

# CS-417 INTRODUCTION TO ROBOTICS AND INTELLIGENT SYSTEMS

### **Motion Planning**

Ioannis Rekleitis

## Outline

- The robots I worked with
- Path Planning
  - Visibility Graph
  - Bug Algorithms
  - Potential Fields
  - Skeletons/Voronoi Graphs
  - C-Space

CS-417 Introduction to Robotics and Intelligent Systems

















### **Carnegie Mellon University**







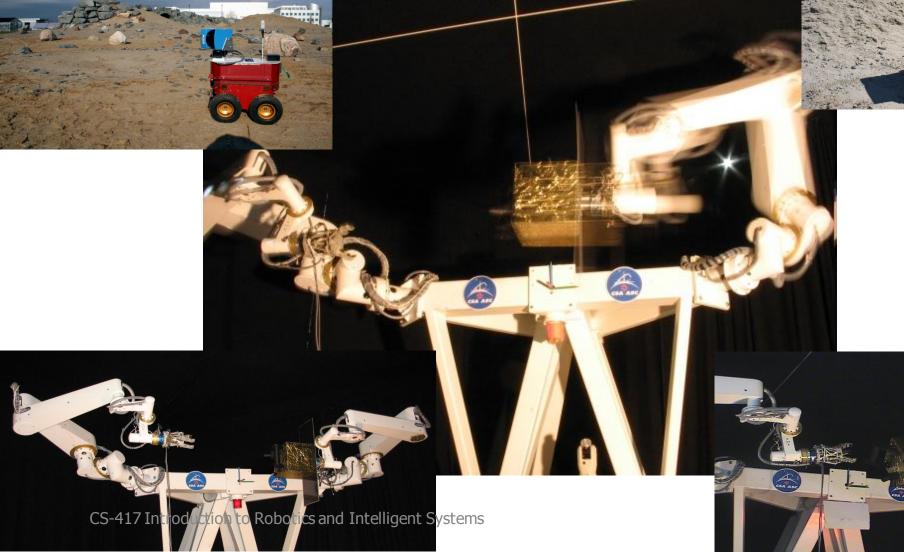






### **Canadian Space Agency**





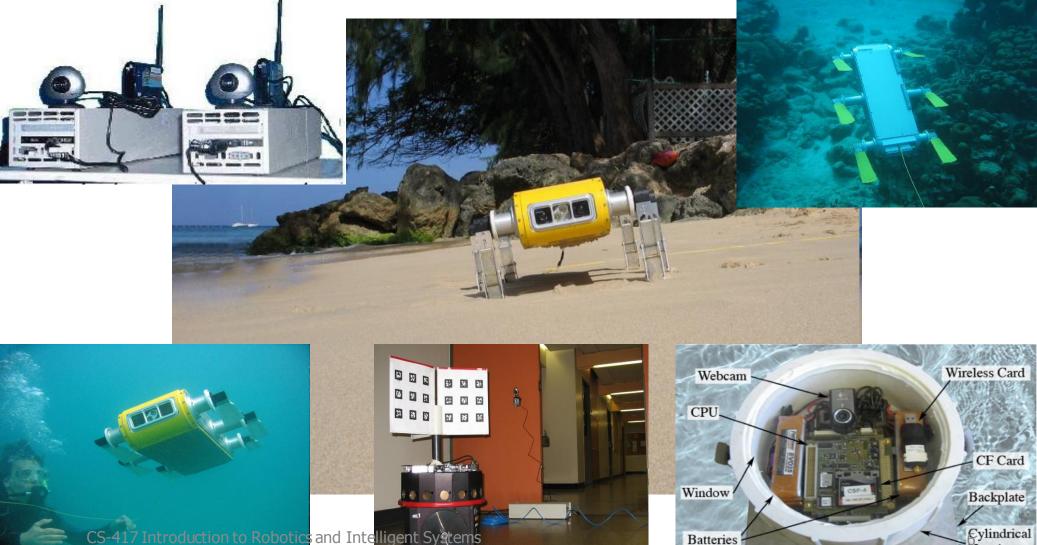


### **McGill University**



Intelligent Machines

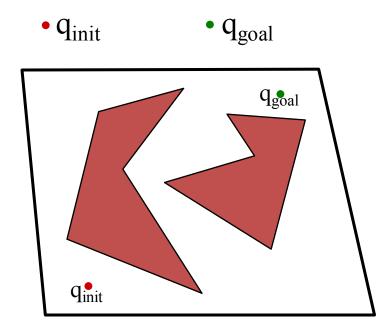
Housing

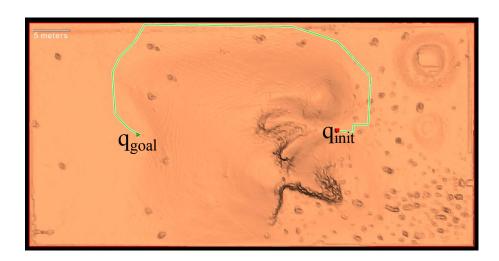


ntroduction to Robotics and Intelligent Systems

## **Motion Planning**

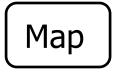
- The ability to go from **A** to **B** 
  - Known map Off-line planning
  - Unknown Environment –Online planning
  - Static/Dynamic Environment



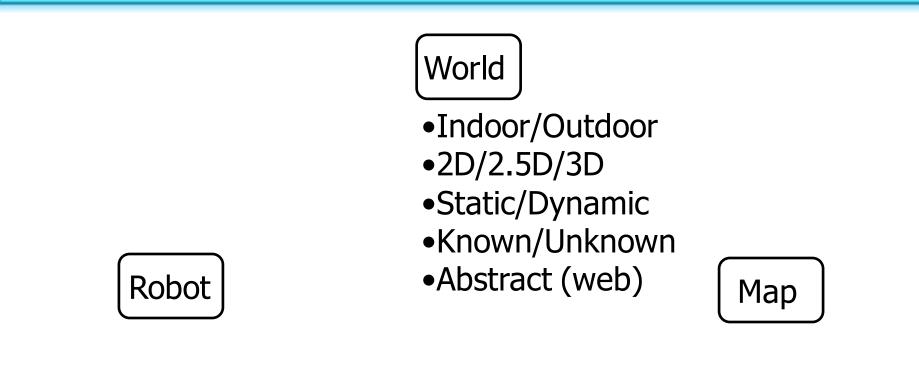












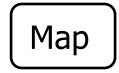






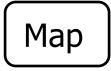
- Mobile
  - ≻Indoor/Outdoor
  - >Walking/Flying/Swimming
- Manipulator
- •Humanoid
- Abstract

CS-417 Introduction to Robotics and Intelligent Systems



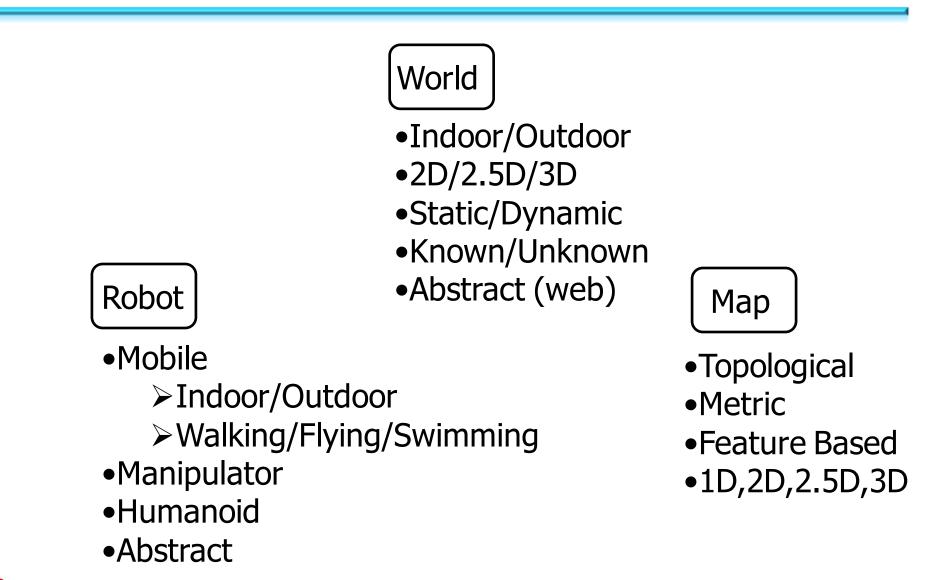






- Topological
- •Metric
- •Feature Based
- •1D,2D,2.5D,3D

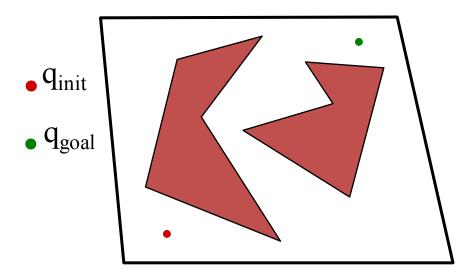




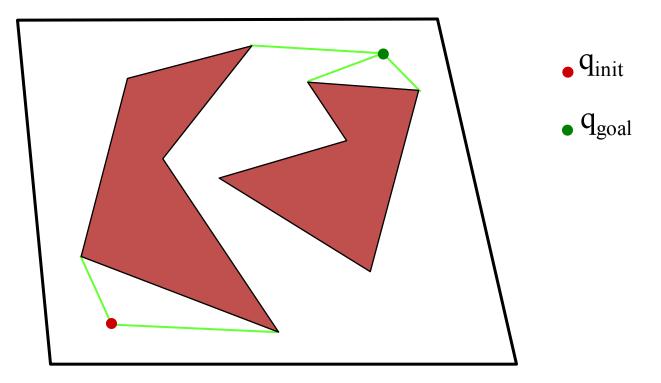
CS-417 Introduction to Robotics and Intelligent Systems

## **Path Planning: Assumptions**

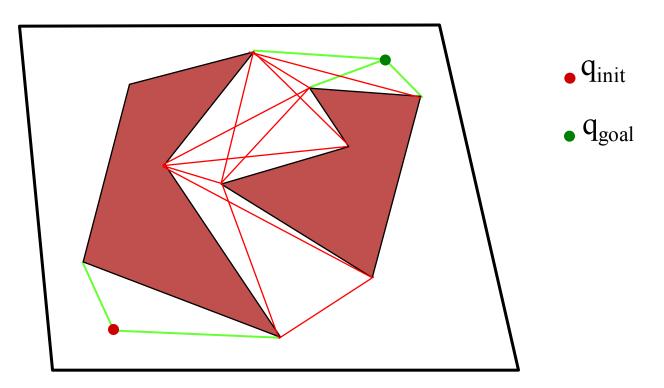
- Known Map
- Roadmaps (Graph representations)
- Polygonal Representation



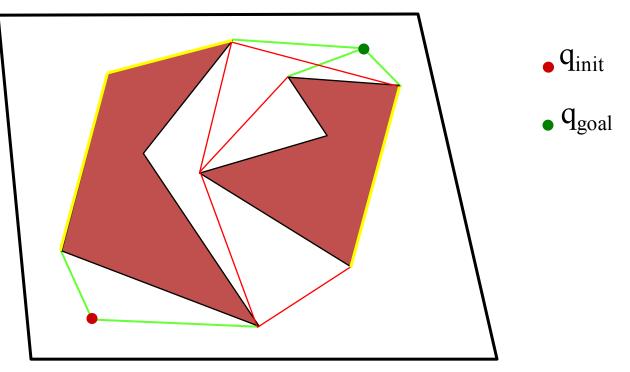
• Connect Initial and goal locations with all the visible vertices



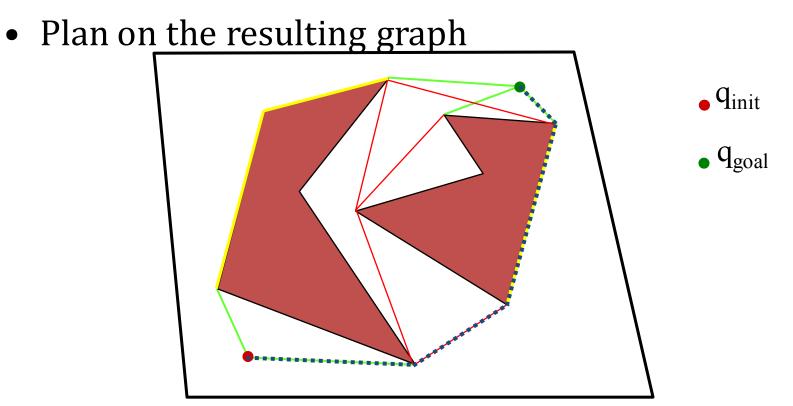
- Connect initial and goal locations with all the visible vertices
- Connect each obstacle vertex to every visible obstacle vertex



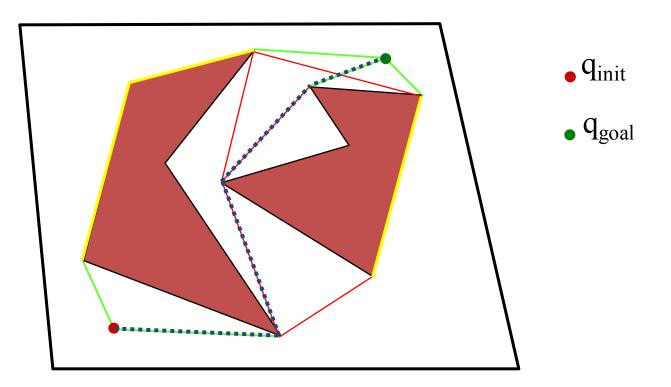
- Connect initial and goal locations with all the visible vertices
- Connect each obstacle vertex to every visible obstacle vertex
- Remove edges that intersect the interior of an obstacle



- Connect initial and goal locations with all the visible vertices
- Connect each obstacle vertex to every visible obstacle vertex
- Remove edges that intersect the interior of an obstacle

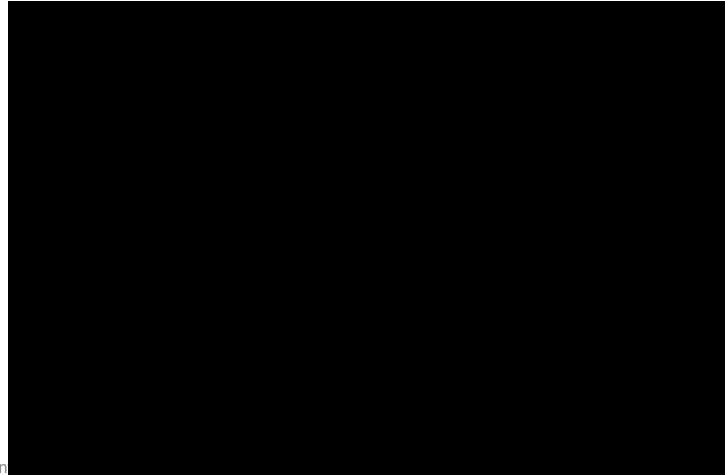


- An alternative path
- Alternative name: "Rubber band algorithm"



### **Major Fault**

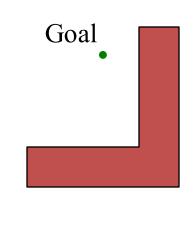
- Point robot
- Path planning like that guarantees to hit the obstacles





# Limited-knowledge path planning

- Path planning with limited knowledge
  - Insect-inspired "bug" algorithms



- known direction to goal
- otherwise local sensing
  - walls/obstacles encoders
- •"reasonable" world
- 1. finitely many obstacles in any finite disc
- 2. a line will intersect an obstacle finitely many times



### Not truly modeling bugs...

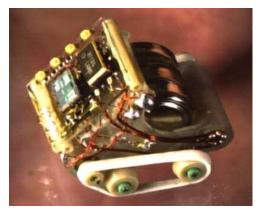
Insects do use several cues for navigation:



visual landmarks

polarized light

chemical sensing



neither are the current bug-sized robots

they're not ears...

Other animals use information from

magnetic fields \_\_\_\_\_

electric currents

temperature CS-417 Introduction to Robotics and Intelligent Systems



bacteria



migrating bobolink<sup>21</sup>

## **Bug Strategy**

Insect-inspired "bug" algorithms



otherwise only local sensing

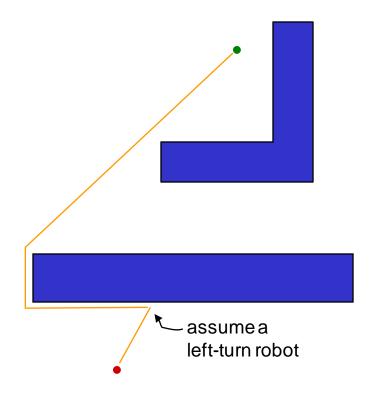
walls/obstacles encoders

"Bug 0" algorithm

1) head toward goal

2) follow obstacles until you can head toward the goal again

3) continue



### **Does It Work?**



# Bug 1

#### Insect-inspired "bug" algorithms

- known direction to goal
- otherwise only local sensing walls/obstacles encoders

"Bug 1" algorithm

1) head toward goal



# Bug 1

#### Insect-inspired "bug" algorithms

- known direction to goal
- otherwise only local sensing walls/obstacles encoders

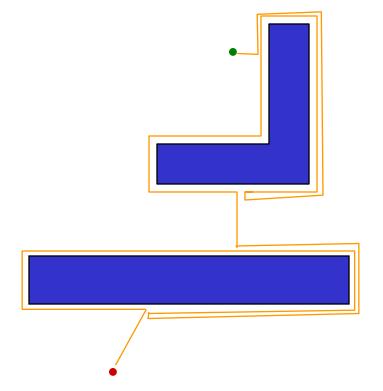
"Bug 1" algorithm

1) head toward goal

2) if an obstacle is encountered, circumnavigate it *and* remember how close you get to the goal

# Bug 1

#### Insect-inspired "bug" algorithms



- known direction to goal
- otherwise only local sensing walls/obstacles encoders

"Bug 1" algorithm

1) head toward goal

2) if an obstacle is encountered, circumnavigate it *and* remember how close you get to the goal

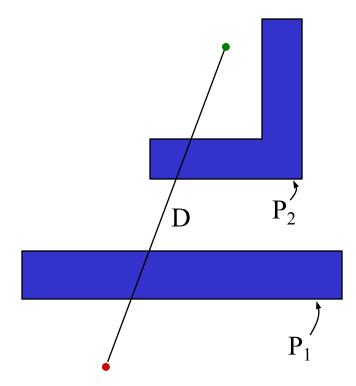
3) return to that closest point (by wall-following) and continue

Vladimir Lumelsky & Alexander Stepanov Algorithmica 1987



## **Bug 1 analysis**

#### Distance Traveled



What are bounds on the path length that the robot takes?

Available Information:

D = straight-line distance from start to goal

 $P_i$  = perimeter of the *i* th obstacle

Lower and upper bounds?

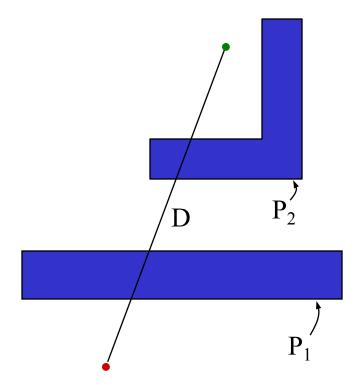
Lower bound:

Upper bound:



## **Bug 1 analysis**

#### Distance Traveled



What are bounds on the path length that the robot takes?

Available Information:

D = straight-line distance from start to goal

 $P_i$  = perimeter of the *i* th obstacle

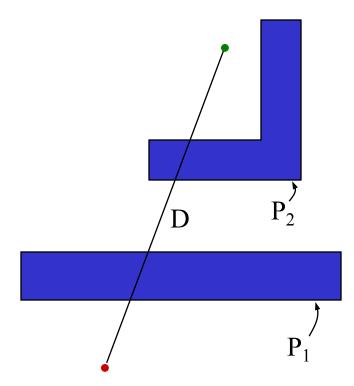
Lower and upper bounds?

Lower bound: D

Upper bound:

## **Bug 1 analysis**

#### Distance Traveled



CS-417 Introduction to Robotics and Intelligent Systems

What are bounds on the path length that the robot takes?

Available Information:

 $D = \mbox{straight-line}\ \mbox{distance}\ \mbox{from}\ \mbox{start}\ \mbox{to}\ \mbox{goal}$ 

 $P_i$  = perimeter of the *i* th obstacle

Lower and upper bounds?

Lower bound: D

Upper bound:  $D + 1.5 \Sigma P_{ii}$ 

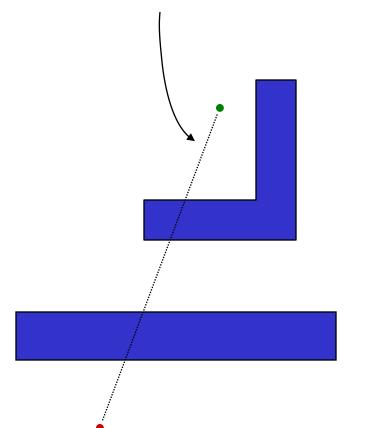
How good a bound? How good an algorithm?

### **Bug Mapping**





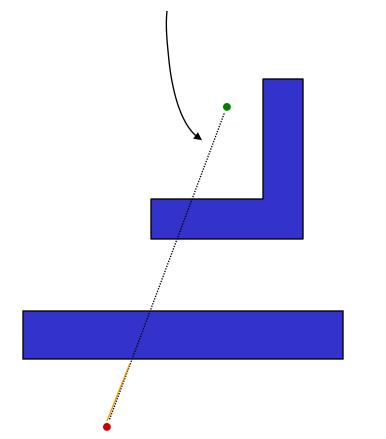
Call the line from the starting point to the goal the *s-line* 



#### "Bug 2" algorithm



Call the line from the starting point to the goal the *s-line* 

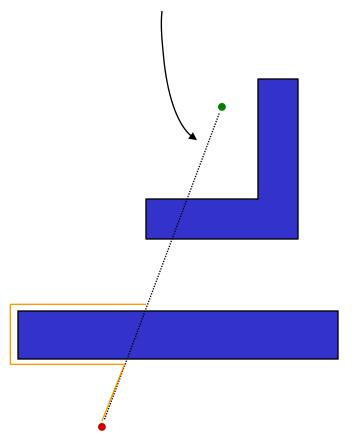


#### "Bug 2" algorithm

1) head toward goal on the *s*-line



Call the line from the starting point to the goal the *s-line* 



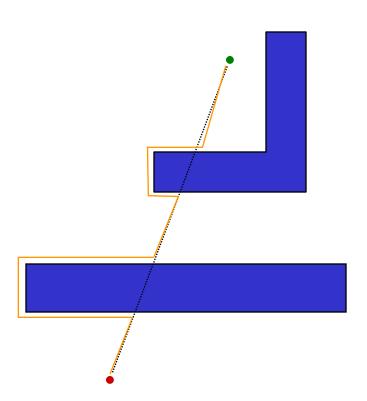
#### "Bug 2" algorithm

1) head toward goal on the *s*-line

2) if an obstacle is in the way, follow it until encountering the sline again.





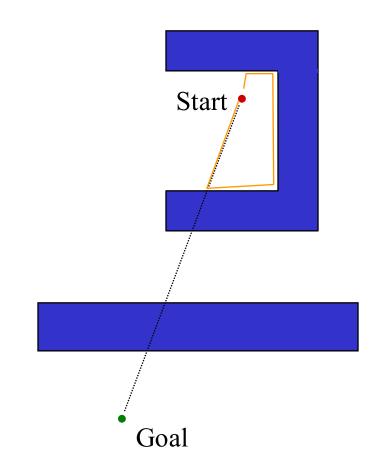


#### "Bug 2" algorithm

1) head toward goal on the *s*-line

2) if an obstacle is in the way, follow it until encountering the sline again.

3) Leave the obstacle and continue toward the goal



#### "Bug 2" algorithm

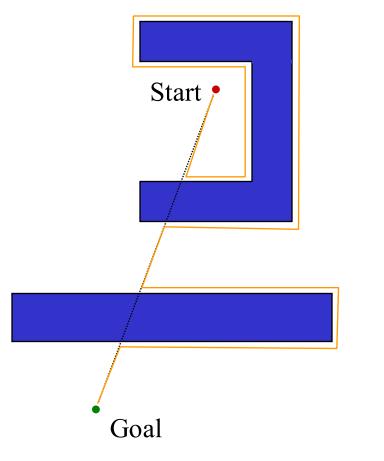
1) head toward goal on the *s*-line

2) if an obstacle is in the way, follow it until encountering the sline again *closer to the goal*.

3) Leave the obstacle and continue toward the goal

## **Bug 2 analysis**

#### Distance Traveled



What are bounds on the path length that the robot takes?

Available Information:

 $D = \mbox{straight-line}\ \mbox{distance}\ \mbox{from}\ \mbox{start}\ \mbox{to}\ \mbox{goal}$ 

 $P_i$  = perimeter of the *i* th obstacle

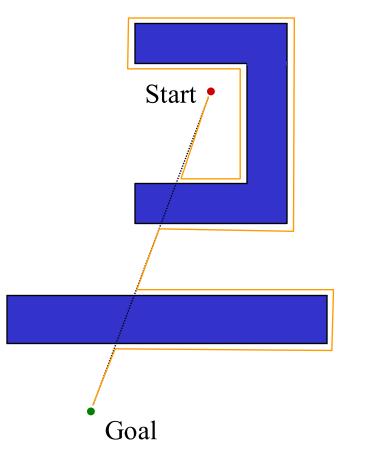
Lower and upper bounds?

Lower bound:

Upper bound:

# **Bug 2 analysis**

### Distance Traveled



What are bounds on the path length that the robot takes?

Available Information:

 $D = \mbox{straight-line}\ \mbox{distance}\ \mbox{from}\ \mbox{start}\ \mbox{to}\ \mbox{goal}$ 

- $P_i$  = perimeter of the *i* th obstacle
- $N_i$  = number of s-line intersections with the *i* th obstacle

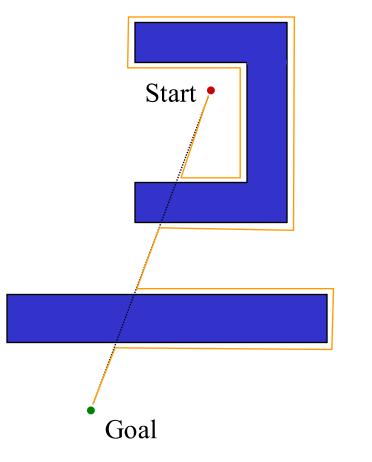
Lower and upper bounds?

Lower bound:

Upper bound:

# **Bug 2 analysis**

#### Distance Traveled



What are bounds on the path length that the robot takes?

Available Information:

 $D = \mbox{straight-line}\ \mbox{distance}\ \mbox{from}\ \mbox{start}\ \mbox{to}\ \mbox{goal}$ 

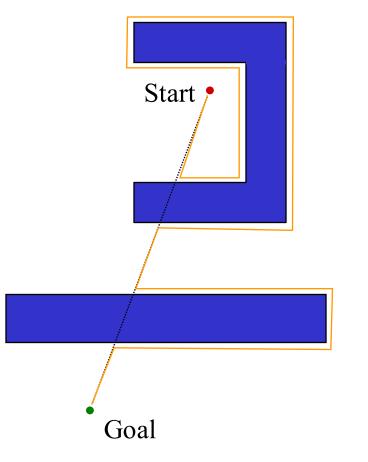
- $P_i$  = perimeter of the *i* th obstacle
- $N_i$  = number of s-line intersections with the *i* th obstacle
- Lower and upper bounds?

Lower bound: D

Upper bound:

# **Bug 2 analysis**

### Distance Traveled



What are bounds on the path length that the robot takes?

Available Information:

 $D = \mbox{straight-line}\ \mbox{distance}\ \mbox{from}\ \mbox{start}\ \mbox{to}\ \mbox{goal}$ 

- $P_i$  = perimeter of the *i* th obstacle
- $N_i$  = number of s-line intersections with the *i* th obstacle
- Lower and upper bounds?

Lower bound: D

Upper bound:

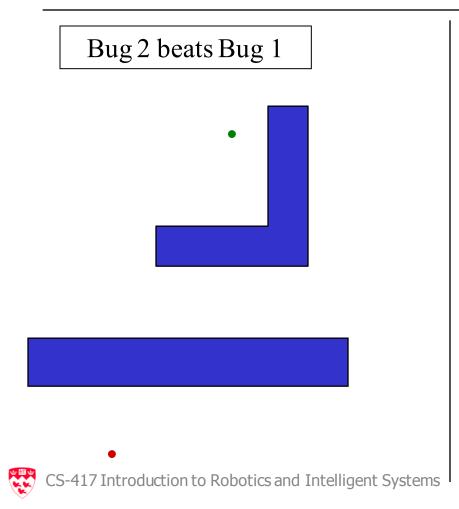
 $D + 0.5 \Sigma N_i P_i$ 



### head-to-head comparison

or thorax-to-thorax, perhaps

What are worlds in which Bug 2 does better than Bug 1 (and vice versa)?

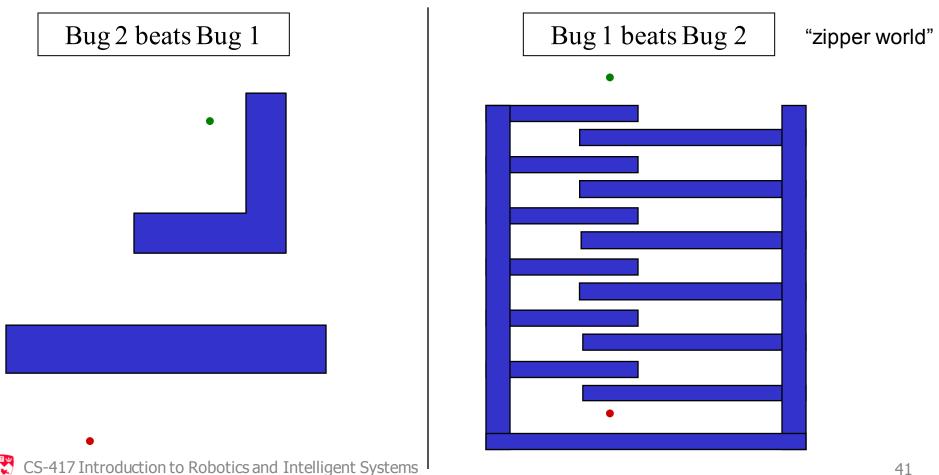


Bug 1 beats Bug 2

### head-to-head comparison

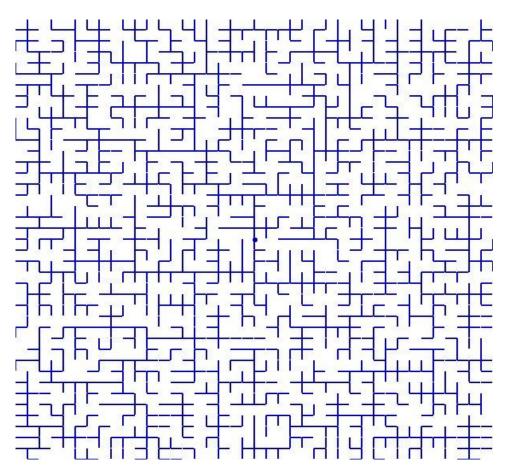
or thorax-to-thorax, perhaps

What are worlds in which Bug 2 does better than Bug 1 (and vice versa)?



# **Other bug-like algorithms**

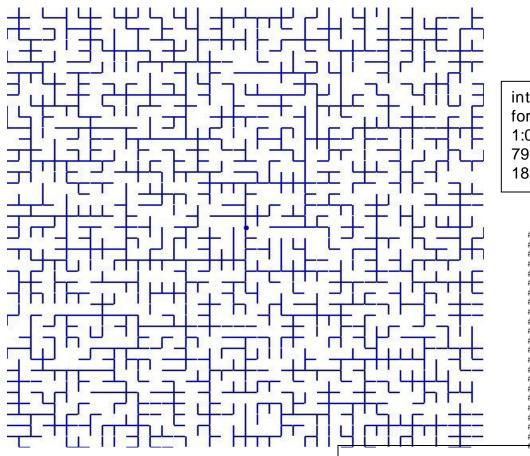
#### The Pledge maze-solving algorithm



- 1. Go to a wall
- 2. Keep the wall on your right
- 3. Continue until out of the maze

# **Other bug-like algorithms**

#### The Pledge maze-solving algorithm



1) Go to a wall

2) Keep the wall on your right

3) Continue until out of the maze

 $int a[1817]; main(z,p,q,r) \{ for(p=80;q+p-80;p=2*a[p]) \\ for(z=9;z--;)q=3&(r=time(0)+r*57)/7,q=q?q-1?q-2?1-p\%79?-1:0:p\%79-77?1:0:p<1659?79:0:p>158?-79:0,q?!a[p+q*2]?a[p+=a[p+=q]=q:0:0;for(;q++1817;)printf(q\%79?"%c":"%c\n"," #"[!a[q-1]]); \}$ 

#### IOCCC random maze generator

|       | #  | ## | ŦĦ      | #Ħ         | ##  |     | • •• |    |    | ## | Ŧ# | ## |    |    |    |     | ## | #1 | ##      | • •• |    |    | ### |     |     | ŦĦŦ |     |    | ### | Ŧ# | ΠΠ |    |     |     |     | Ŧ#  |    |    | ##  | ##        | ### | #### |     |
|-------|----|----|---------|------------|-----|-----|------|----|----|----|----|----|----|----|----|-----|----|----|---------|------|----|----|-----|-----|-----|-----|-----|----|-----|----|----|----|-----|-----|-----|-----|----|----|-----|-----------|-----|------|-----|
| ŧ     |    |    |         |            |     | #   | #    | ## | ŧ  |    |    |    |    | #  | #  | #   |    |    |         |      | #  | #  |     | #   | #   |     | #   | #  |     |    |    | #  |     | ŧ   | ŧ   |     |    | #  |     |           |     | #    | ŧ ; |
| ŧ     | #  | ŧ  | ##:     | ##         | #   | ##  | #    | ## | ## | #  | #  | ## | †# | #  | #  | ##  | 4# | #1 | ##:     | # :  | #  | #  | ##  | ##  | ##  | ##  | #   | ## | 4## | ## | #  | ## | ##  | ŧ # | ŧ 1 | ##  | #  | ## | ### | #         | #   | ###  | ŧ ; |
| ŧ     | #  |    |         |            |     | #   | #    | #  | ŧ  | #  | #  | ŧ  | ŧ  |    |    |     | #  |    |         |      | #  |    |     |     |     | #   | #   | #  |     | #  |    |    | ŧ   | ŧ   |     | į   | #  |    | #   |           | #   | #    | į   |
| ŧ     | #  | ## | ##:     | #          | #   | #   | #    | #  | ŧ  | #  | #  | ŧ  | ŧ# | ## | #  | ##  | 4# | #1 | ¥ ;     | ¥ :  | ## | ## | ##  | ##  | #   | #   | #   | ## | ##  | #  | ## | #  | ##  | ŧ # | ### | ##  | #  | #  | ##  | ##        | ##  | #### | ŧ ; |
| ŧ     | #  |    |         |            | #   |     |      |    |    | #  |    |    |    |    | #  |     |    |    | i       | #    | #  | #  | #   |     | #   |     | #   |    |     |    |    |    | ŧ   | ŧ   |     | ł   | ŧ  | #  | #   |           |     |      | i   |
| ŧ     | #  | ŧ  | ##:     | #          | ##  | ##: | ¥#   | #  | ## | #  | #  | ## | ŧ# | #  | ## | ### | ¥# | 4  | ##:     | ¥ :  | #  | ## | ##  | #   | #   | #   | #   | ## | ##  | #  | ## | ## | + # | ### | ŧ ‡ | ##  | ## | #  | #   | ##        | ### | ###  | ##: |
| ŧ     |    |    | i       | #          | #   |     |      | ŧ  | ŧ  | #  | #  | ŧ  | ŧ  | #  |    |     |    |    | i       | #    | #  |    |     | #   | #   | #   |     | #  | #   | #  | #  |    |     | #   | # # | ŧ   |    |    |     | #         |     | ŧ    | ŧ ; |
| ##    | #  | ## | ##:     | ##         | ŧ#  | #;  | ¥#   | ## | ŧ  | #  | #  | ŧ  | ŧ  | ## | ## | ### | ¥# | #1 | ##:     | ¥ :  | ## | ## | ##  | ### | ### | ### | ### | ## | ##  | ## | #  | ## | ##  | ŧ # | ### | ##  | #  | #  | #   | #         | ##  | ###  | ŧ ; |
| ŧ     |    |    | į       | #          |     |     | #    |    |    |    | #  |    |    |    | #  | #   |    |    | į       | ¥ :  | #  | #  | #   |     |     | #   |     | #  |     |    |    | #  | + # | ŧ # | ŧ 4 | ŧ   | #  | #  | #   | #         | #   |      | į   |
| ##    | ## | ŧ  | ##      | #          | ##  | ##; | ¥#   | #  | ## | ## | ## | ## | ŧ  | ## | #  | #   | #  | 4  | ##      | ÷    | #  | #  | #   | #   | ##  | ¥## | ##  | #  | #   | #  | ## | ## | ÷ # | ŧ # | ŧ i | ŧ   | ## | #  | #   | #         | ##  | +### | ŧ   |
| ŧ     |    |    | į       | #          |     | #   |      | ŧ  | ŧ  |    |    | #  | ŧ  |    |    |     | #  |    | į       | ¥ :  | #  |    | #   | #   |     | ##  | ##  | #  | #   |    |    |    | ŧ   | ŧ   |     |     | #  | #  | #   | #         |     | #    |     |
| ##    | ## | ŧ  | ¥ ;     | ##         | ŧ#  | #   | #    | ## | ## | #  | #  | ## | ŧ  | #  | ## | +#+ | ¥# | 4  | ¥ ;     | ÷    | #  | ## | ;## | ##  | ##  | 4## | ##  | ## | ¥#  | #  | ## | #  | ##  | ### | ### | ŧ   | #  | ## | ;## | ##        | #   | ###  | ŧ   |
| ŧ     | #  | ŧ  | ¥ ;     | #          |     |     | #    |    |    |    | #  |    |    | #  | #  | #   |    | ŧ  | ¥ ;     | ŧ    |    | #  | #   |     |     | #   | #   |    | #   | #  |    | #  |     |     |     | ł   | #  |    |     | #         | #   | ŧ    | ŧ   |
| ŧ     | #  | ŧ  | ¥ ;     | ##         | +#+ | ##: | ¥#   | ## | ## | #  | #  | ## | ŧ  | ## | #  | ##  | ¥# | 4  |         |      | ## | ŧ# | #   | #   | ##  | ##  | #   | ## | +## | ## | ## | #  | ##  | ŧ # | ### | ##  | #  | ## | ŧ#  | ##        | ##  | ###  | ##  |
| ŧ     | #  | #  | ¥.      |            | #   |     |      |    |    |    |    | #  | ŧ  | #  |    |     |    | 1  | ŧ       |      |    | #  |     | #   |     |     |     | #  | #   |    |    |    | #   | ŧ   |     |     | #  | #  |     |           |     | #    |     |
| ŧ     | #  | ł  | ¥ :     | ##         | ##  | #:  | ¥#   | ## | ŧ  | ## | +# | ## | ŧ  | #  | ## | ### | ¥# | #1 | ¥ :     | ¥ :  | ## | ## | ##  | +#+ | +#+ | ##  | #1  | ## | #   | #  | ## | #  | ##  | ### | +#+ | ŧ   | ## | #  | ##  |           | ##  | ###  | ŧ   |
| ŧ     | #  | į  | ÷ ;     | #          |     |     | #    |    |    | #  | #  |    |    | #  |    |     | #  |    |         | ÷.   | #  |    |     |     |     | #   |     |    | #   | #  | #  |    |     |     | 4   | ŧ   | #  |    | #   |           | #   | #    |     |
| ŧ     | #  | 4  | ¥ :     | <br>##     | ##  | #:  | ¥#   | 4  | ŧ# | #  | #  | ŧ  | ŧ  | #  | #  | #1  | ¥# |    | ¥ ;     | ¥ :  | #  | ## | ##  | #   | #1  | +## | +#+ | ## |     |    |    | ## | ##  | ### | ŧ i | ÷ . | ŧ  | #  | #   | #         | #   | ###  | ŧ : |
| ŧ     |    | ł  | ¥       |            |     |     | #    |    |    |    | "  | £  | ŧ  | #  | #  |     |    |    | ;<br>;; |      |    | #  | #   | #   |     |     |     | #  |     | "  |    |    | ±   | ŧ   |     |     | #  | #  | #   | #         |     |      |     |
| ŧ     | #  | ł  | ¥ :     | #          | #   | #:  |      |    | ŧ# | ## | ## | #  |    | ·  | ## | ##  | #  |    |         | ·    |    |    |     | ;## | ##  | ##  | ##  | ## | ##  | #  | #  | #  | . 7 | ### | ŧ 4 |     |    |    | ##  | <i>i#</i> | #   | ###  | . 1 |
| ŧ     | #  | ł  | <br># : | <br>#      | #   | #   |      | ć  |    | #  |    | "  |    |    | ## |     | #  |    |         |      | #  |    |     | #   |     | #   |     |    | #   | #  | #  |    |     |     |     |     |    |    |     |           | #   | #    | 1   |
| r<br> |    |    | τ.      | п<br>11 11 | π.  |     | и.и  | ш. |    |    | ш  | ш  | ш  |    |    |     |    |    |         |      |    |    |     |     |     |     |     |    |     |    |    |    |     |     |     |     |    |    |     |           |     | #### |     |

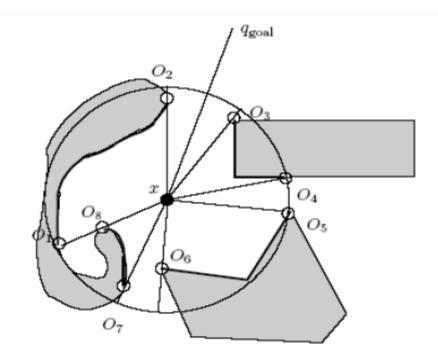
mazes of unusual origin

CS-417 Introduction to Robotics and Intelligent Systems

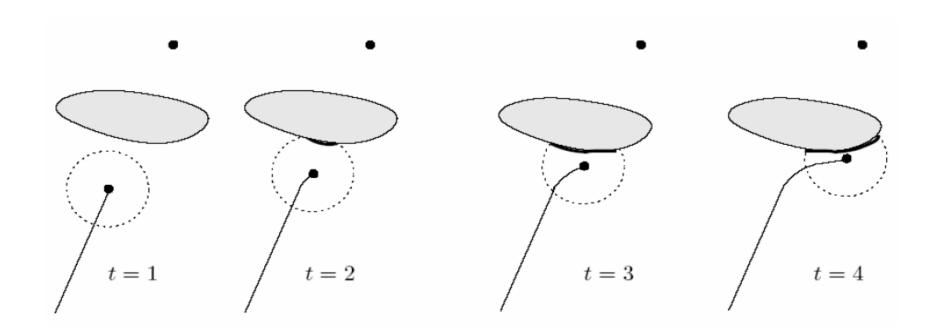
discretized RRT

## **Tangent Bug**

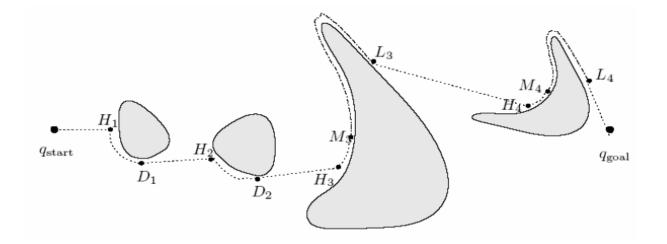
- Limited Range Sensor
- Tangent Bug relies on finding endpoints of finite, continues segments of the obstacles



### **Tangent Bug**



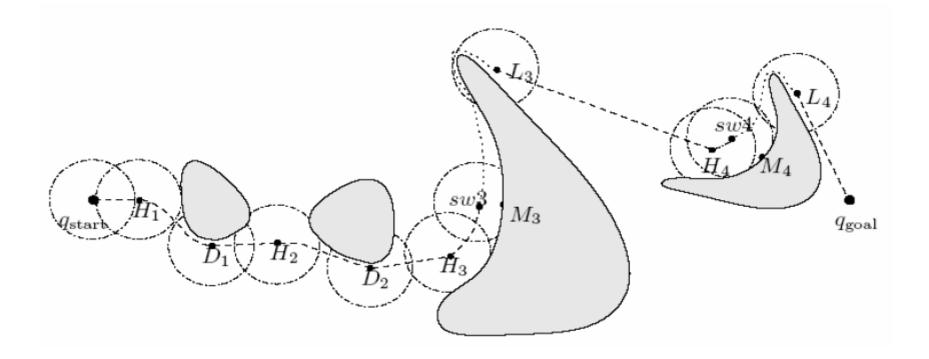
### **Contact Sensor Tangent Bug**



- 1. Robot moves toward goal until it hits obstacle 1 at H1
- 2. Pretend there is an infinitely small sensor range and the direction which minimizes the heuristic is to the right
- 3. Keep following obstacle until robot can go toward obstacle again
- 4. Same situation with second obstacle
- 5. At third obstacle, the robot turned left until it could not increase heuristic
- 6. D\_followed is distance between M3 and goal, d\_reach is distance between robot and goal because sensing distance is zero

CS-417 Introduction to Robotics and Intelligent Systems

### **Limited Sensor Range Tangent-Bug**





### **Infinite Sensor Range Tangent Bug**

