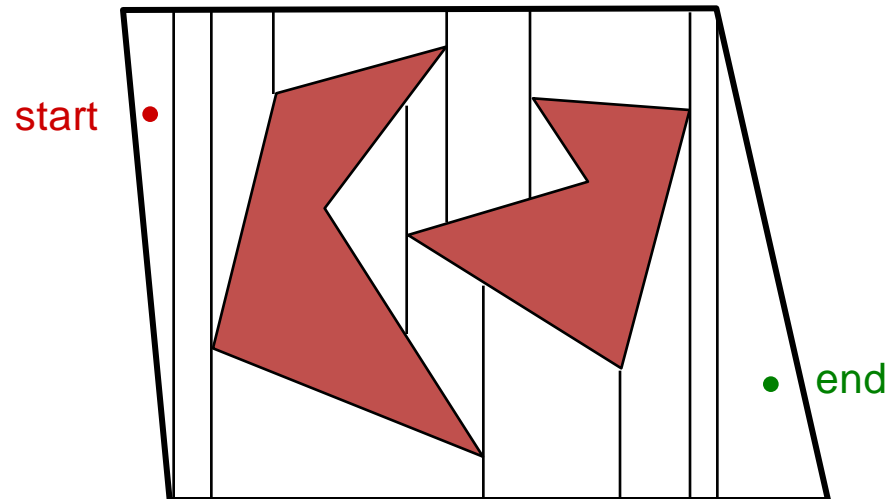
Exact cell decomposition  
sweepline algorithm

Running time?



# Spatial decompositions

- Dividing free space into pieces and using those...



Exact cell decomposition  
sweepline algorithm

Running time?

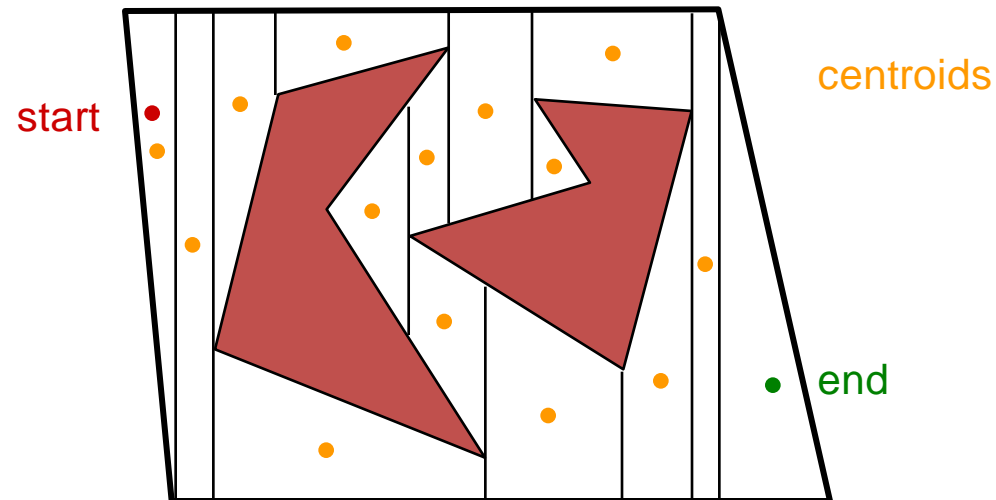
Path?

$O(N \log(N))$



# Spatial decompositions

- Dividing free space into pieces and using those...



Exact cell decomposition  
sweepline algorithm

Running time?

$O(N \log(N))$

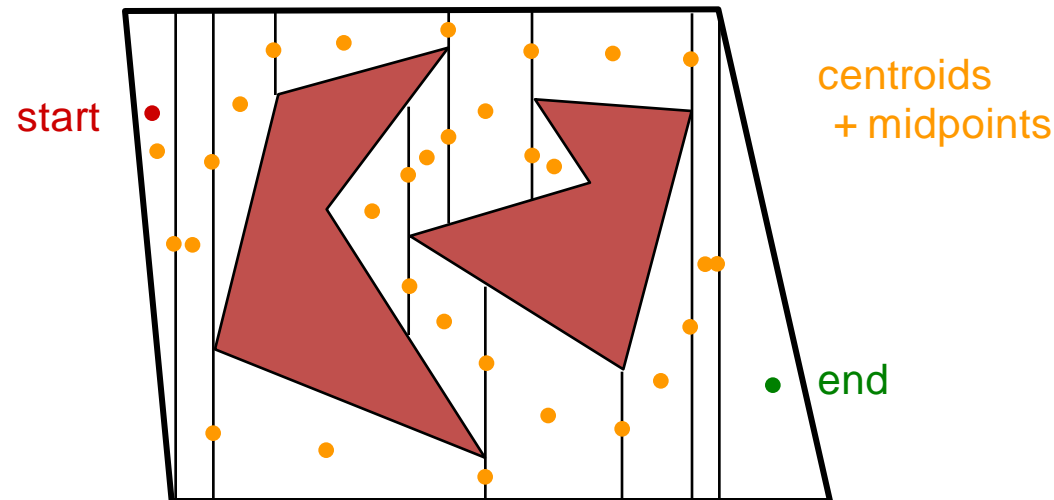
Path?

via centroids



# Spatial decompositions

- Dividing free space into pieces and using those...



Exact cell decomposition  
sweepline algorithm

Running time?

$O(N \log(N))$

Path?

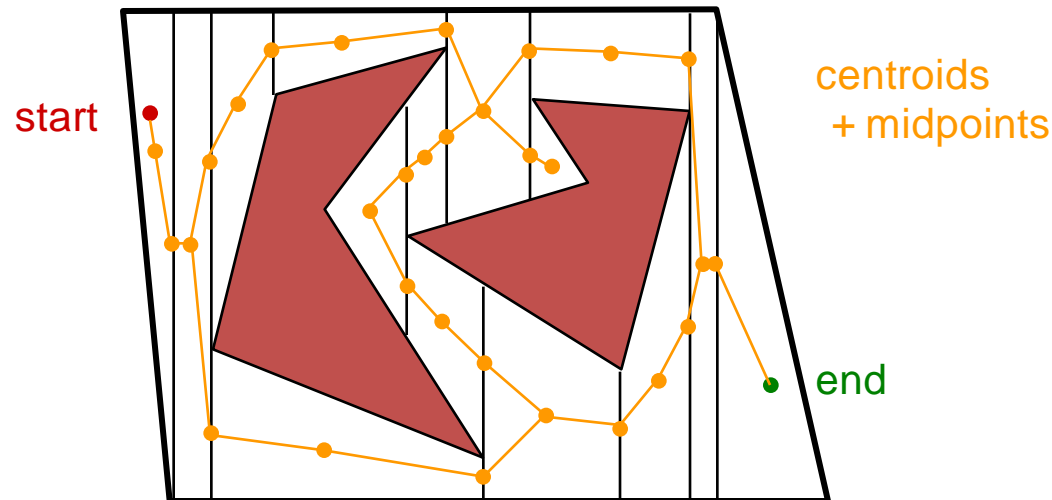
via centroids  
+ edge midpoints





# Spatial decompositions

- Dividing free space into pieces and using those...



Exact cell decomposition  
sweepline algorithm

Running time?

$O(N \log(N))$

Path?

via centroids  
+ edge midpoints  
+ graph search

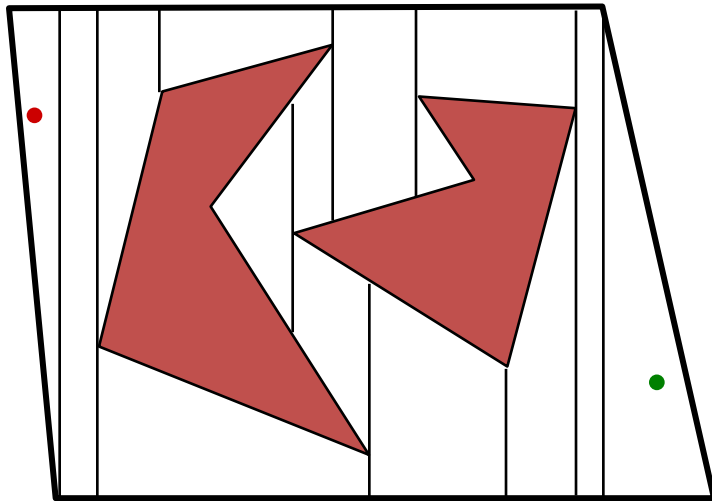
Why?

why else?

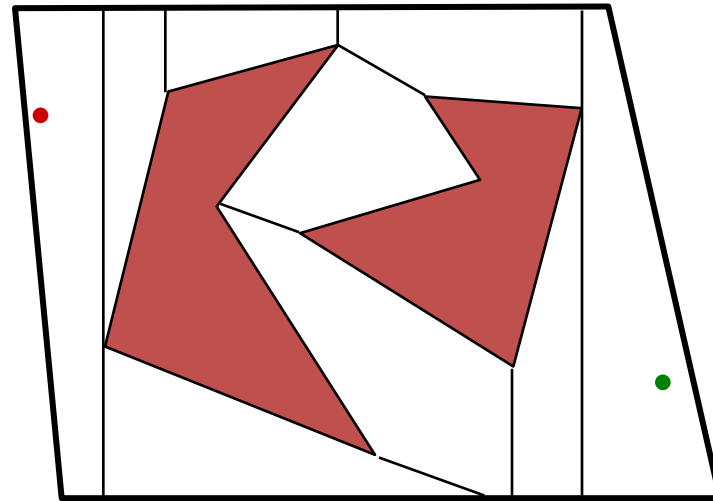


# Optimality

- Obtaining the *minimum* number of convex cells is NP-complete.



15 cells



9 cells

Trapezoidal decomposition is exact and complete, but not optimal -- even among convex subdivisions.

# Cell-Decomposition Methods

---

Two families of methods:

- Exact cell decomposition
- Approximate cell decomposition

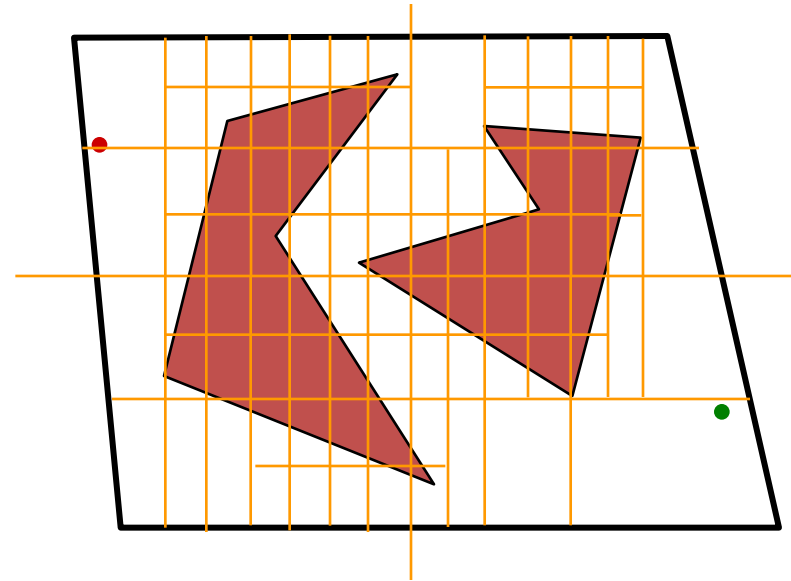
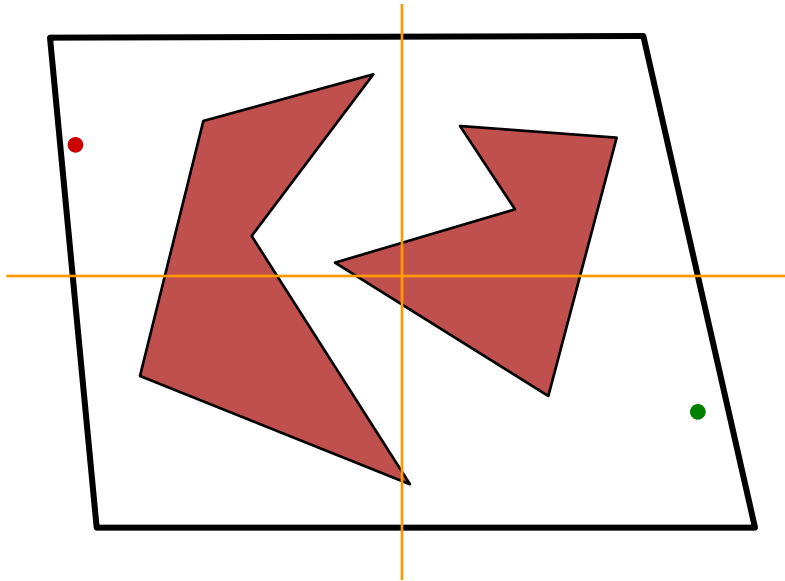
F is represented by a collection of non-overlapping cells whose union is contained in F

Examples: quadtree, octree,  $2^n$ -tree



# further decomposing...

- Approximate cell decomposition



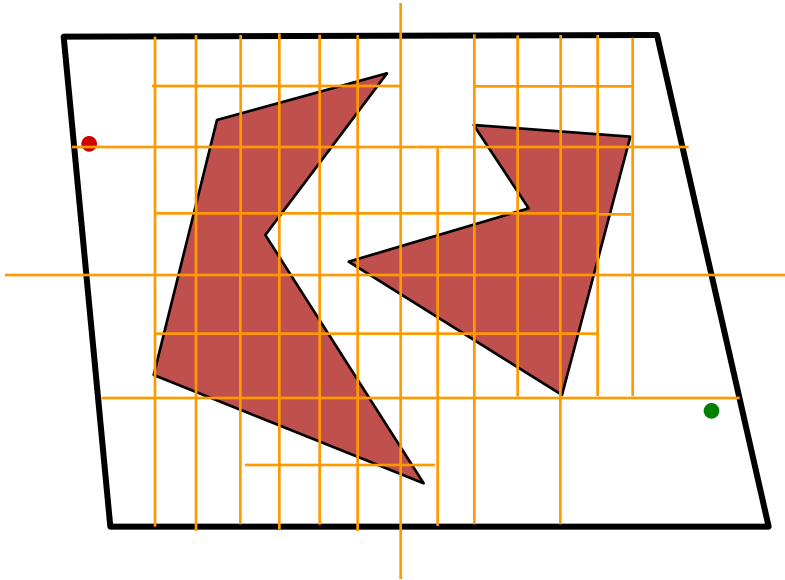
Quadtrees:

recursively subdivides each *mixed* obstacle/free (sub)region into four quarters...

# further decomposing...

---

- Approximate cell decomposition



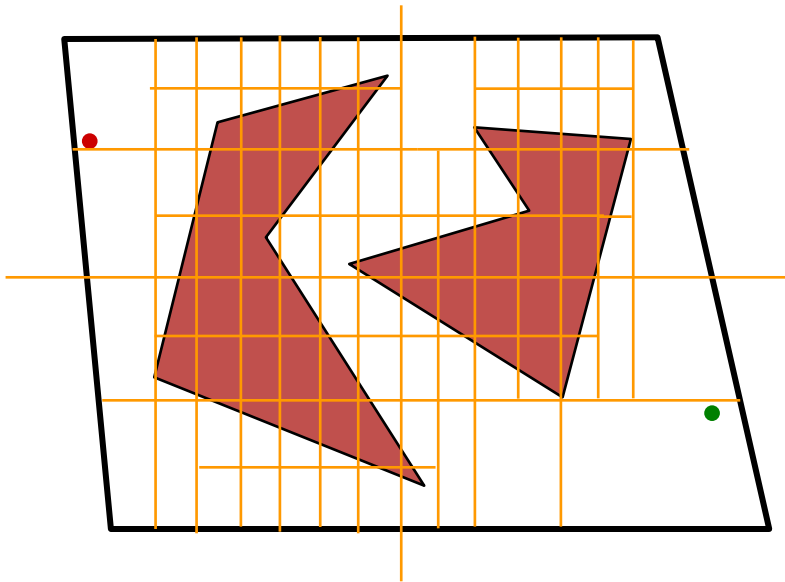
Quadtree:

recursively subdivides each *mixed* obstacle/free (sub)region into four quarters...

# further decomposing...

---

- Approximate cell decomposition

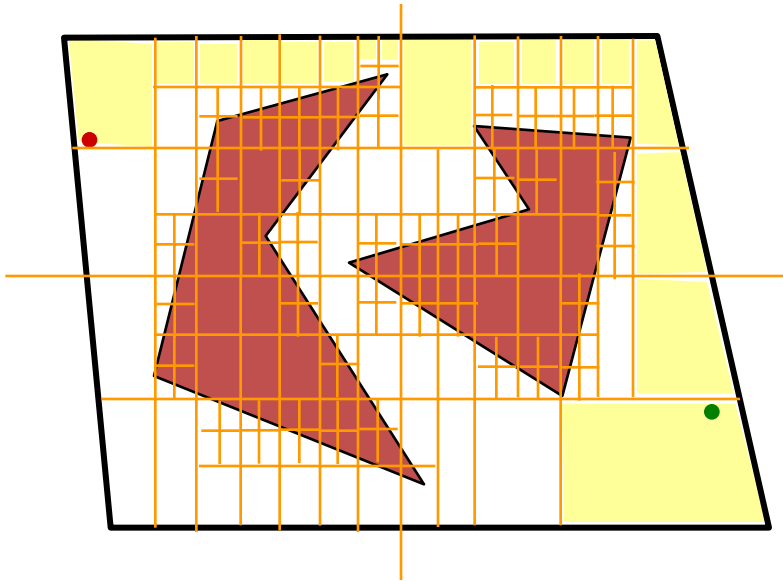


Quadtree:

recursively subdivides each *mixed* obstacle/free (sub)region into four quarters...

# further decomposing...

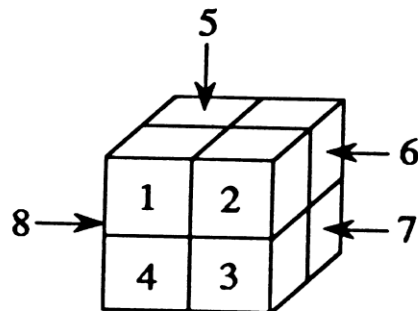
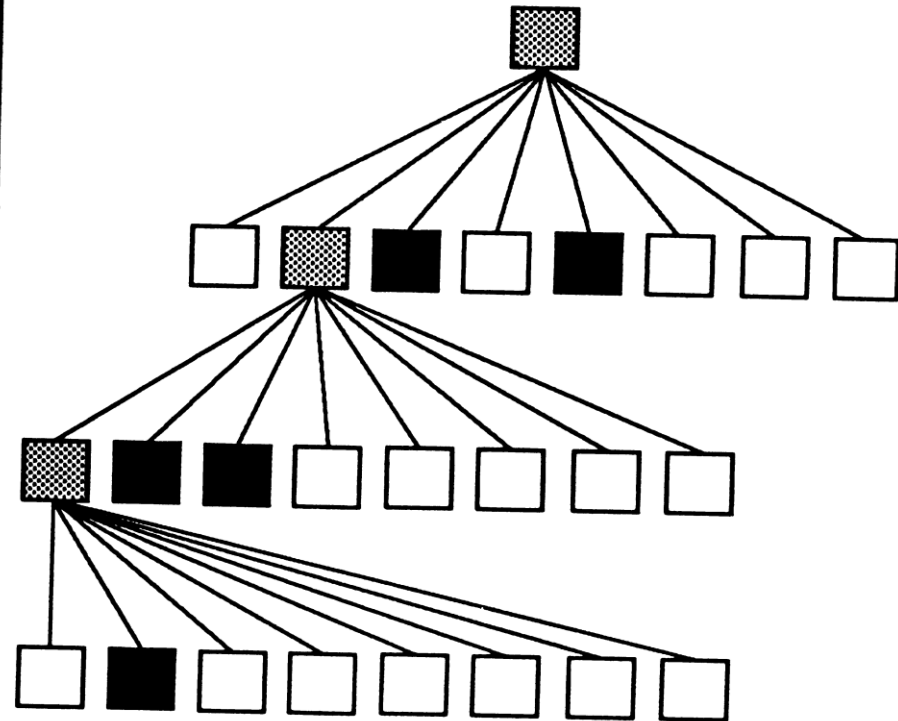
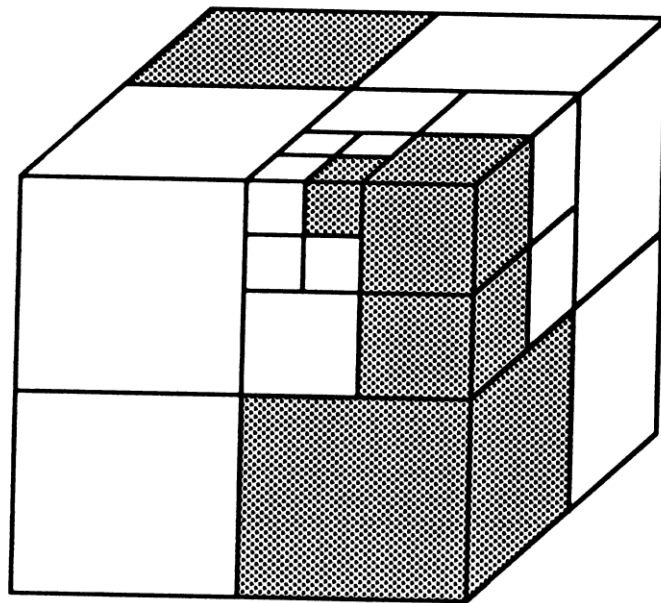
- Approximate cell decomposition



Quadtree

Again, use a graph-search algorithm to find a path from the start to goal

# Octree Decomposition



 EMPTY cell    MIXED cell    FULL cell

