Burmester Problem

The module deals with the Burmester problem, which aims at finding the geometric parameters of a four-bar linkage required for a prescribed set of finitely separated poses [1]. It is well-known that a revolute-revolute (RR) dyad can be synthesized exactly for up to five prescribed poses. The synthesis problem discussed here pertains to both four and five poses.

The four-pose problem is known to admit infinitely many solutions, each solution dyad being given by a pair of corresponding cubics, the *centrepoint* and the *circlepoint* curves. The five-pose problem, on the other hand, leads to a quartic equation, which is most conveniently solved numerically—the closed form solution of the quartic can be found using Ferrari's formula, but this is too cumbersome to be of ay use. Extensive research has been reported on the solution of the Burmester problem with different approaches. Bottema and Roth [2], McCarthy [3] and Hunt [4] solved the problem by intersecting two centrepoint curves of two four-pose problems, for two four-poses subsets out of the given five-pose set, to obtain the centrepoints. Beyer [5] and Lichtenheldt [6] reported a method based on projective geometry, while Modler [7, 8, 9, 10] investigated various special cases. Al-Widyan and Angeles [11] developed a robust algorithm to synthesize four-bar linkages, in which circlepoints and centrepoints were found through the intersections of the four possible contours of the four-pose problems. Sandor and Erdman applied complex numbers [12], while Ravani and Roth [13], Hayes and Paul [14] solved the problem via the kinematic mapping. Of all works cited, most focus on revolute-revolute (RR) dyads, few investigating the problem of prismatic-revolute (PR) dyads. To the authors' knowledge, the problem of four-bar linkage synthesis leading to PR dyads is not included in commercially available design software, such as LINCAGES [15].

In view of this, a planar four-bar linkage synthesis method addressing the problem of the determination of PR dyads is desirable. Such a method should detect the presence of PR dyads within the prescribed set of poses, and determine either the PR or the RR dyads composing the desired linkage.

Problem Formulation

The Burmester problem reads: A rigid body, attached to the coupler link of the four-bar linkage, as shown in Fig. 1a, is to be guided through a discrete set of m poses, given by $\{\mathbf{r}_j,\ \theta_j\}_0^m$, where $\mathbf{r}_j$ is the position vector of a landmark point R of the body at the jth pose and θ_j is the corresponding angle of a line of the body, as depicted in Fig. 2. The problem consists in finding the joint centers A_0 and B that define the BA_0R *dyad* of the guiding four-bar linkage, dyad $B^*A_0^*R$ being determined likewise. Given that A_0 and A_0^* describe circles centred at B and B^*, respectively, the former are termed the *circle points*, the latter the *centre points* of the dyads.

If a PR dyad exists for the prescribed poses, as depicted in Fig. 1b, the centrepoint B^* is located at infinity. For $m = 3, 4$, a discrete set of circlepoint solutions exists, besides the direction corresponding to the prismatic joint. For RR dyads, we have:

- In the case of $m = 3$, the problem leads to a system of three algebraic equations in four unknowns, the coordinates of points A_0 and B. Hence, infinitely many solutions are available; these solutions defining two related loci: the *centrepoint* and the *circlepoint* (cubic) curves.
- In the case of $m = 4$, a system of four algebraic equations exists for four unknowns, the coordinates of points A_0 and B. The problem is thus determined, but nonlinear, and admits up to four different solutions, i.e., four different dyads. The pairwise combination of these dyads then leads to up to six distinct linkages.

Within the module, we developed a general synthesis method for five-pose synthesis, applicable to problems admitting either RR or PR dyads.

The relevant background material is available in two publications:

1. Al-Widyan, K., Angeles, J., and Cervantes-Sánchez, J. J., 2002. "A numerical roboust algorithm to solve the five-pose burmester problem". *Proc. of DETC'2002*, MECH–34270.
2. Angeles, J., and S.P. Bai, 2005. "Some Special Cases of The Burmester Problem For Four and Five Poses". *Proc. of DETC'2005*, MECH–84871, to appear

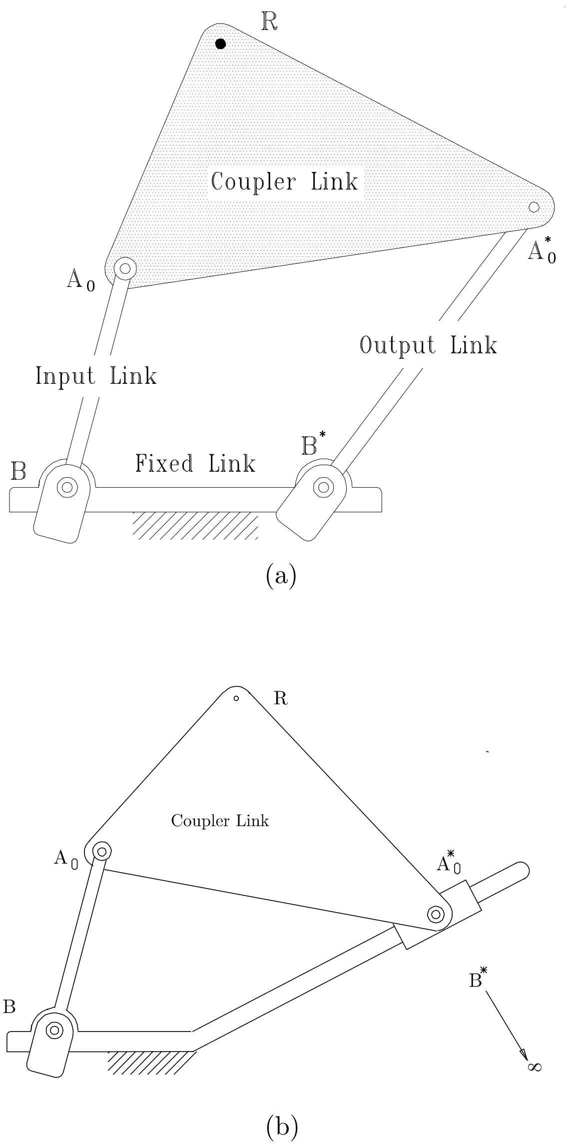

Figure 1: Four-bar linkages with: (a) Revolute-Revolute dyads only; and (b) one Prismatic-Revolute dyad

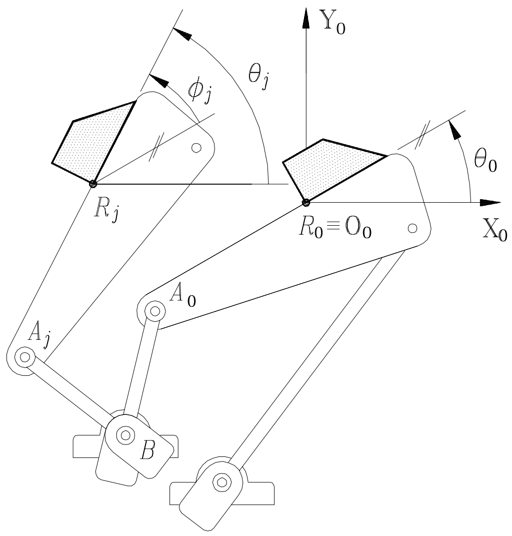

Figure 2: Two finitely-separated poses of a rigid body carried by the coupler link of a four-bar linkage

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