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THE DESIGN OF A NOVEL MECHANICAL TRANSMISSION FOR SPEED REDUCTION

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ABSTRACT

The design of a novel transmission for speed reduction is reported in this paper. The transmission is based on the layout of *pure-rolling* indexing cam mechanisms, for it is intended to eliminate backlash and friction, which are the main drawbacks of gear transmissions. The design relies on a unified methodology for indexing cam mechanisms, whereby the same procedure is used to design planar, spherical, and spatial mechanisms. Two versions of the speed reducer, that we term Speed-o-Cam, are described here, namely, planar, to couple shafts of parallel axes, and spherical, to couple shafts of intersecting axes. By means of a cascade of two planar versions of Speed-o-Cam, moreover, coaxial shafts can be coupled. Moreover, contrary to other alternatives to gear transmissions, such as harmonic drives or epicyclic transmissions, Speed-o-Cam caters to shafts with either parallel or intersecting axes; in the latter case, moreover, the angle at which the axes intersect can vary from 0° to 180° continuously. The design with shaft axes at 0° leads to an *internal cam*, i.e., one with its axis external to the axes of the follower and the roller in contact; for shaft axes at 180°, the design leads to an *external cam*, i.e., one with its axis lying between those of the follower and the roller in contact.

INTRODUCTION

The technology of gear-train manufacturing, production, and utilization is well established (Dudley, 1962). However, most recent technological developments in the realm of gear transmissions pertain to materials and manufacturing processes that have led to highly accurate machined surfaces. On the contrary, the basic principle of gear transmission has remained virtually untouched, since the advent of the involute tooth. Indeed, only recently have fundamental developments been reported (Phillips, 1994; 1999) in connection with gear-tooth geometry. To be sure, developments in the geometric modelling of tooth shapes have been achieved (Tsai and Chin, 1987; Litvin, Hsiao, Wang, and Zhou, 1994), that should lead to more efficient production methods and gear-transmission operations. In spite of this progress, however, the inherent drawbacks of gear transmissions still remain, which has prompted the development of alternative transmission devices such as the *harmonic drive* and the *direct drive*. Harmonic drives (Tuttle and Seering, 1993) consist of a flexible element, called the *flexspline*, that has the form of an elliptic gear, and meshes with an internal gear called the *wave generator*. The flexspline allows for a backlash-free operation, but its flexibility brings about inherent kinematic errors, besides hysteretic dynamical effects due to the viscoelasticity of the flexible materials involved. Direct drives, on the other hand, require extremely massive motors for the kind of low speeds at which they are usually required to operate (Asada and Youcef-Toumi, 1987).

Upon considering the foregoing alternatives to gears, along with their drawbacks, we decided to try to replace gear trains with cam mechanisms. In the methodology that we adopted, only line contact is considered, for this methodology is based on the three-dimensional version of the Aronhold-Kennedy Theorem, as described below. Furthermore, the cam mechanisms at the core of the family of speed reductions under discussion are of the oscillating roller-follower type. A unified methodology on the synthesis of cam mechanisms was introduced in González-Palacios and Angeles (1993). This methodology was exploited extensively to develop the software package USyCaMs (González-Palacios and Angeles, 1997), which proved an invaluable tool in the design of the transmissions reported in this paper.

In USyCaMs we introduced a displacement program called the *cycloidal-modified*, which can be varied smoothly from cycloidal to linear motion with the aid of a coefficient that changes from one to zero. The results obtained with this function motivated us to investigate the possibility of transmitting a *linear*—in the algebraic sense—displacement program as needed in speed reducers.

CAM-SYNTHESIS METHODOLOGY

The speed reducer introduced here, Speed-o-Cam, has the morphology of an indexing cam mechanism (ICM), i.e., a cam mechanism whose follower bears a *periodic* geometry, repeating itself N times. The mechanism is thus said to have N *stages* or *indexing steps*. An ICM is designed so as to allow the production of a periodic, nonreversing speed of the follower when the cam rotates at a constant angular speed.

While Speed-o-Cam stems from the concept of ICM, its distinguishing feature is the type of periodic speed produced on the follower, namely, a constant speed that is a multiple or a submultiple of the cam speed. As a matter of fact, when Speed-o-Cam is used to actually reduce speed—the mechanism is reversible, just as gear transmissions are—the speed-reduction ratio is $1/N$, for an N-stage layout. When Speed-o-Cam is used as a speed amplifier, the speed-amplification ratio is N.

The procedure adopted to synthesize Speed-o-Cam is an instance of the unified methodology developed by the authors to synthesize ICM (González-Palacios and Angeles, 1993). In fact, simple cam mechanisms can also be synthesized with this methodology, while taking into account that these produce a reversing speed, different from ICM. We outline below the synthesis procedure leading to Speed-o-Cam in its two versions.

Within the framework of the methodology under description, we are interested in the synthesis of mechanisms composed of four links, as mentioned above, whereby the frame, the cam, the follower, and the roller are numbered from 1 to 4, in this order. Moreover, the cam and the follower are coupled to the frame via revolute pairs, the roller in the most general case of skew axes of rotation of the cam and the follower being coupled to the follower via a cylindrical pair and to the cam via a higher pair. The contact surface between the cam and the roller is a general ruled surface on the cam and a one-sheet hyperboloid of revolution on the roller (González-Palacios and Angeles, 1993). Obviously, in the case of planar mechanisms, the roller surface is a cylinder and in the case of spherical mechanisms it is a cone. In both cases, the roller is coupled to the follower by means of a revolute pair.

The cam surface of contact is determined by the position vector $\mathbf{r}$ defining a ruled surface, which is given by

$$\mathbf{r}(\psi,\lambda) = \mathbf{p}(\psi) + \lambda\mathbf{e}(\psi) \tag{1}$$

where ψ and λ are the parameters of the ruled surface.

Furthermore, the geometric parameters defining the cam mechanism are

1. The distance a_1 between the input and output axes;
2. the angle α_1 between the above two axes;
3. the distance a_3 between the output and roller axes;
4. the angle α_3 between the above two axes;
5. the distance a_4 between the roller axis and its generatrix; and
6. the angle α_4 between the above two axes.

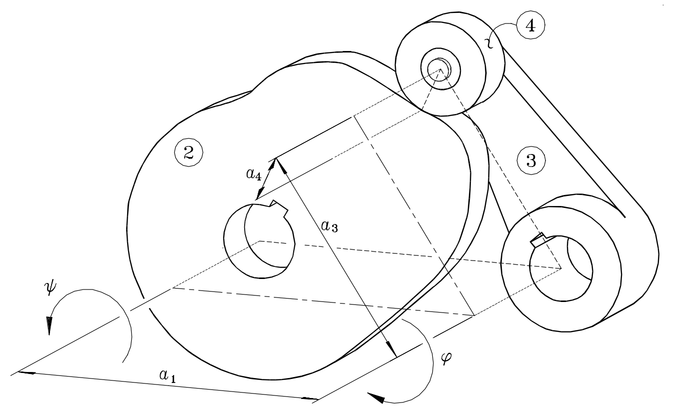


Figure 1. The general layout of a planar cam mechanism with a roller-follower

Note that, in general, the above parameters need not be constant; some are, in fact, functions of the angle of rotation of the cam. Furthermore, the above parameters, as well as the contact surface on the cam and the roller hyperboloid of revolution, are determined by both the geometric relations dictated by the Aronhold-Kennedy Theorem in three-dimensional space (Beggs, 1959; Phillips and Hunt, 1964; Beggs, 1966) and the relation between the input and the output angles, ψ and ϕ, respectively. In addition, the aforementioned methodology is applicable to indexing mechanisms, which requires the specification of the number N of indexing steps per turn of the cam. In the case at hand, the input-output relation is simply given by

$$\phi = \frac{1}{N}\psi \tag{2}$$

and hence, the speed reduction is the reciprocal of N, and is given by

$$\phi'(\psi) = \frac{1}{N} \tag{3}$$

A PLANAR SPEED REDUCER

In the discussion below we will make reference to the layout displayed in Fig. 1, which shows a general planar cam mechanism with a roller follower. In that figure, angle ψ is the input variable, angle $\phi = \phi(\psi)$ being the output variable, a prescribed function of ψ. Based on eq.(1), the position vectors defining the contact surface of the cam for the planar Speed-o-Cam is given as a special case of the general synthesis equations derived in (González-Palacios and Angeles, 1993).

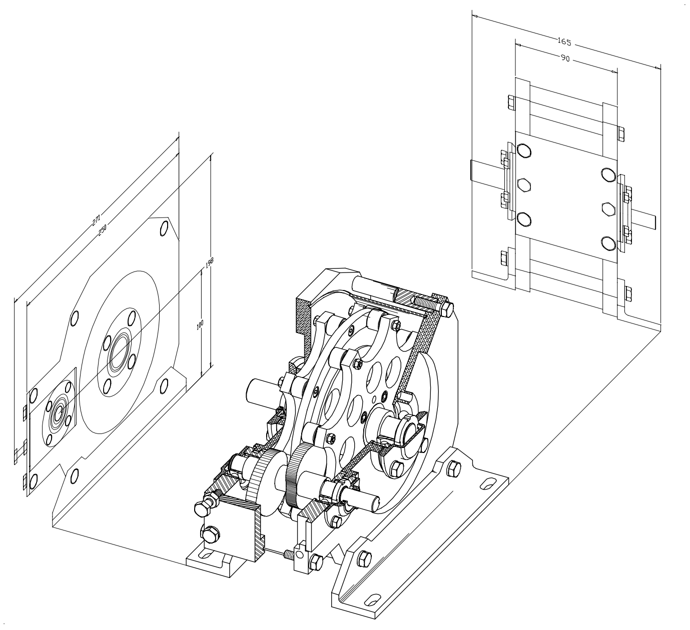


Figure 2. Assembly of the Speed-o-Cam planar prototype

If we substitute angle ϕ as given by eq.(3) in those equations, the position vector $\mathbf{r}_c$ of the contact point, for a given value of ψ, becomes

$$\mathbf{r}_c = \begin{bmatrix} k_1 \cos\psi + (k_2 - a_4)\cos(\psi - k_3) \\ -k_1 \sin\psi - (k_2 - a_4)\sin(\psi - k_3) \\ \lambda \end{bmatrix} \tag{4}$$

where coefficients k_i, for $i = 1, 2, 3$, are defined as

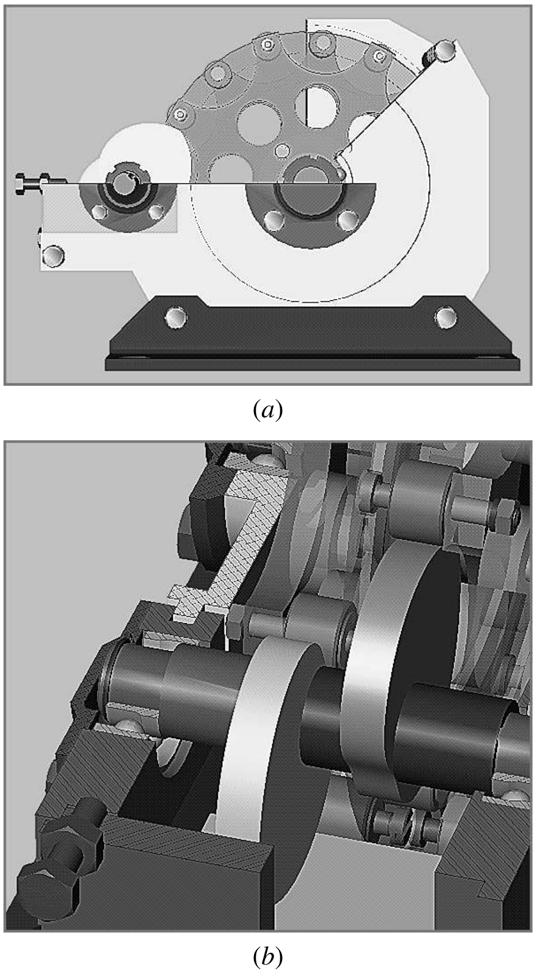
$$k_1 = \frac{a_1}{1+N} \tag{5a}$$

$$k_2 = \sqrt{(a_3\cos\phi + a_1 - k_1)^2 + a_3{}^2 \sin^2\phi} \tag{5b}$$

$$k_3 = \arctan\left(\frac{a_3 \sin\phi}{a_3\cos\phi + a_1 - k_1}\right) \tag{5c}$$

and, to avoid undercutting, a_3 should be bounded as (González-Palacios and Angeles, 1993)

$$a_3 < \frac{a_1}{1 + 1/N} \tag{5d}$$



(a)

(b)

Figure 3. Planar Speed-o-Cam (a) Front and (b) generic views

An isometric view of the planar Speed-o-Cam prototype with two bidimensional projections is shown in Fig. 2, while in Fig. 3 two still frames of its solid model are displayed.

A SPHERICAL SPEED REDUCER

Similar to Fig. 1, Fig. 4 shows the general layout of a spherical cam mechanism with a roller-follower. While this element is cylindrical in the planar case, in the case at hand the roller is conical.

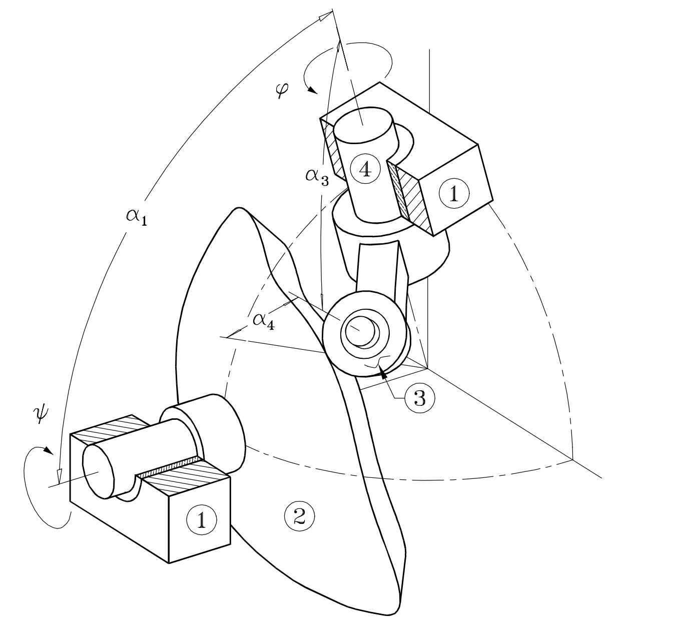


Figure 4. The general layout of a spherical cam mechanism with a roller-follower

According to eq.(1), the position vectors defining the contact surface of the cam for the spherical Speed-o-Cam is given as

$$\mathbf{r}_c = \lambda \begin{bmatrix} -k_7 \sin\psi + \sin(k_5 - \alpha_4) \sin k_6 \cos\psi \\ -k_7 \cos\psi - \sin(k_5 - \alpha_4) \sin k_6 \sin\psi \\ \cos k_1 \cos(k_5 - \alpha_4) - \sin k_1 \sin(k_5 - \alpha_4) \cos k_6 \end{bmatrix} \tag{6}$$

with coefficients k_i, for $i = 1, \ldots, 7$, defined as

$$k_1 = \arctan\left(\frac{\sin\alpha_1}{\cos\alpha_1 - N}\right) \tag{7a}$$

$$k_2 = \cos(\alpha_1 - k_1)\cos\phi \sin\alpha_3 + \cos\alpha_3 \sin(\alpha_1 - k_1) \tag{7b}$$

$$k_3 = \sin\alpha_3 \sin\phi \tag{7c}$$

$$k_4 = \cos\alpha_3 \cos(\alpha_1 - k_1) - \cos\phi \sin\alpha_3 \sin(\alpha_1 - k_1) \tag{7d}$$

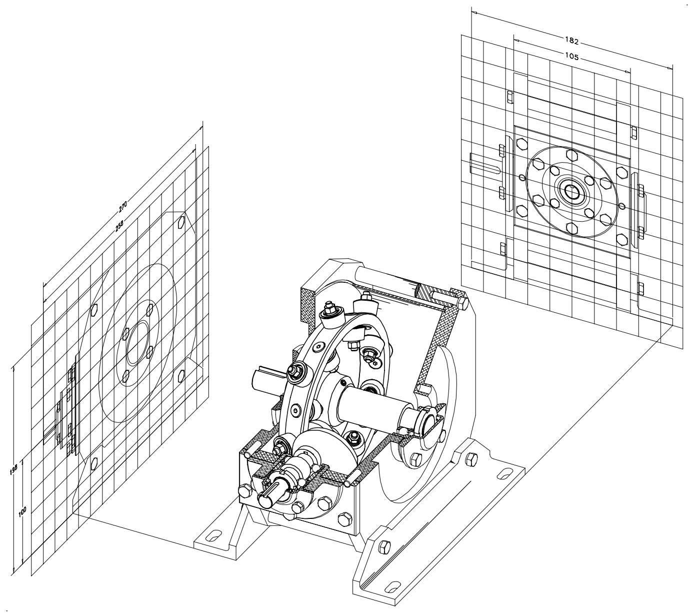


Figure 5. Assembly of the spherical prototype

$$k_5 = \arctan\left(\frac{\sqrt{k_2^2 + k_3^2}}{k_4}\right) \tag{7e}$$

$$k_6 = \arctan\left(\frac{k_3}{k_2}\right) \tag{7f}$$

$$k_7 = \sin k_1 \cos(k_5 - \alpha_4) + \cos k_1 \sin(k_5 - \alpha_4) \cos k_6 \tag{7g}$$

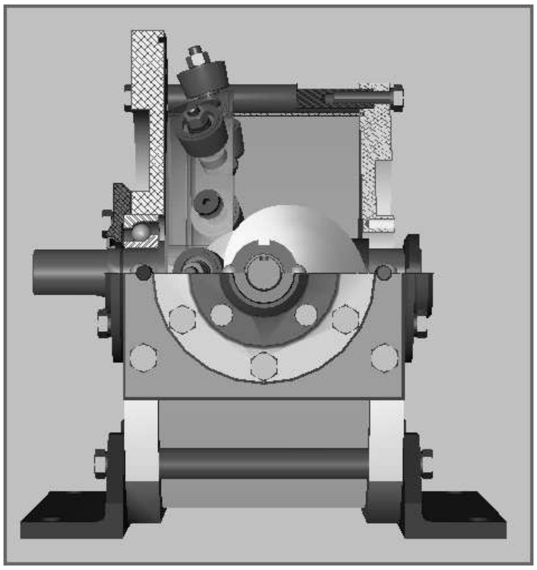
and, to avoid undercutting, α_3 should be bounded as (González-Palacios and Angeles, 1993)

$$\alpha_3 < \arctan\left(\frac{\sin\alpha_1}{\cos\alpha_1 + 1/N}\right) \tag{7h}$$

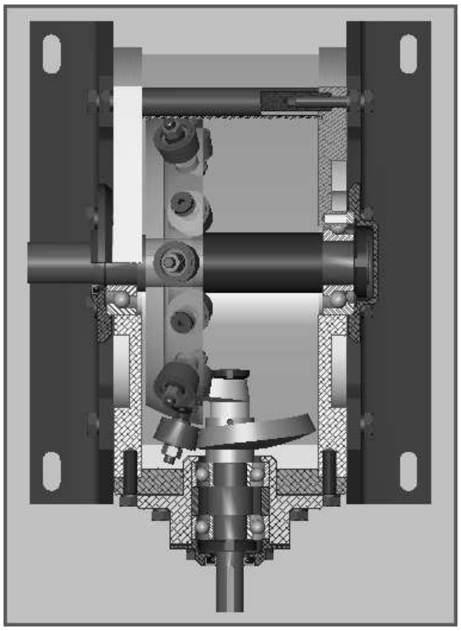
An isometric view of the spherical Speed-o-Cam prototype with two bidimensional projections is shown in Fig. 5, while in Figs. 6 and 7 four still frames of its solid model are shown.

THE PRESSURE ANGLE

One means of measuring the quality of the force transmission in mechanisms involving higher pairs is the pressure angle μ. The pressure angle is defined as that between the direction of the force applied onto the follower and the direction of the velocity vector of the follower-point of application. The expressions for the



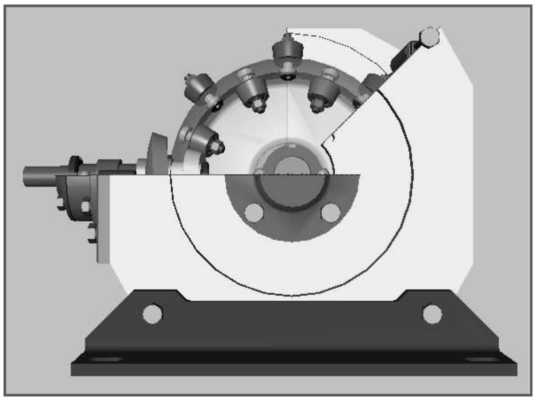
(a)



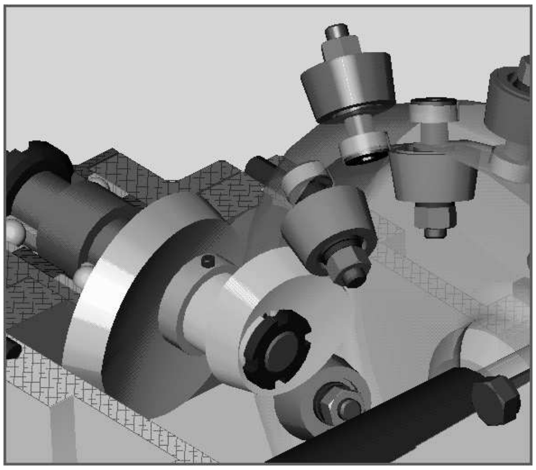
(b)

Figure 6. a) Side and b) top solid-model views of the spherical Speed-o-Cam

pressure angle of the planar and spherical Speed-o-Cam are derived from a generalized formula derived in (González-Palacios and Angeles, 1993). Thus, for the planar Speed-o-Cam the expression of the pressure angle is



(a)



(b)

Figure 7. a) Front and b) generic solid-model views of the spherical Speed-o-Cam

$$\tan\mu = \frac{a_3(\phi' - 1) - a_1\cos(\phi)}{a_1\sin(\phi)} \tag{8}$$

while, for the spherical Speed-o-Cam, we have

$$\tan\mu = \frac{(\phi' - \cos\alpha_1)\sin\alpha_3 - \sin\alpha_1\cos\alpha_3\cos\phi}{\sin\alpha_1\sin\phi} \tag{9}$$

In the design of cam mechanisms, we distinguish two kinds of actuating forces at the contact between the cam and the follower, namely, the force that transmits the motion to the follower and the force that opposes this motion. The action of each of these forces is termed *positive action* (PA) and *negative action* (NA), respectively. Moreover, we call *positive motion* that under which

Table 1. Limiting values of the pressure angle

N	Planar		Spherical	
	μ_{min}	μ_{max}	μ_{min}	μ_{max}
	degrees			
4	−24.9	12.9	−6.2	8.4
5	−17.9	18.6	−2.5	12.0
6	−12.8	18.6	−0.2	15.2
8	−5.6	32.0	2.7	20.5
10	−0.5	38.8	4.6	25.3
12	3.6	44.5	6.4	30.1
16	10.3	53.1	9.5	38.8
20	15.8	59.2	12.4	45.3
40	35.6	73.6	24.4	64.1
100	62.1	83.3	48.5	78.7

both PA and NA are present in the transmission. Because we designed Speed-o-Cam with positive motion, we have two cams attached to the input shaft and two followers attached to the output shaft.

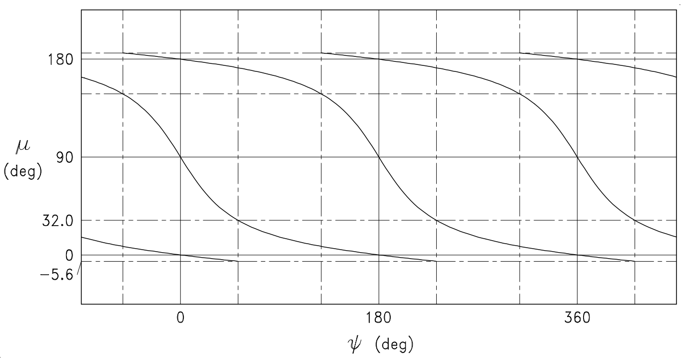


Figure 8. Pressure angle for a planar Speed-o-Cam with $N = 8$

Based on the definition of the pressure angle, it is desired then, to keep the pressure angle as close as possible to zero for PA and to 180° for NA. The pressure-angle distribution for the planar Speed-o-Cam with $N = 8$ is shown in Fig. 8, where we notice that the minimum and maximum values of the pressure angle for PA are $-5.6°$ and $32.0°$, respectively. Similarly, the pressure angle distribution for the spherical Speed-o-Cam, is shown in Fig 9. In these figures, the pressure angle appears as a double-valued relation, the reason being that we have, in fact, two values of the pressure angle under the layout adopted: one for each of the two conjugate cams.

Furthermore, in order to visualize the variation of the pressure angle with respect to N, the bounds of the pressure angle for PA are shown in Table 1. In that table, we used the values of the geometric parameters of the mechanisms that lead to maximum values of pressure angle with the limiting values of a_3 and α_3 from eqs.(5d & 7h), respectively. That is, for the tabulated values of N, we used

$$\frac{a_3}{a_1} = \frac{1}{1+N}, \quad \alpha_1 = 90°, \quad \alpha_3 = \arctan\left(\frac{1}{1/N}\right) = \arctan(N) \tag{10}$$

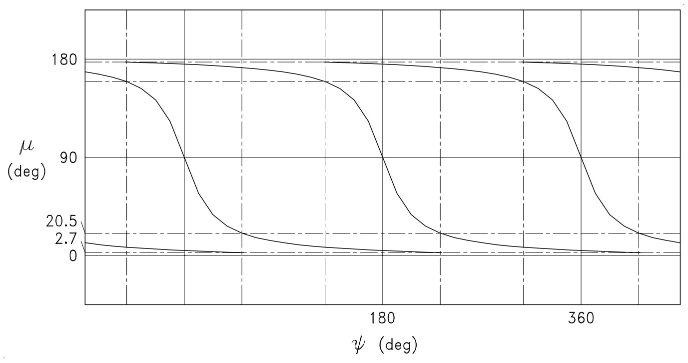


Figure 9. Pressure angle for a spherical Speed-o-Cam with $N = 8$ and $\alpha_1 = 90°$

PRACTICAL ASPECTS

The number N that gives the speed-reduction ratio as $1/N$ is limited by the pressure angle that a specific application can tolerate. We bounded the pressure angle for PA within $\pm 35°$. Thus, for the planar Speed-o-Cam, $N = 8$ is the maximum value allowed, whereas for the spherical Speed-o-Cam, with $\alpha_1 = 90°$, N can go up to 12. Note that the values for a_3 and α_3 were defined close to the bounds given in expressions (5d) and (7h), respectively. Moreover, reversibility implies that Speed-o-Cam can be used for any application where gears are currently used. Therefore, upon reversing the mechanism, a speed amplifier is obtained.

The mounting of the input shaft—the high-speed shaft in Figs. 2 and 5—deserves special mention, for it was designed for assembly: the special mounting, shown in Fig. 10 for the planar Speed-o-Cam prototype allows for submillimetric translations to compensate for angular offsets in the planar version and for linear—in the geometric sense—offsets of this shaft with respect to the output shaft in the spherical version.

The low friction in Speed-o-Cam is to be highlighted as a major advantage of this transmission: low friction means that the life of a speed reducer with plastic elements can be greatly increased if it is based on Speed-o-Cam. The alternative would be plastic gears, which are normally of the spur type, and hence, exposed to friction and, as a consequence, to excessive wear. Friction losses, of course, cannot be completely eliminated. These are still present in Speed-o-Cam, but confined to the rollers mounted on the follower.

Moreover, although Speed-o-Cam does not allow for coaxial shafts in one reduction step, the cascading of two such steps can readily accommodate this type of shaft layout.

When comparing Speed-o-Cam with gears, notice that the former requires more moving parts than a gear train, which undeniably affects the cost of Speed-o-Cam. However, notice that, contrary to gears, which require special machine tools for their cutting, Speed-o-Cam can be machined with general-purpose CNC machine tools. One more item for comparison is the structural stiffness of Speed-o-Cam vs. that of gear trains: a gear is as stiff as the stiffest of its teeth, while the stiffness of Speed-o-Cam is enhanced by the essentially convex shape of its cams, the flexibility of Speed-o-Cam thus being confined to the roller bearings on the follower. In order to enhance the stiffness of the planar prototype of Speed-o-Cam, we added two supporting plates to the roller-carrying disk, as shown in Figs, 2, 3, and 11–13.

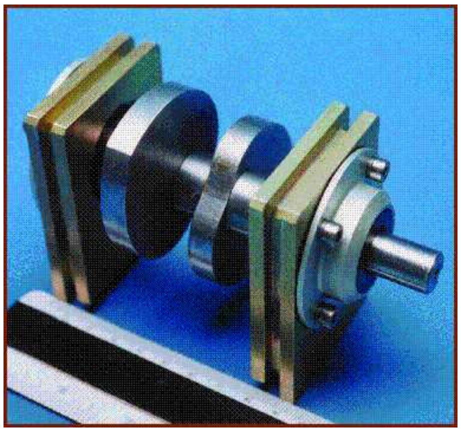


Figure 10. The shaft carrying the conjugate cams

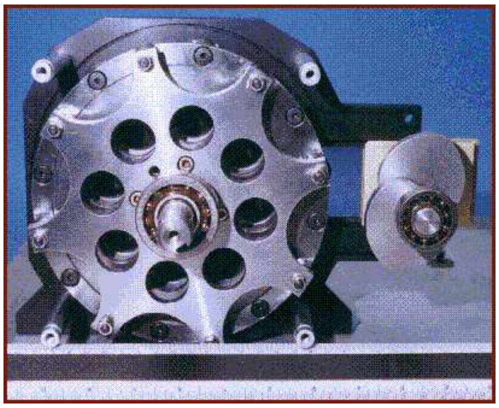


Figure 11. Roller-carrier and conjugate-cam shaft upon assembling

CONCLUSIONS

We introduced here a novel mechanical transmission, Speed-o-Cam, for speed reduction. The morphology of this transmission stems from that of pure-rolling indexing cam mechanisms, that was proposed by the authors elsewhere (González-Palacios and Angeles, 1993). Hence, Speed-o-Cam is aimed at the elimination of backlash and friction. Currently we are conducting tests on the planar Speed-o-Cam that will allow us to quantify how this transmission performs in these respects.

Finally, Speed-o-Cam caters to parallel, intersecting, and coaxial shafts. We have built one planar prototype of Speed-o-Cam, photographs of which are shown in Figs. 10—13; a second prototype, of the spherical type, is currently under production.

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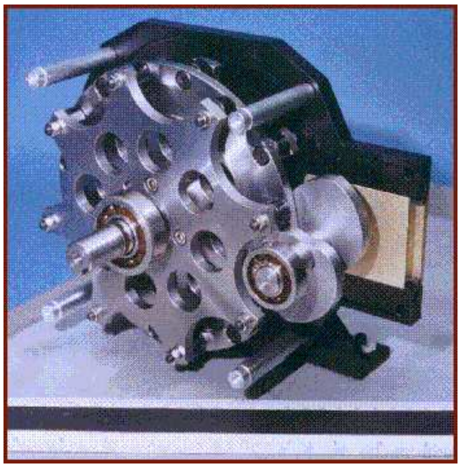

Figure 12. Assembled roller-carrier and conjugate-cam shaft

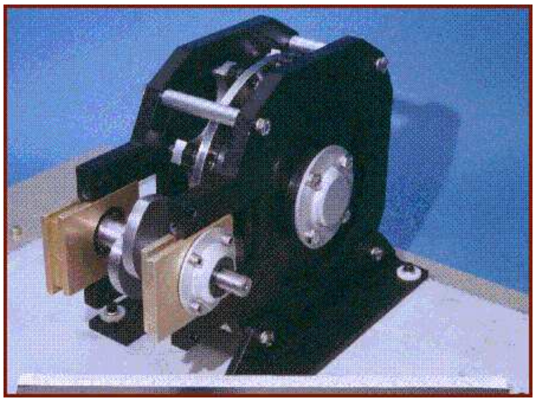

Figure 13. Assembled planar Speed-o-Cam prototype

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