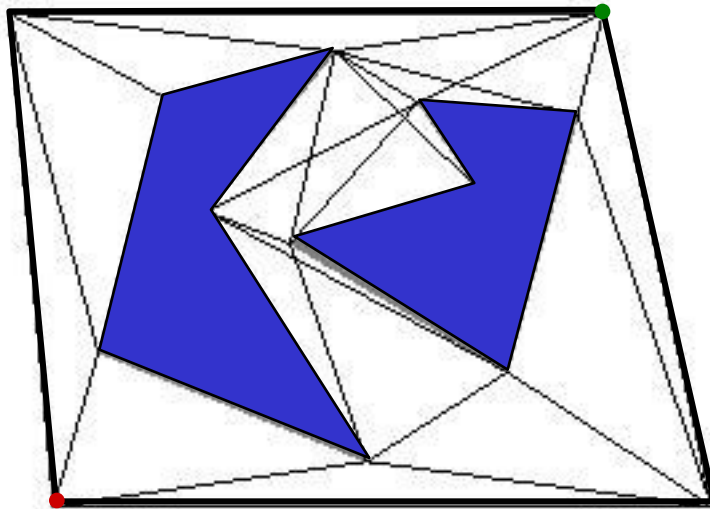
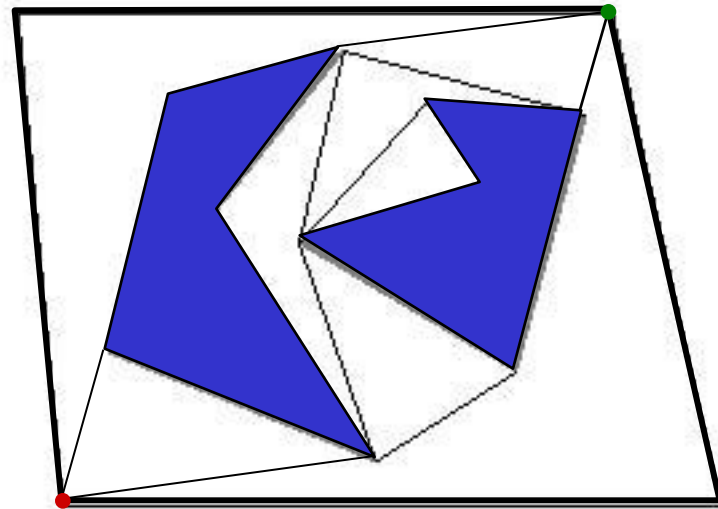


# Roadmaps

## Vertex Visibility Graph



Full visibility graph

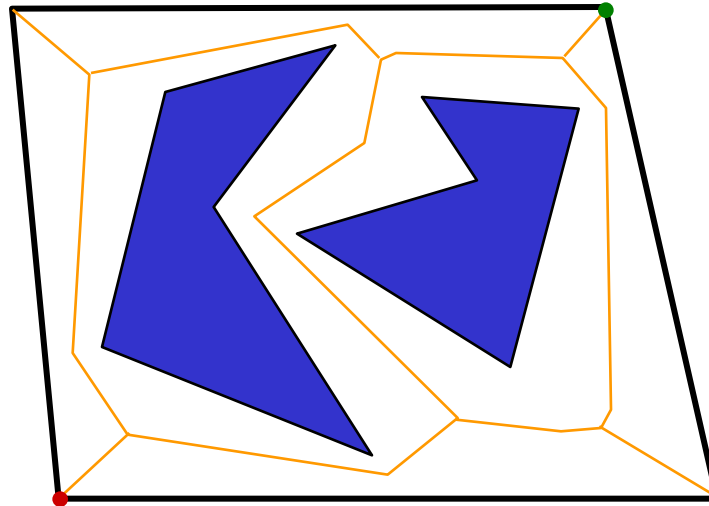


Reduced visibility graph, i.e., not including segments that extend into obstacles on either side.

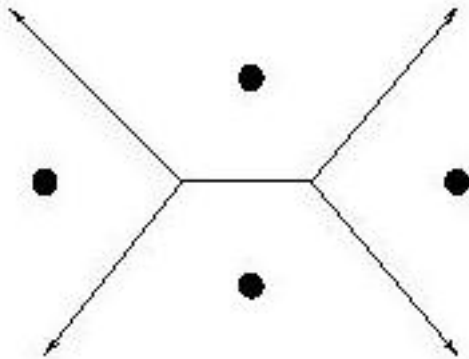
(but keeping endpoints' roads)

what else might we do ?

# An alternative roadmap



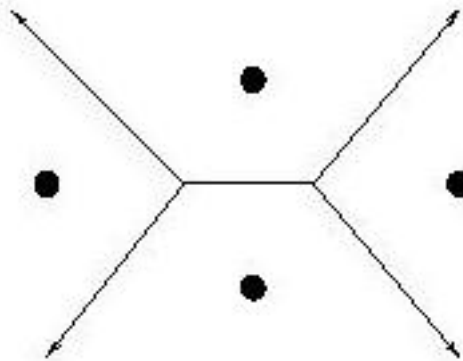
# Voronoi diagrams



These line segments make up the **Voronoi diagram** for the four points shown here.

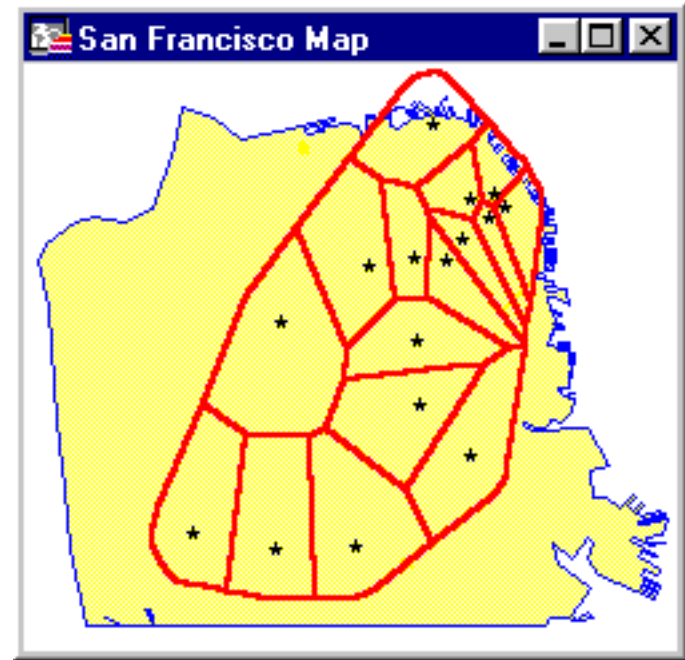
Solves the “Post Office Problem”

# Voronoi diagrams



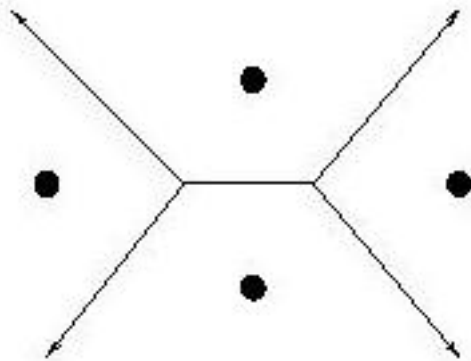
These line segments make up the **Voronoi diagram** for the four points shown here.

Solves the “Post Office Problem”

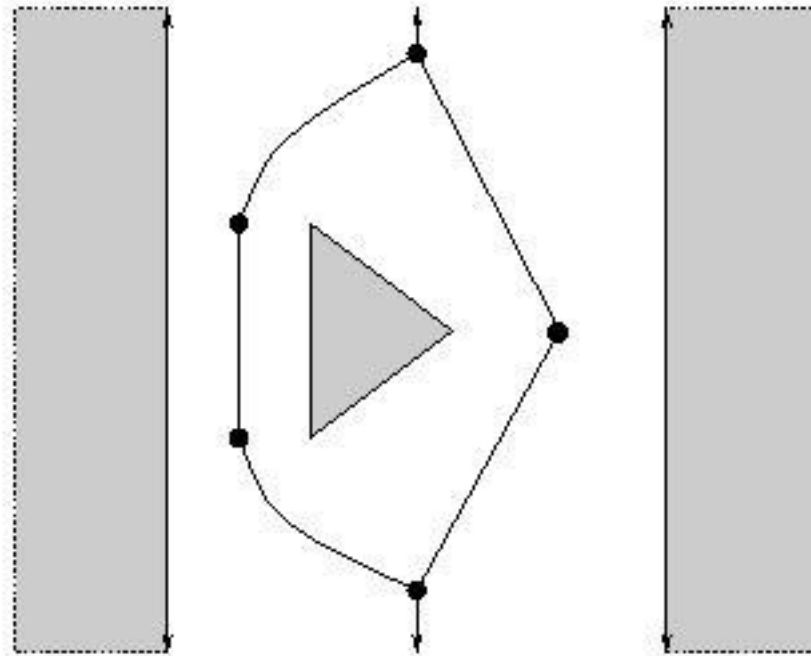


or, perhaps, more important problems...

# Voronoi diagrams



“true” Voronoi diagram  
(isolates a set of points)



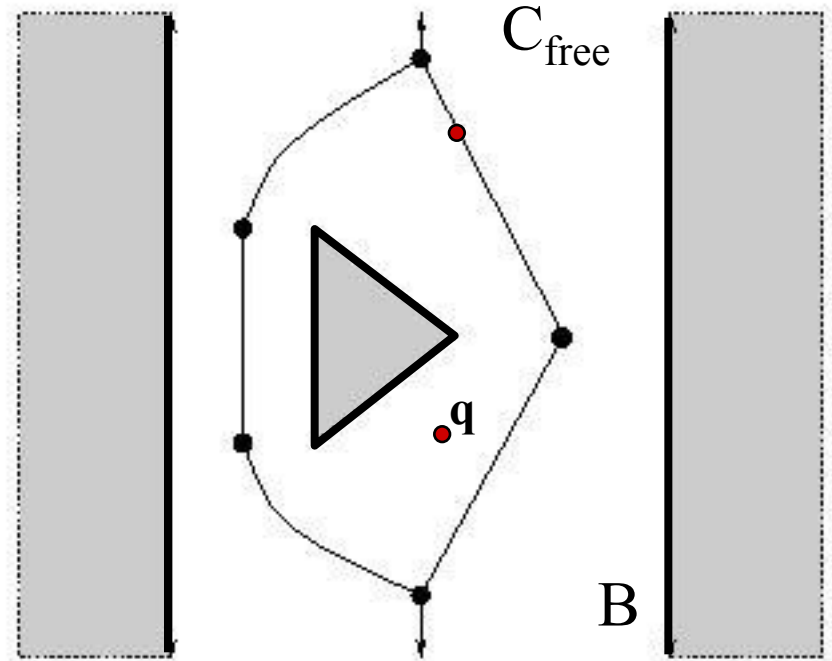
generalized Voronoi diagram

What is it?

# Voronoi diagrams

Let  $B$  = the boundary of  $C_{\text{free}}$  .

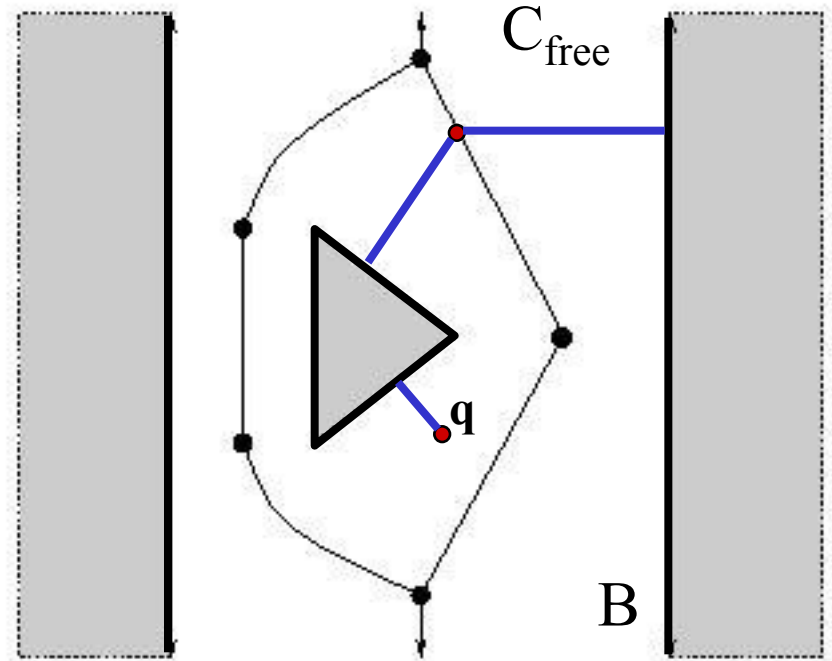
Let  $\mathbf{q}$  be a point in  $C_{\text{free}}$  . (  $\bullet$  )



# Voronoi diagrams

Let  $B$  = the boundary of  $C_{\text{free}}$  .

Let  $q$  be a point in  $C_{\text{free}}$  .

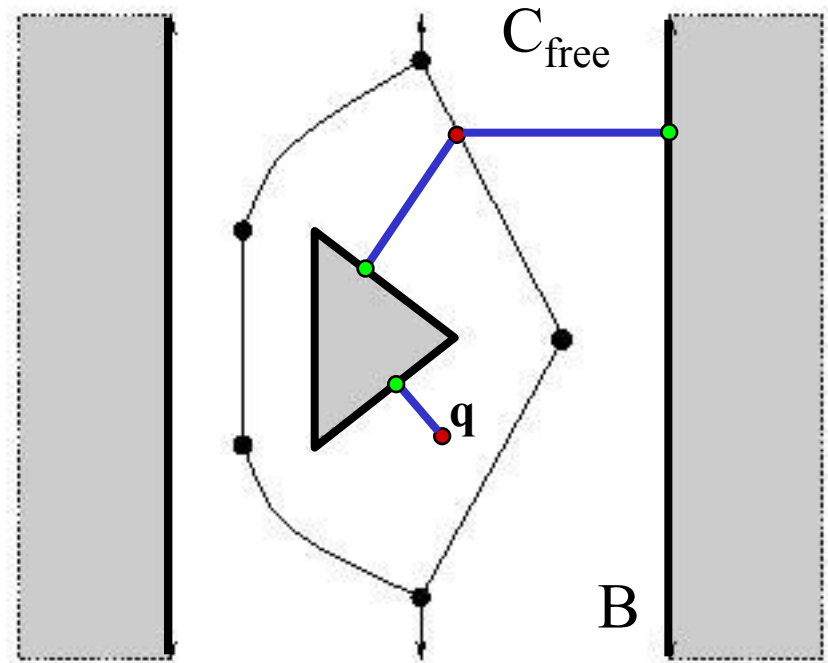


Define *clearance*( $q$ ) =  $\min \{ |q - p| \}$ , for all  $p \in B$

# Voronoi diagrams

Let  $B$  = the boundary of  $C_{\text{free}}$  .

Let  $q$  be a point in  $C_{\text{free}}$  .



Define *clearance*( $q$ ) =  $\min \{ |q - p| \}$ , for all  $p \in B$

Define *near*( $q$ ) =  $\{ p \in B \text{ such that } |q - p| = \textit{clearance}(q) \}$



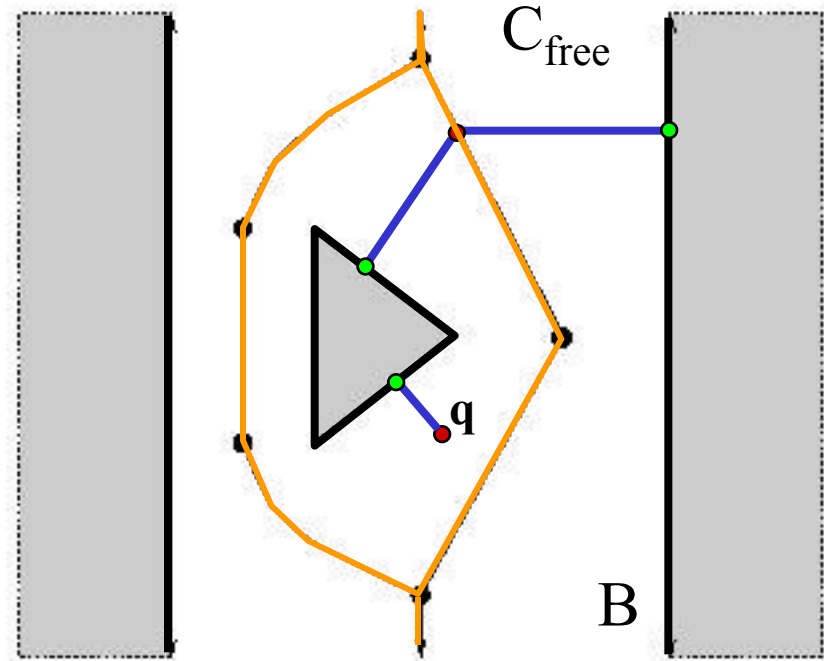
# Voronoi diagrams

## Evaluation

- + maximizes distance from obstacles
- + reduces to graph search
- + can be used in higher-dimensions
- nonoptimal
- real diagrams tend to be noisy

Let  $B$  = the boundary of  $C_{\text{free}}$  .

Let  $q$  be a point in  $C_{\text{free}}$  .



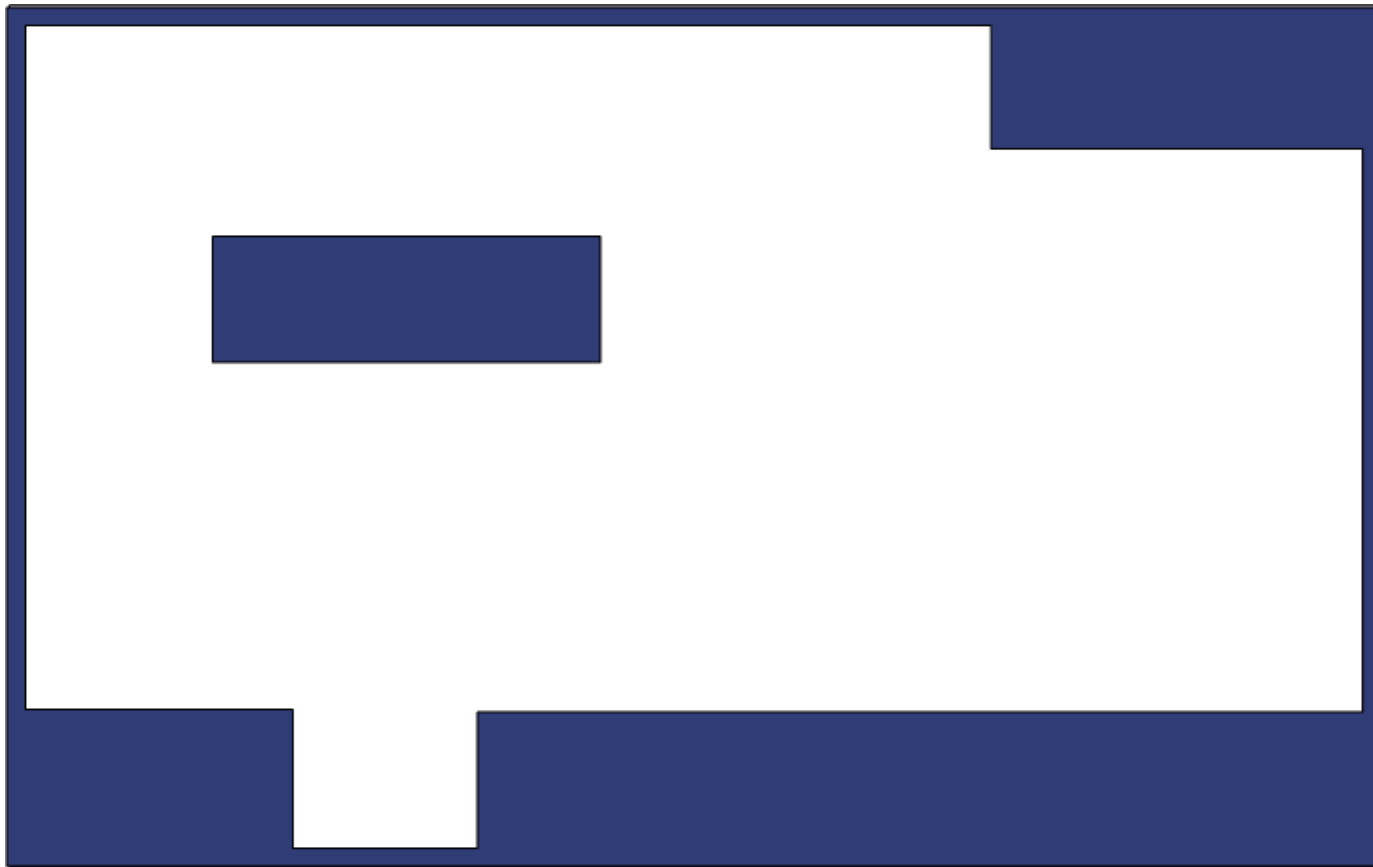
Define *clearance*( $q$ ) =  $\min \{ |q - p| \}$ , for all  $p \in B$

Define *near*( $q$ ) =  $\{ p \in B \text{ such that } |q - p| = \textit{clearance}(q) \}$

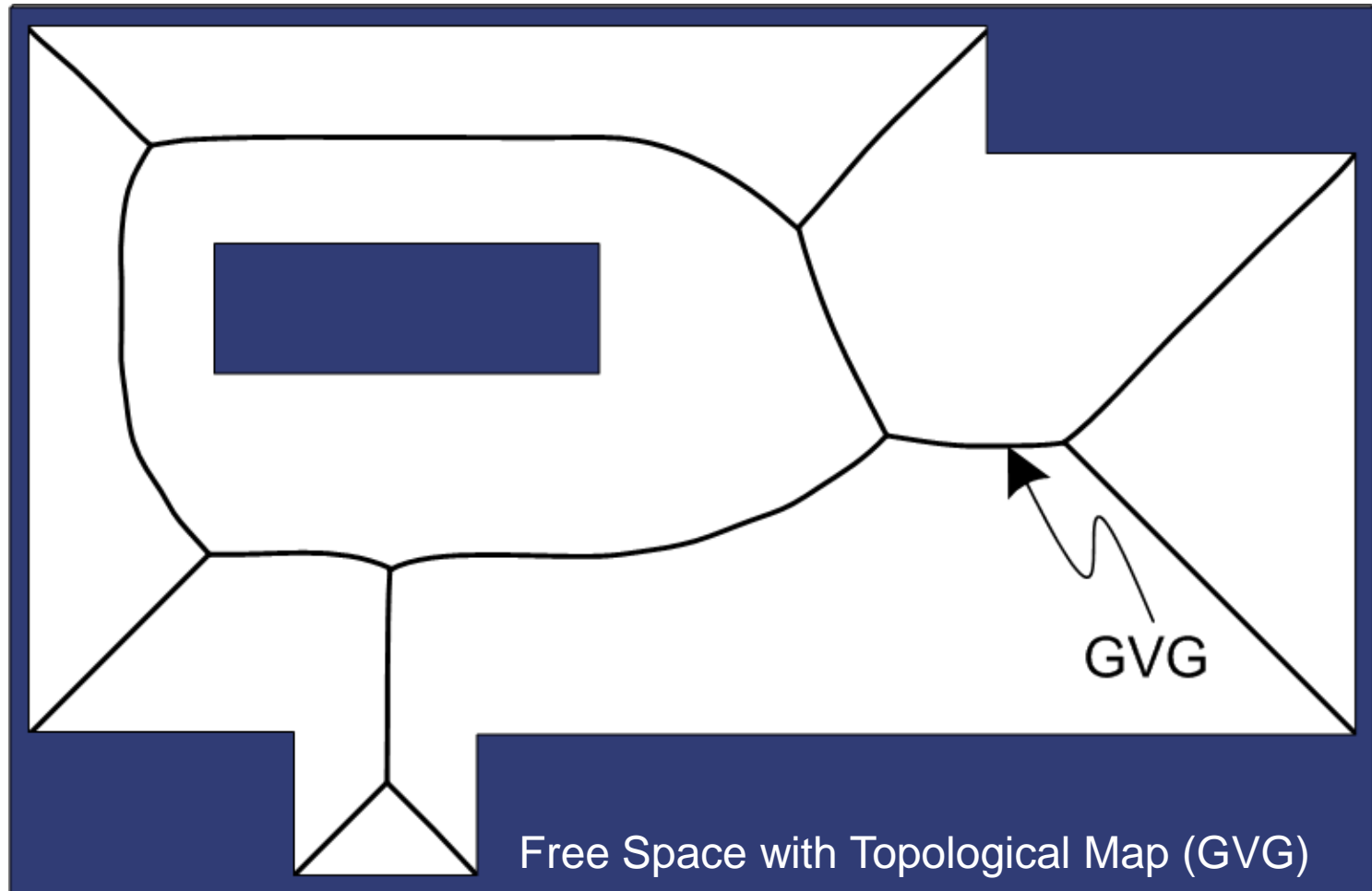
$q$  is in the *Voronoi diagram* of  $C_{\text{free}}$  if  $| \textit{near}(q) | > 1$

number of  
set elements

# Generalized Voronoi Graph (GVG)

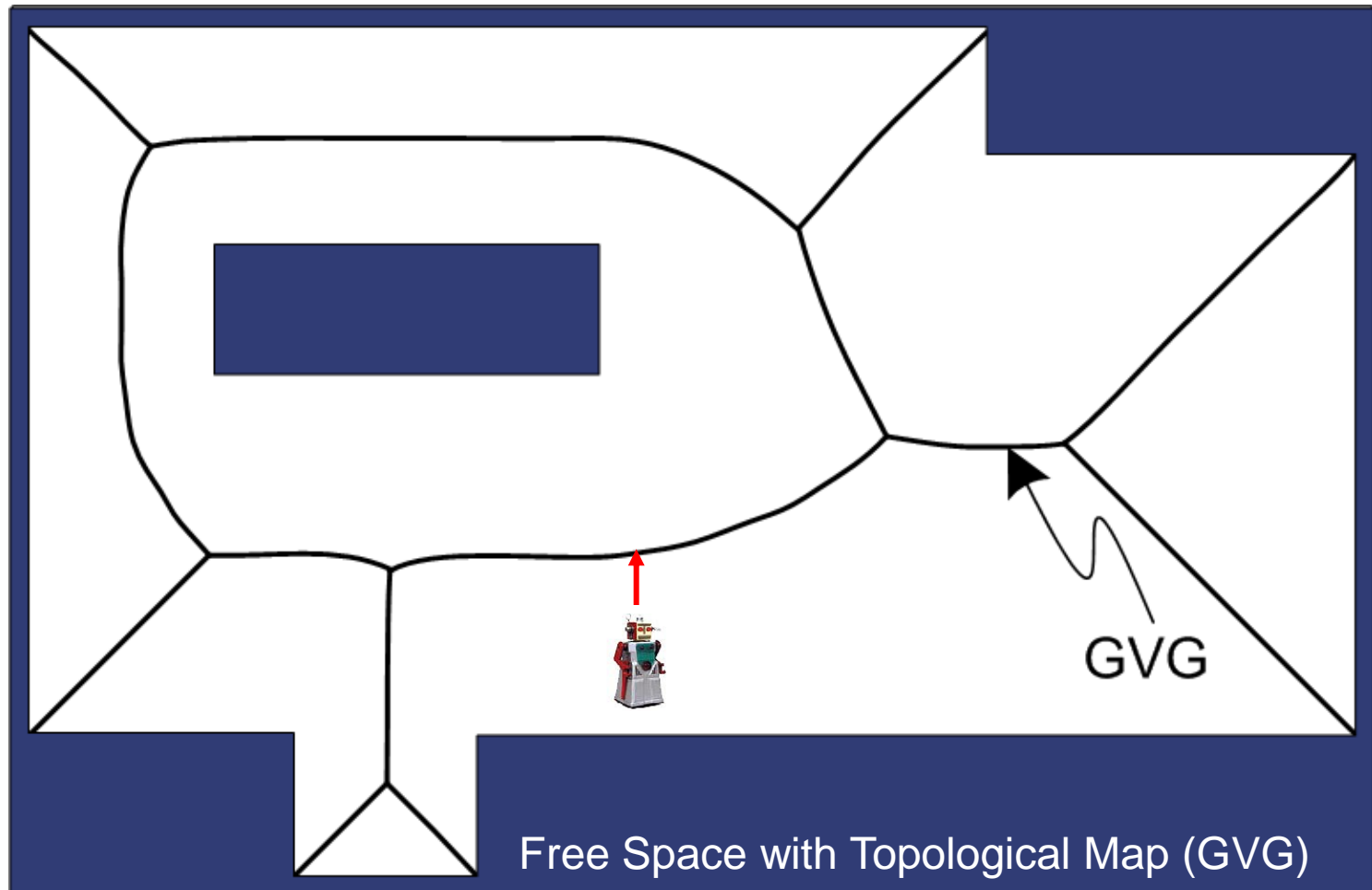


# Generalized Voronoi Graph (GVG)



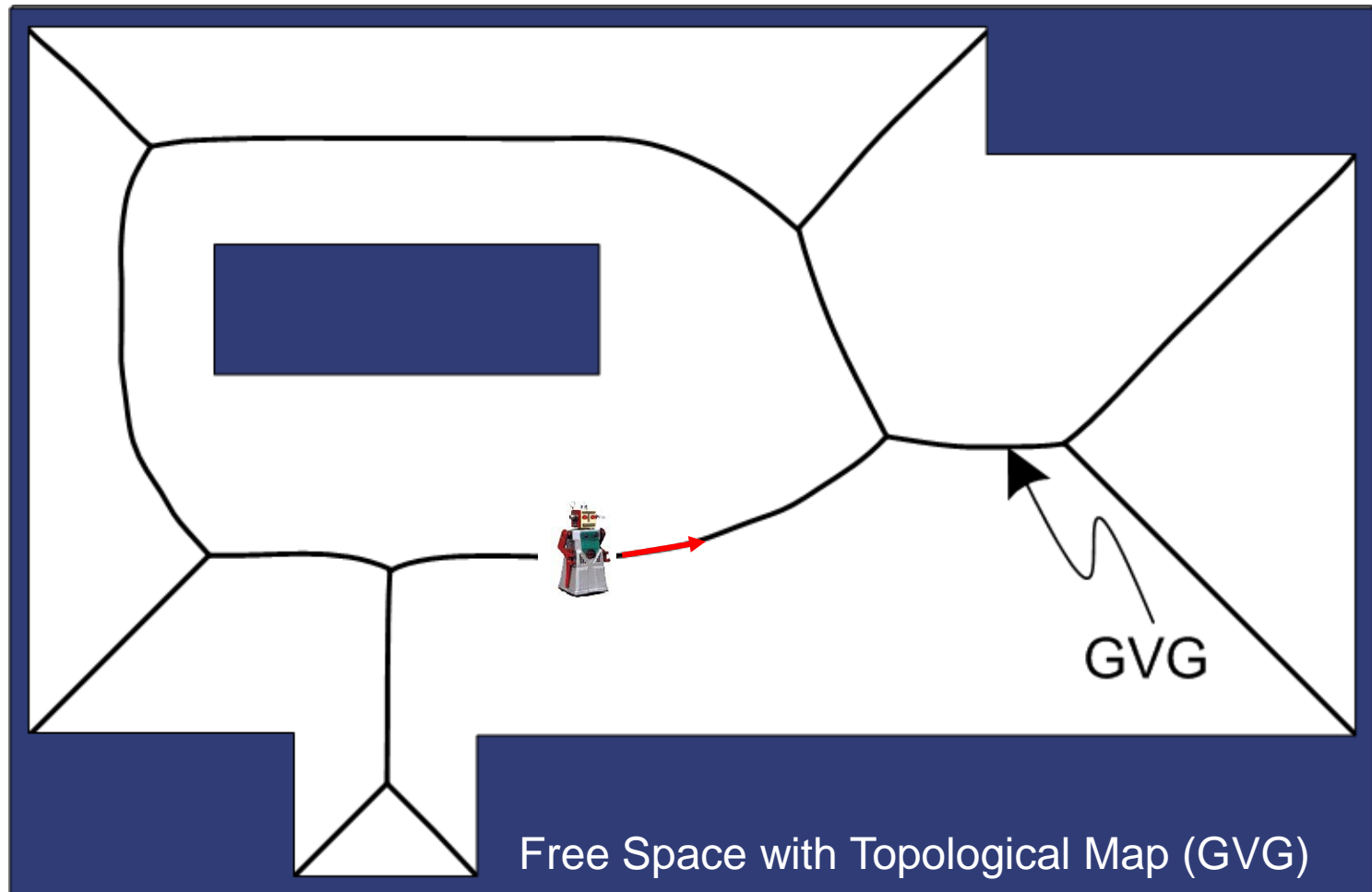
# Generalized Voronoi Graph (GVG)

- Access GVG



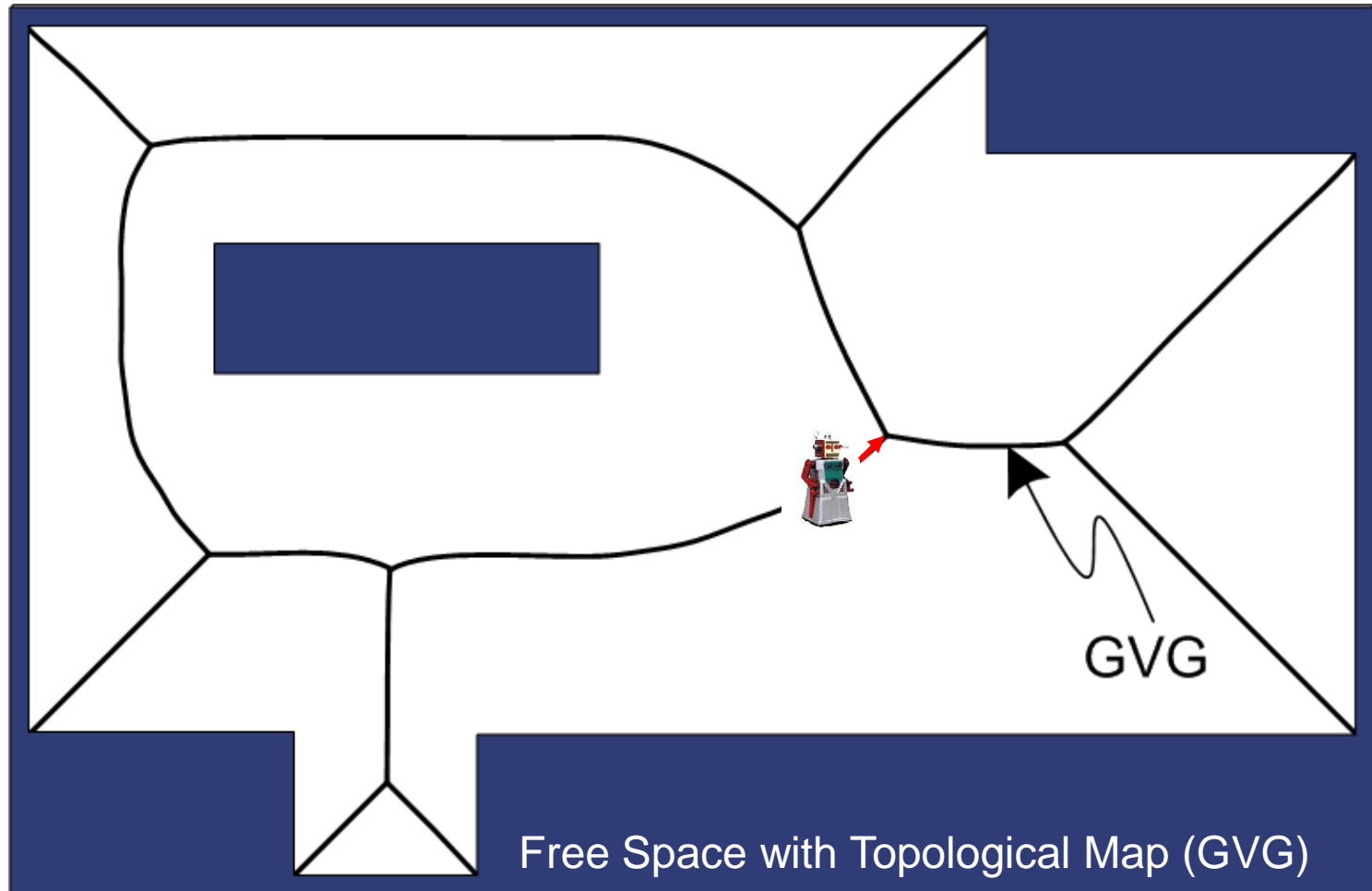
# Generalized Voronoi Graph (GVG)

- Access GVG
- Follow Edge



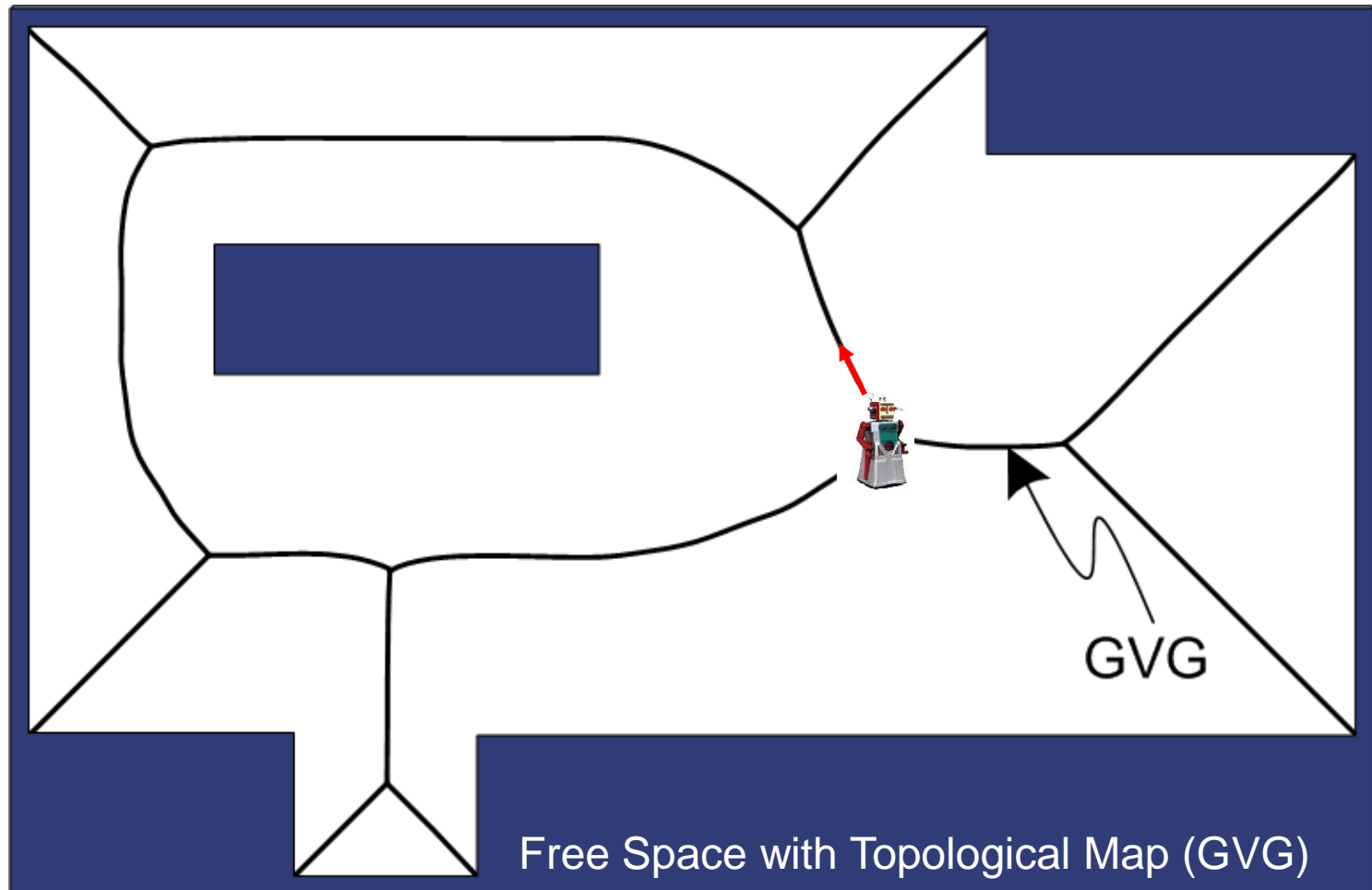
# Generalized Voronoi Graph (GVG)

- Access GVG
- Home to the MeetPoint
- Follow Edge



# Generalized Voronoi Graph (GVG)

- Access GVG
- Home to the MeetPoint
- Follow Edge
- Select Edge



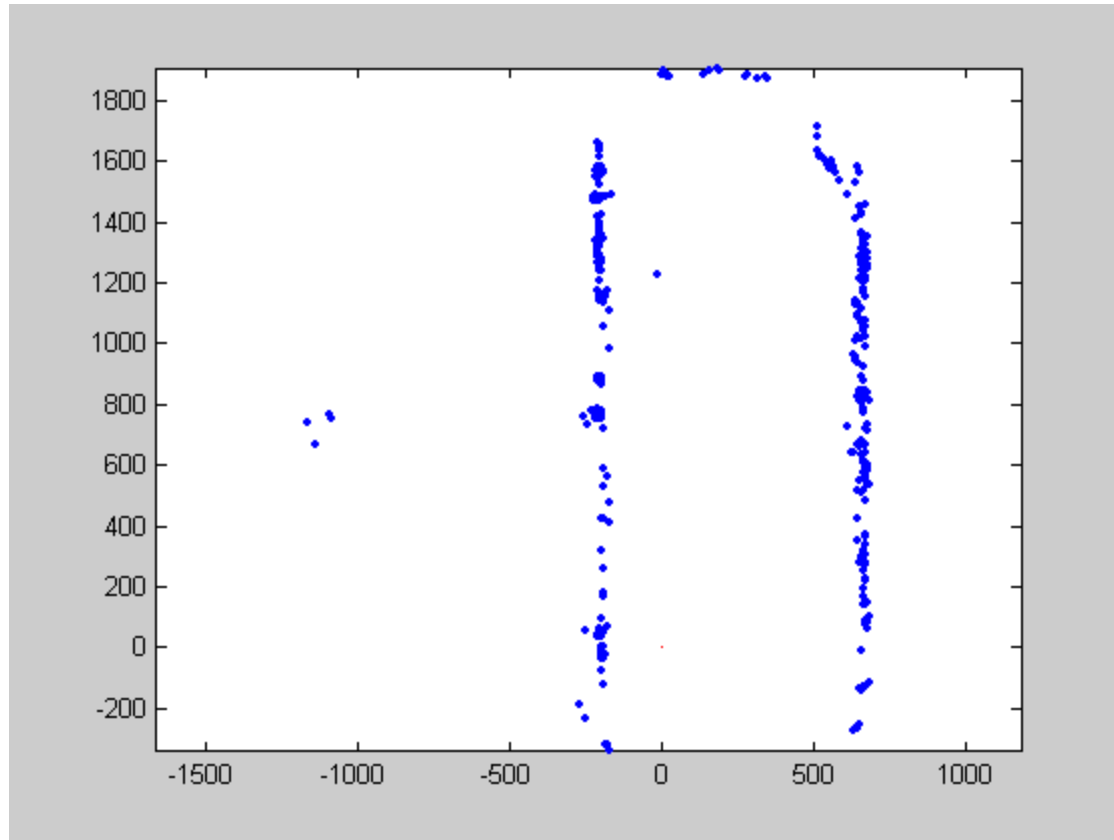
# GVG construction using sonar



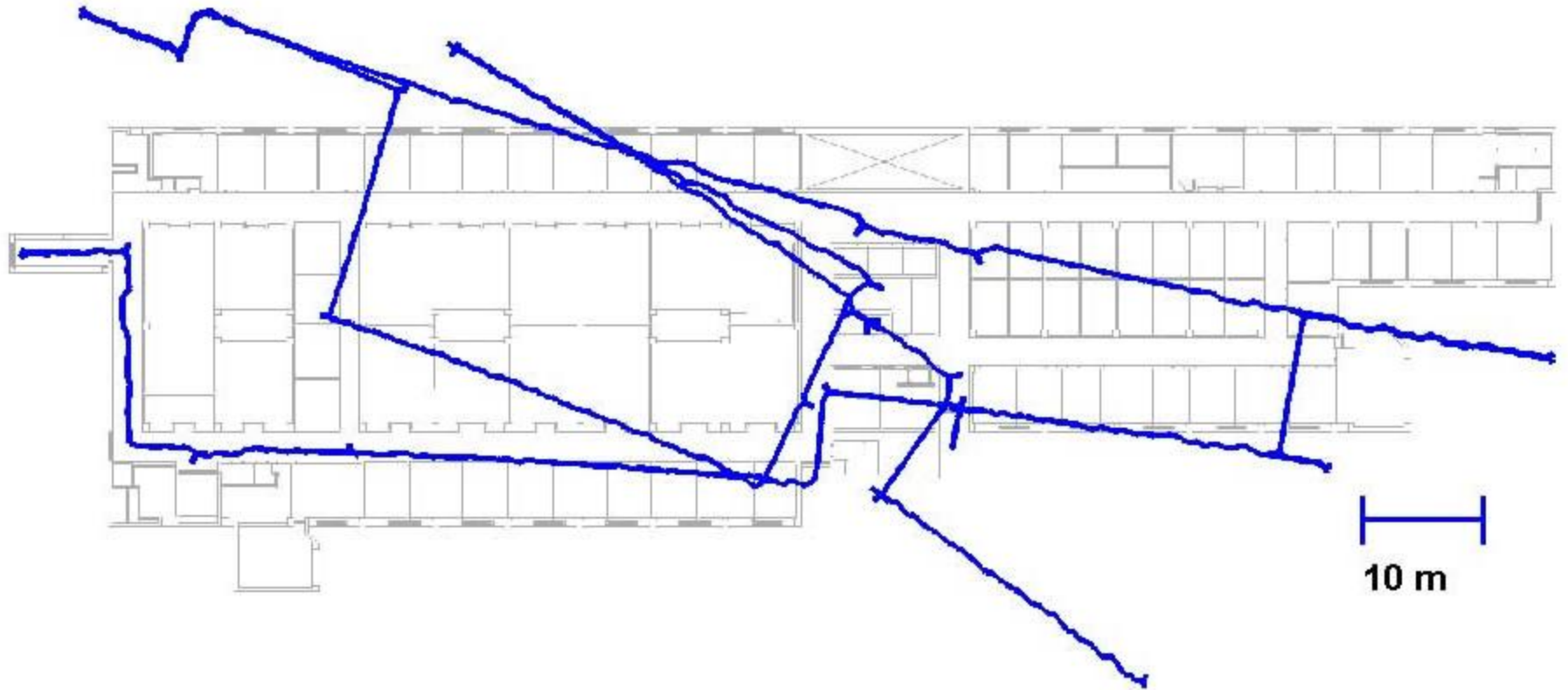
- Nomadic Scout
- Sonar (GVG navigation)
- Camera with omni-directional mirror (feature detection)
- Onboard 1.2 GHz processor



# GVG construction using sonar



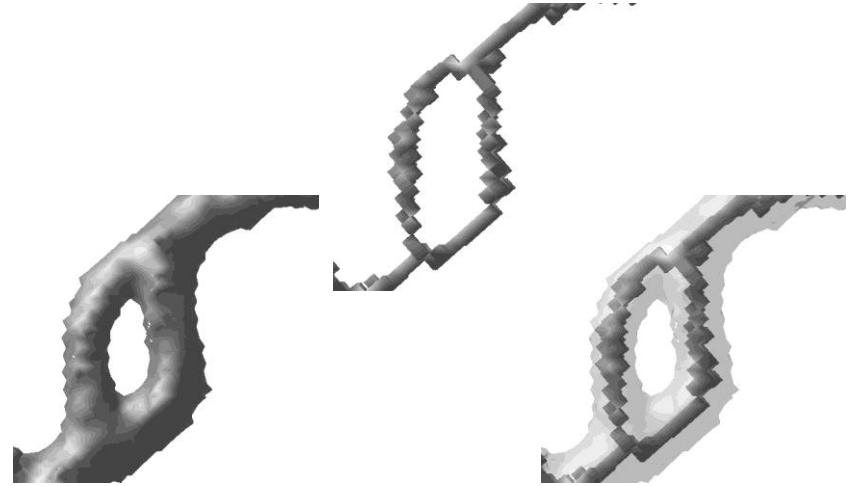
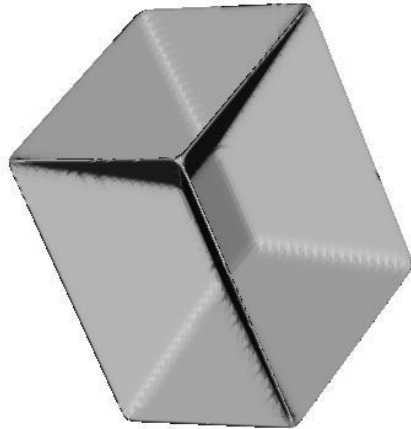
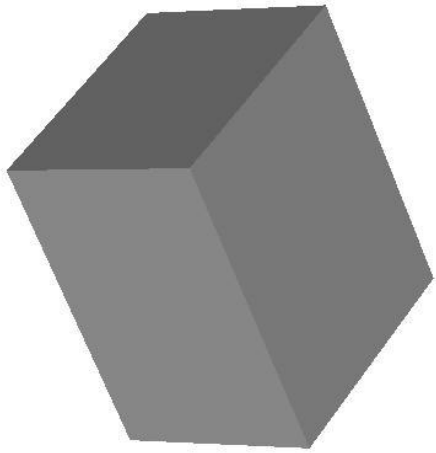
# GVG construction using sonar



# *Slammer* in Action



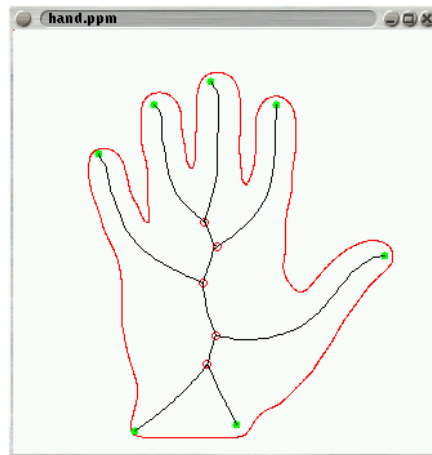
# Voronoi applications



A retraction of a 3d object  
== “*medial surface*”

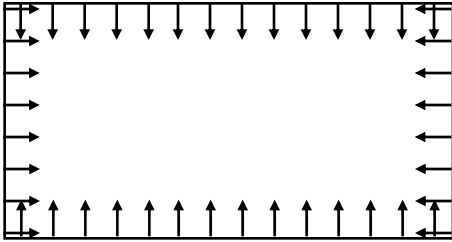
what? ↗

Skeletonizations resulting from  
constant-speed curve evolution

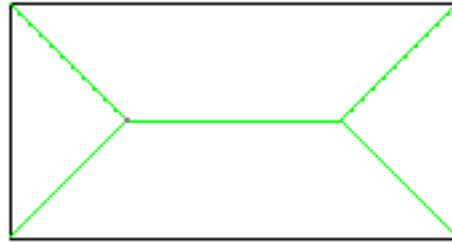


in 2d, it's called  
a *medial axis*

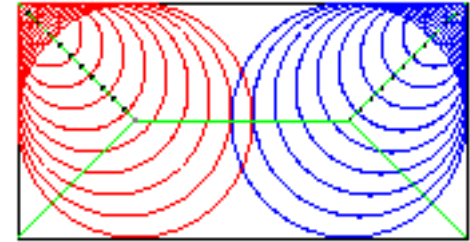
# skeleton $\longleftrightarrow$ shape



curve evolution



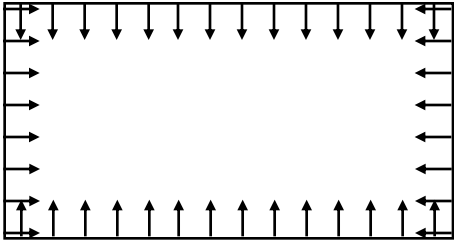
where wavefronts collide



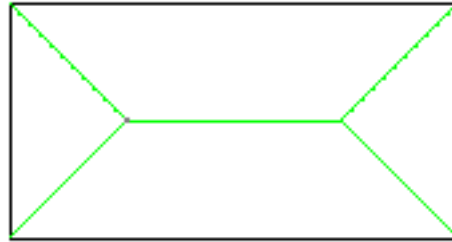
centers of maximal disks

again reduces a 2d (or higher) problem to a question about graphs...

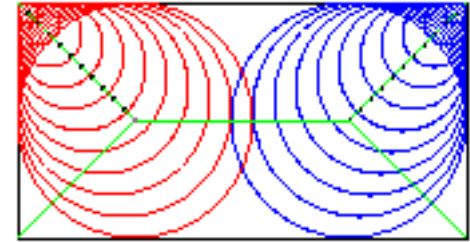
# skeleton $\leftrightarrow$ shape



curve evolution



where wavefronts collide



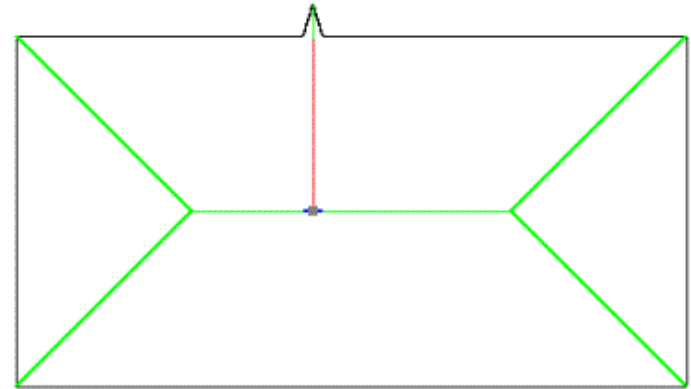
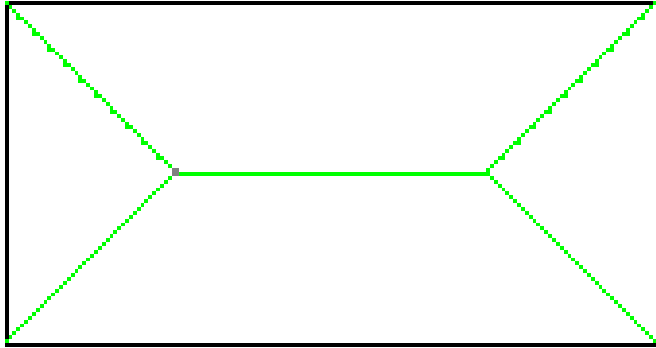
centers of maximal disks

again reduces a 2d (or higher) problem to a question about graphs...



graph matching

# Problems

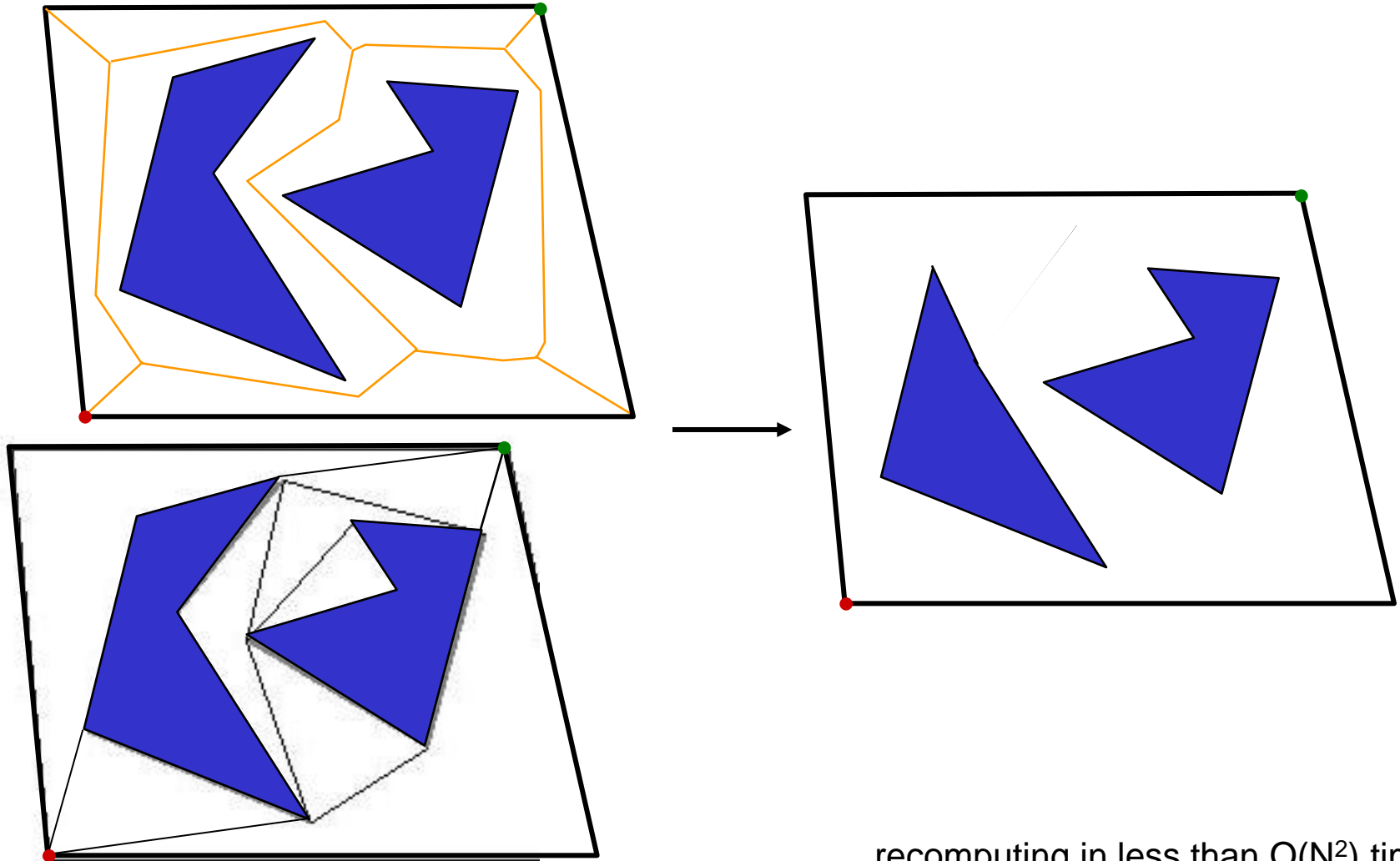


The skeleton is sensitive to small changes in the object's boundary.

- graph isomorphism (and lots of other graph questions) : NP-complete

# Roadmap problems

If an obstacle decides to roll away... (or wasn't there to begin with)



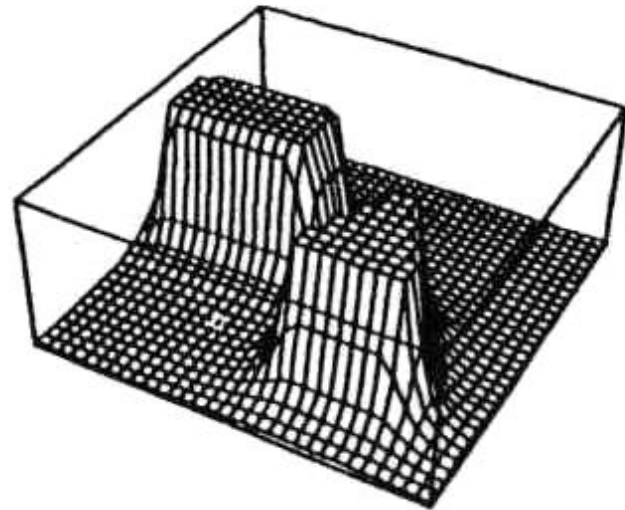
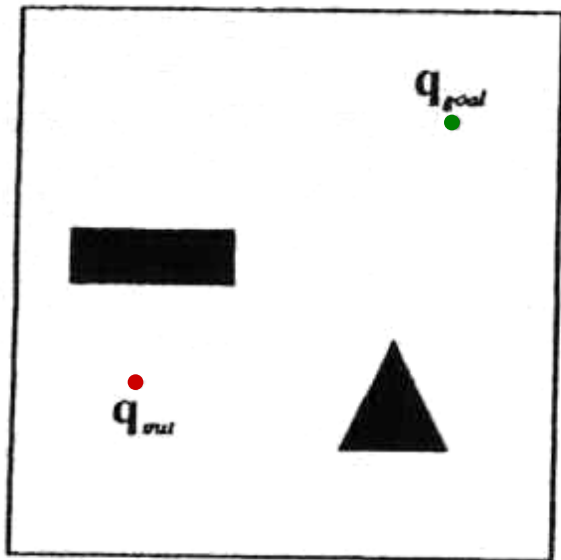
recomputing in less than  $O(N^2)$  time?



# Path Planning

## Potential Field methods

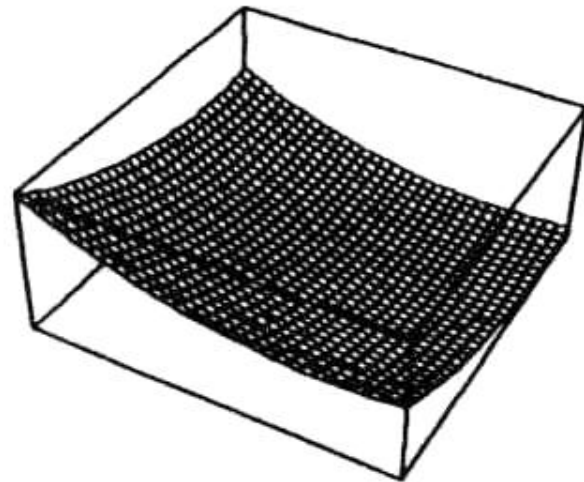
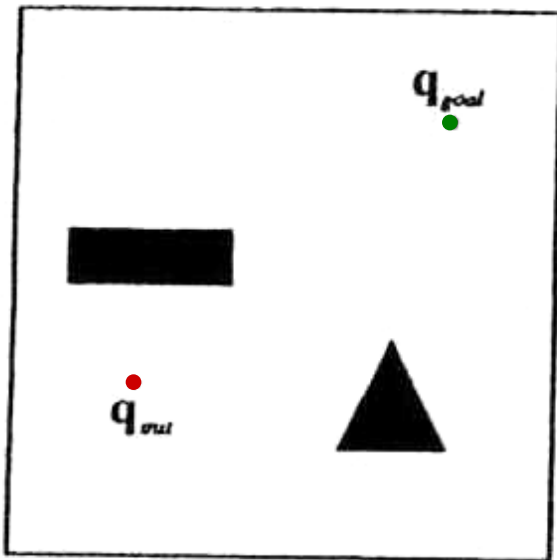
- compute a repulsive force away from obstacles



# Local techniques

## Potential Field methods

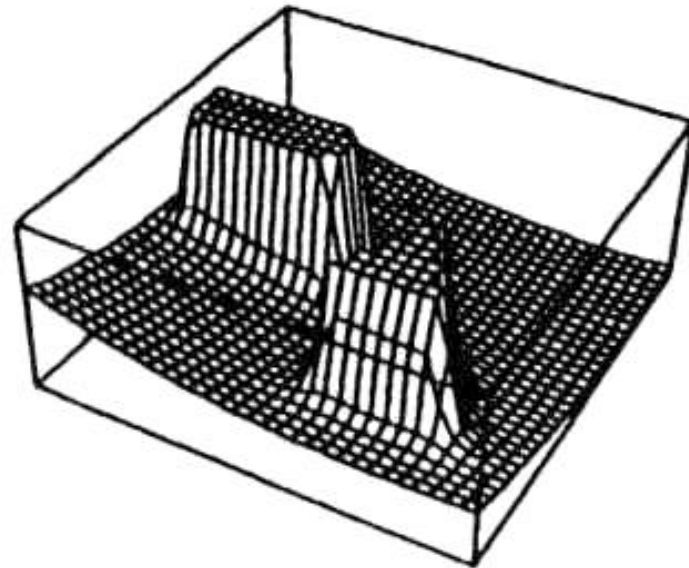
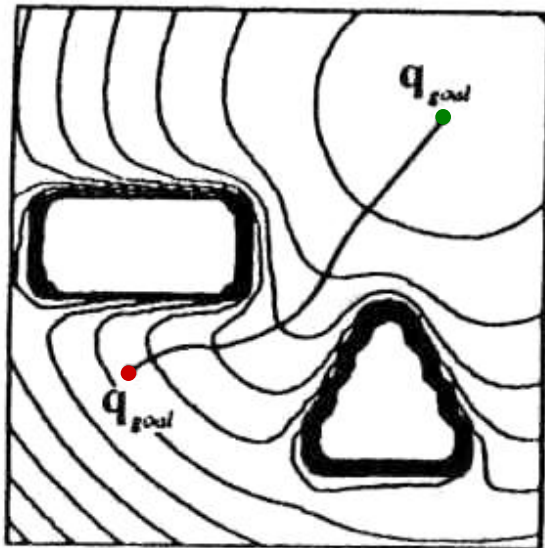
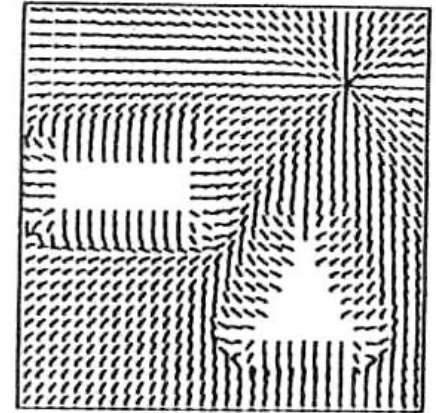
- compute a repulsive force away from obstacles
- compute an attractive force toward the goal



# Local techniques

## Potential Field methods

- compute a repulsive force away from obstacles
  - compute an attractive force toward the goal
- let the sum of the forces control the robot

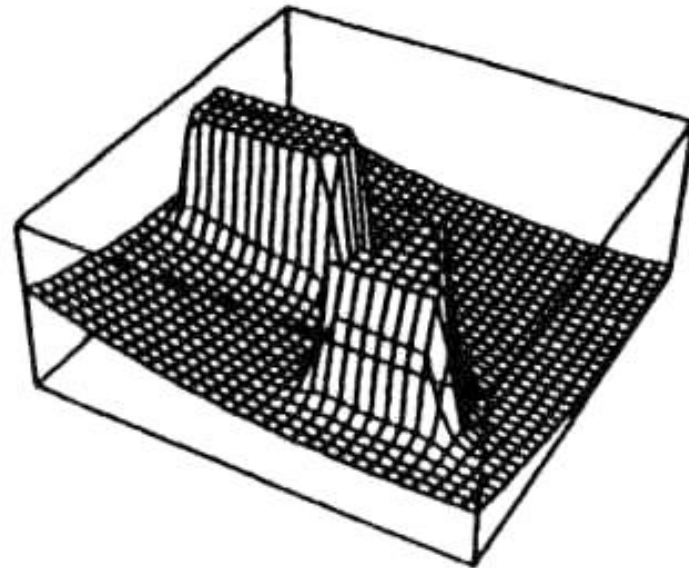
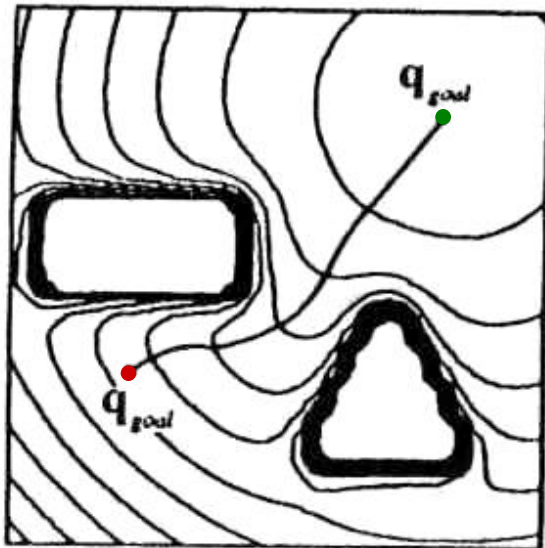
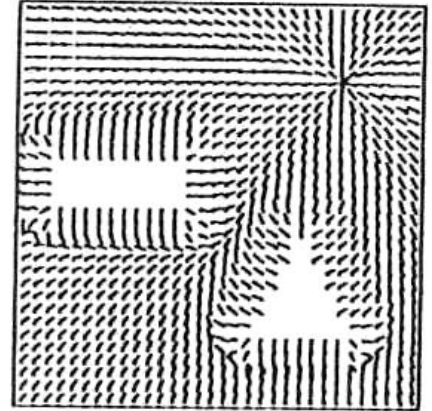


key advantages?

# Local techniques

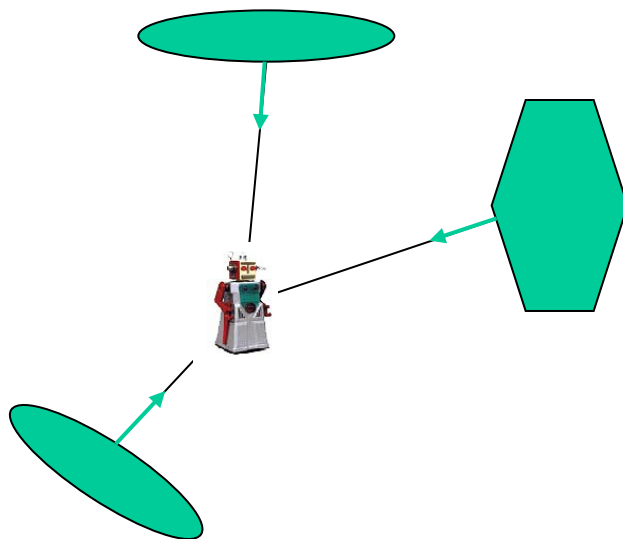
## Potential Field methods

- compute a repulsive force away from obstacles
  - compute an attractive force toward the goal
- let the sum of the forces control the robot



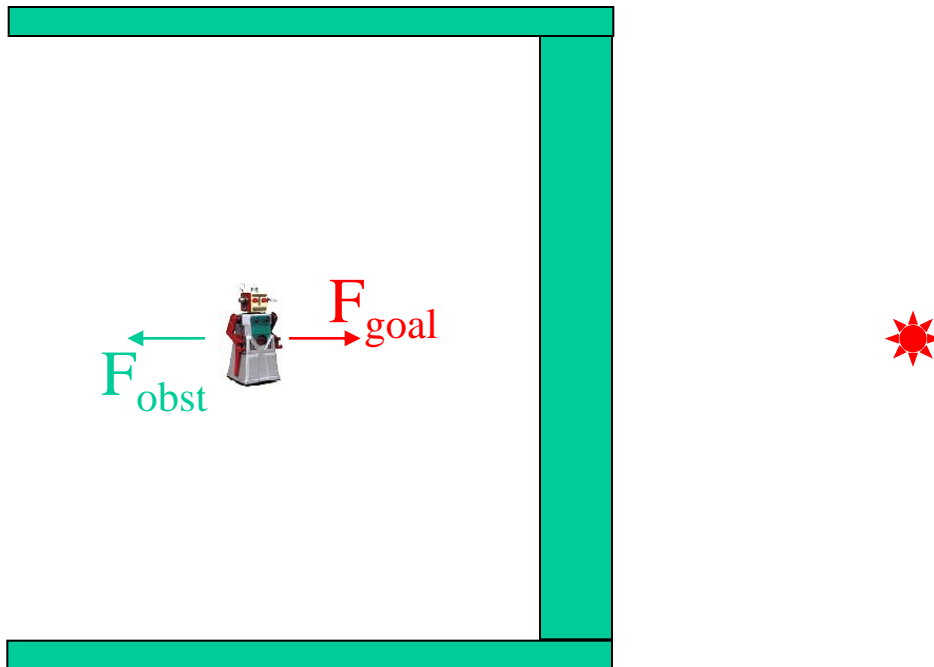
To a large extent, this is computable from sensor readings

# Sensor Based Calculations



Major Problem?

# Local Minima!



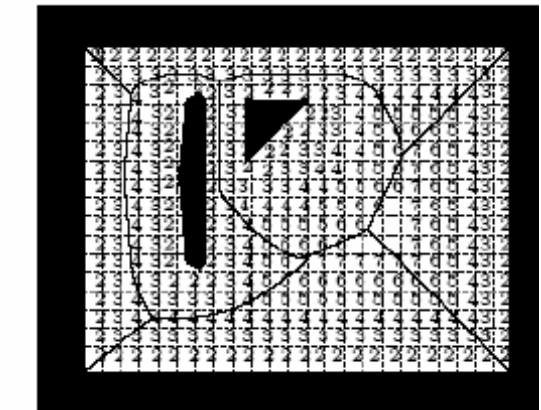
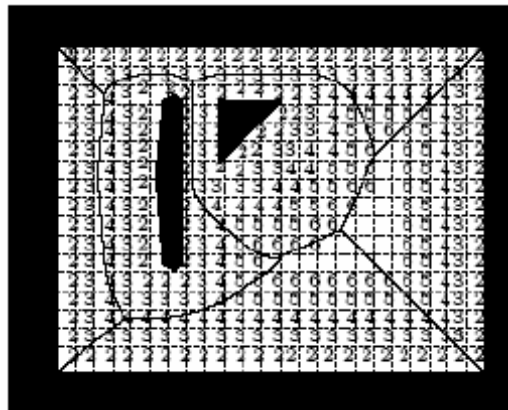
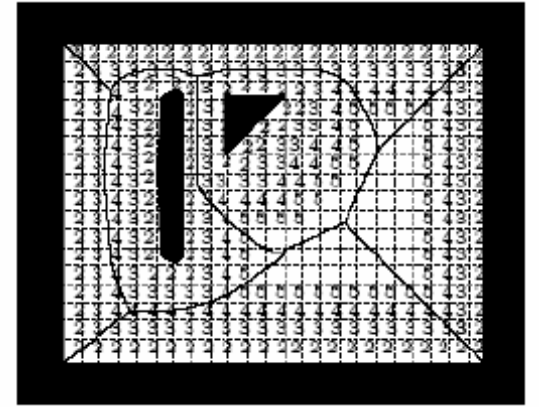
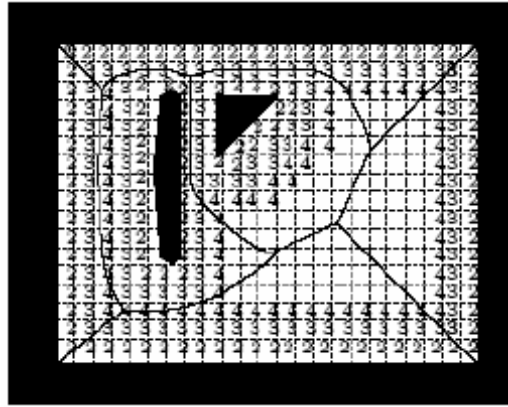
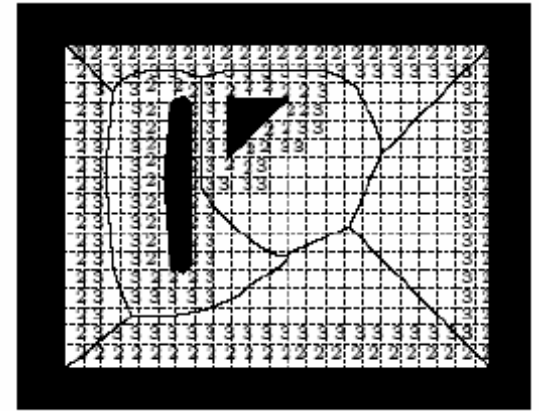
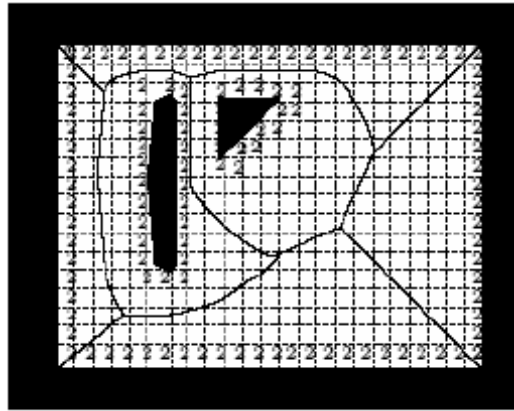
# Simulated Annealing

- Every so often add some random force



# Known Map

## Brushfire Transform



# The Wavefront Planner: Setup

7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
4	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	
3	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

# The Wavefront in Action (Part 1)

- Starting with the goal, set all adjacent cells with “0” to the current cell + 1
  - 4-Point Connectivity or 8-Point Connectivity?
  - Your Choice. We’ll use 8-Point Connectivity in our example

7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
4	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	
3	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	2	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

# The Wavefront in Action (Part 2)

- Now repeat with the modified cells
  - This will be repeated until no 0's are adjacent to cells with values  $\geq 2$
- 0's will only remain when regions are unreachable

7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
4	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	
3	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	
2	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	
1	0	0	0	0	0	0	0	0	0	0	0	0	4	3	3	
0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	2	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

# The Wavefront in Action (Part 3)

- Repeat

7	<b>0</b>	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
4	0	0	0	0	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	0	0	0	
3	0	0	0	0	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	5	5	5	
2	0	0	0	0	0	0	0	0	0	0	0	0	5	4	4	
1	0	0	0	0	0	0	0	0	0	0	0	0	5	4	3	
0	0	0	0	0	0	0	0	0	0	0	0	0	5	4	3	<b>2</b>
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

# The Wavefront in Action (Part 3)

- Repeat

7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
4	0	0	0	0	1	1	1	1	1	1	1	6	6	6	6	
3	0	0	0	0	1	1	1	1	1	1	1	5	5	5	5	
2	0	0	0	0	0	0	0	0	0	0	6	5	4	4	4	
1	0	0	0	0	0	0	0	0	0	0	6	5	4	3	3	
0	0	0	0	0	0	0	0	0	0	0	6	5	4	3	2	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

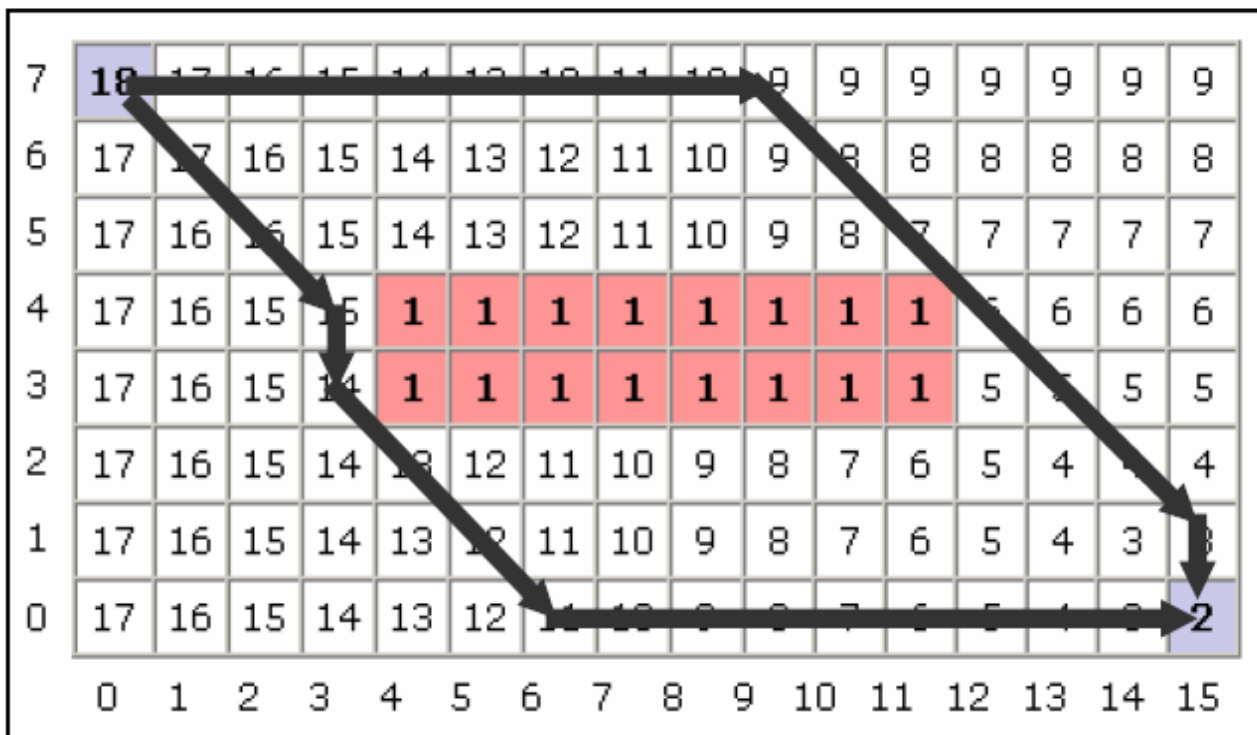
# The Wavefront in Action (Part 3)

- Until Done
  - 0's would only remain in the unreachable areas

7	<b>18</b>	17	16	15	14	13	12	11	10	9	9	9	9	9	9	
6	17	17	16	15	14	13	12	11	10	9	8	8	8	8	8	
5	17	16	16	15	14	13	12	11	10	9	8	7	7	7	7	
4	17	16	15	15	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	6	6	6	
3	17	16	15	14	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	5	5	5	
2	17	16	15	14	13	12	11	10	9	8	7	6	5	4	4	
1	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	
0	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	<b>2</b>
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

# The Wavefront in Action

- To find the shortest path, according to your metric, simply always move toward a cell with a lower number
  - The numbers generated by the Wavefront planner are roughly proportional to their distance from the goal



Two possible shortest paths shown