The Kinematics and Dynamics of Parallel Schönflies-Motion Generators

Jorge Angeles

Department of Mechanical Engineering
Centre for Intelligent Machines
McGill University
Montreal, QC, Canada

What is a Schönflies Motion?

(SMG video)

Objectives

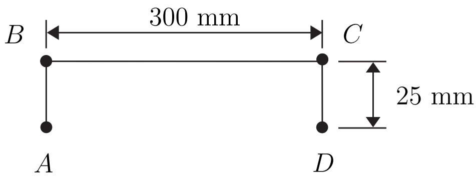
- Design a parallel manipulator, which outperforms current SMG (serial and parallel SCARA)
- Pick and place operations and machining

Objectives

- Design a parallel manipulator, which outperforms current SMG (serial and parallel SCARA)
- Pick and place operations and machining

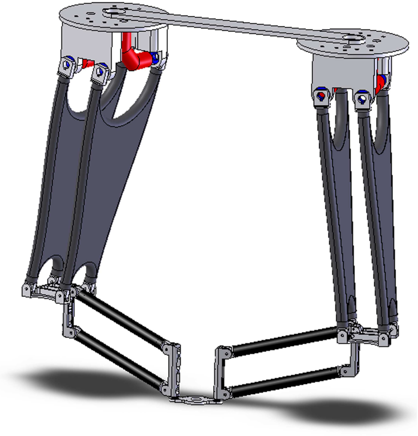
Specifications

- The robot is expected to beat the record-setting 500-ms cycle time

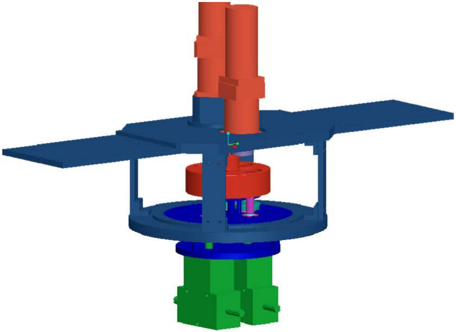


- Kinetostatic robustness
- Identical motors fixed to the base
- Large workspace
- High stiffness

Manipulator Architecture

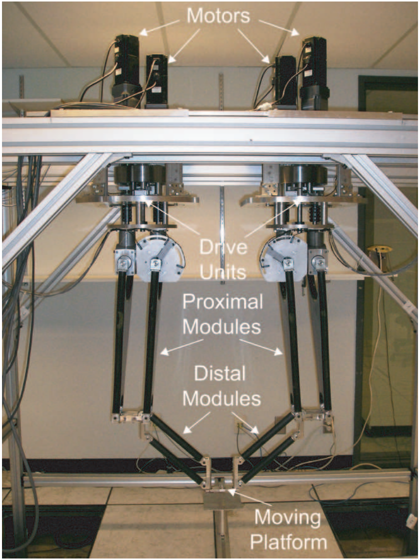


$2R\Pi\Pi R$



Drive Unit

Prototype


Motors
Drive Units
Proximal Modules
Distal Modules
Moving Platform

Animation

(SMG video)

Kinetostatic Design

Kinetostatics

- Mechanical analysis of rigid-body mechanical systems moving under static, conservative conditions
- Relations between the feasible twists—point-velocity and angular velocity—and the constraint wrenches—force and moment—pertaining to the various links of a kinematic chain
- Kinetostatic design $\equiv$ dimensioning of the links under kinetostatic conditions

Kinetostatic Design

Kinetostatics

- Mechanical analysis of rigid-body mechanical systems moving under static, conservative conditions
- Relations between the feasible twists—point-velocity and angular velocity—and the constraint wrenches—force and moment—pertaining to the various links of a kinematic chain
- Kinetostatic design $\equiv$ dimensioning of the links under kinetostatic conditions

Kinetostatic Design

Kinetostatics

- Mechanical analysis of rigid-body mechanical systems moving under static, conservative conditions
- Relations between the feasible twists—point-velocity and angular velocity—and the constraint wrenches—force and moment—pertaining to the various links of a kinematic chain
- Kinetostatic design $\equiv$ dimensioning of the links under kinetostatic conditions

Jacobian matrices

$$\mathbf{At} = \mathbf{B}\dot{\theta} \tag{1}$$

- $\mathbf{t}$ is the twist
- $\dot{\theta}$ is the vector of active joint rates
- $\mathbf{A}$ is the forward Jacobian
- $\mathbf{B}$ is the inverse Jacobian

$$\mathbf{A} \equiv \begin{bmatrix} \mathbf{A}_I \\ \mathbf{A}_{II} \end{bmatrix} \in \mathbb{R}^{6\times 4}, \quad \mathbf{B} \equiv \begin{bmatrix} \mathbf{B}_I & \mathbf{O}_{32} \\ \mathbf{O}_{32} & \mathbf{B}_{II} \end{bmatrix} \in \mathbb{R}^{6\times 4} \tag{2}$$

- Redundancy is introduced to add robustness to the model and to avoid formulation singularities

Jacobian matrices

$$\mathbf{At} = \mathbf{B}\dot{\theta} \tag{1}$$

- $\mathbf{t}$ is the twist
- $\dot{\theta}$ is the vector of active joint rates
- $\mathbf{A}$ is the forward Jacobian
- $\mathbf{B}$ is the inverse Jacobian

$$\mathbf{A} \equiv \begin{bmatrix} \mathbf{A}_I \\ \mathbf{A}_{II} \end{bmatrix} \in \mathbb{R}^{6\times 4}, \quad \mathbf{B} \equiv \begin{bmatrix} \mathbf{B}_I & \mathbf{O}_{32} \\ \mathbf{O}_{32} & \mathbf{B}_{II} \end{bmatrix} \in \mathbb{R}^{6\times 4} \tag{2}$$

- Redundancy is introduced to add robustness to the model and to avoid formulation singularities

Singularity Analysis

Two types of singularities

- Rank($\mathbf{A}$) $< 4 \Rightarrow$ parallel singularity
- Rank($\mathbf{B}$) $< 4 \Rightarrow$ serial singularity

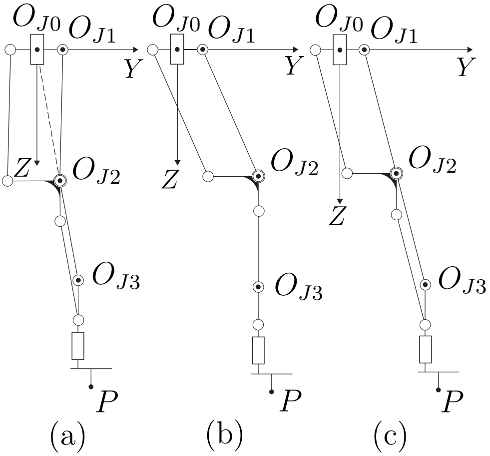
Singularity Analysis

Two types of singularities

- Rank(**A**) $< 4 \Rightarrow$ parallel singularity
- Rank(**B**) $< 4 \Rightarrow$ serial singularity

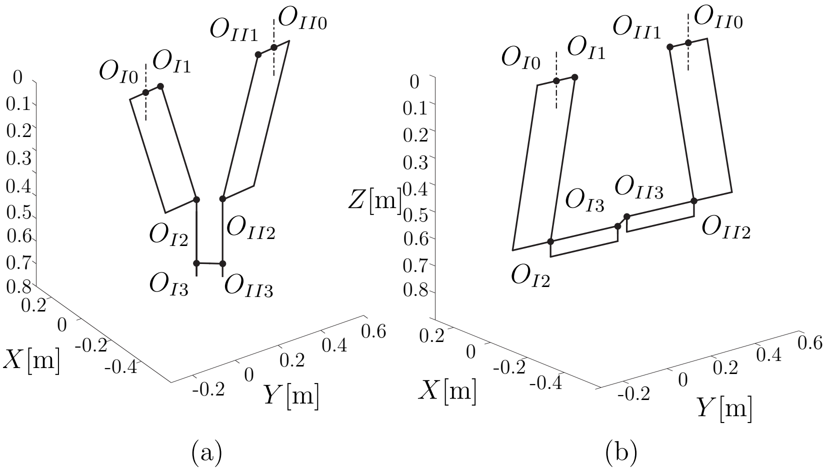
Singularity Analysis

Serial singularities

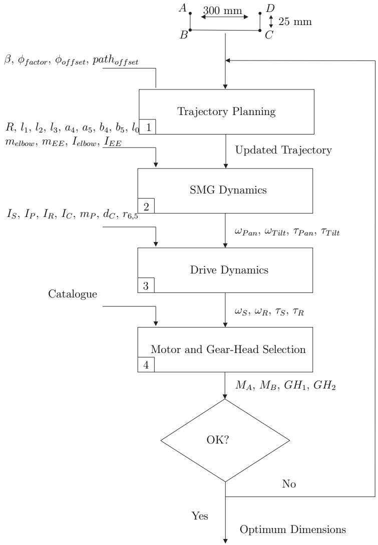


Singularity Analysis

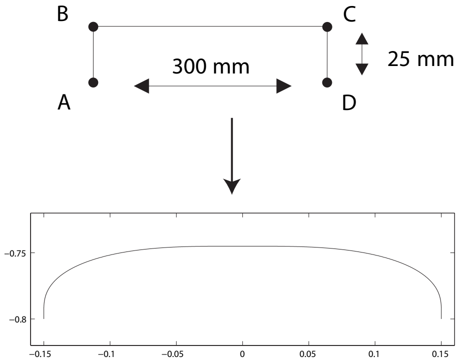
Parallel singularities



Dimensioning

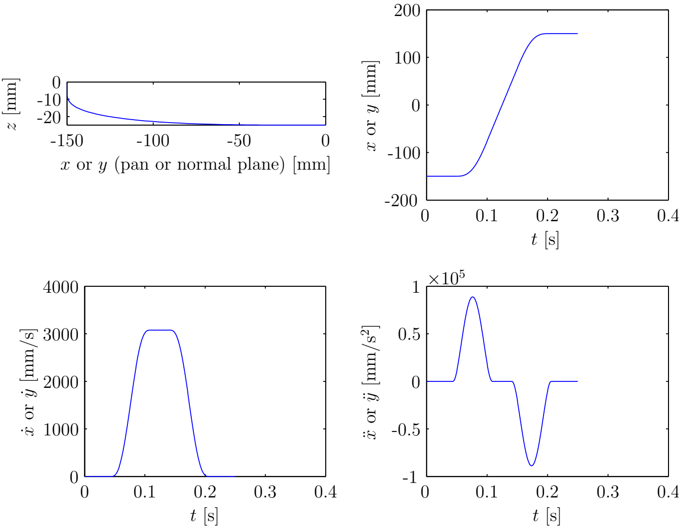


Trajectory Optimization

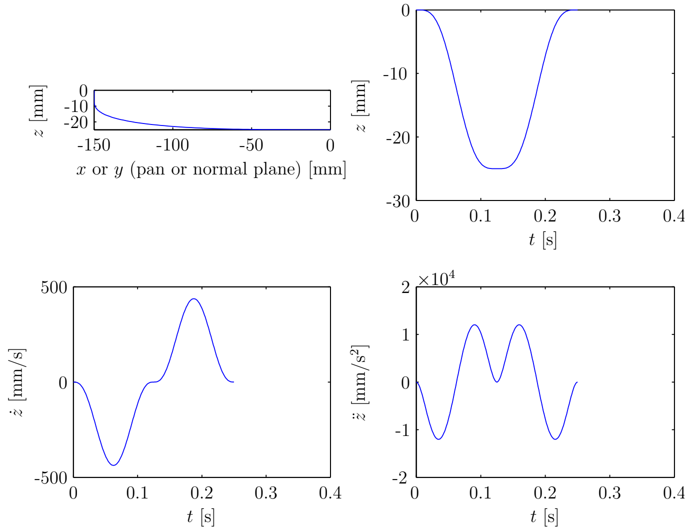


Use a combination of 4-5-6-7 polynomials

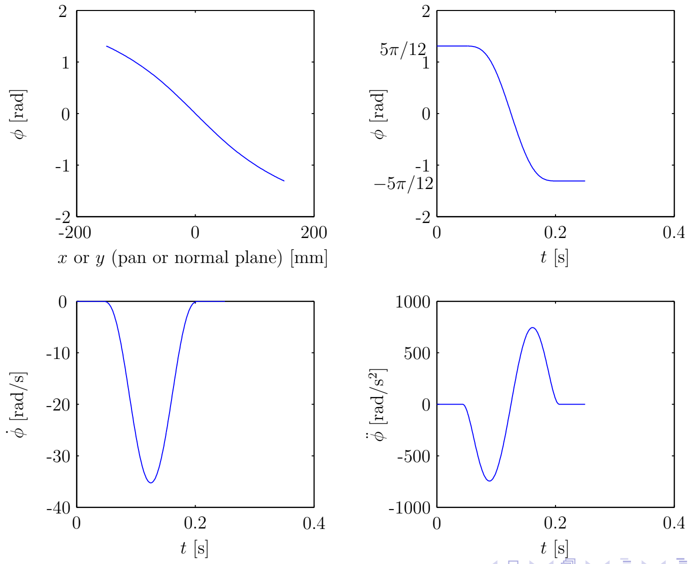
Displacement along X and Y



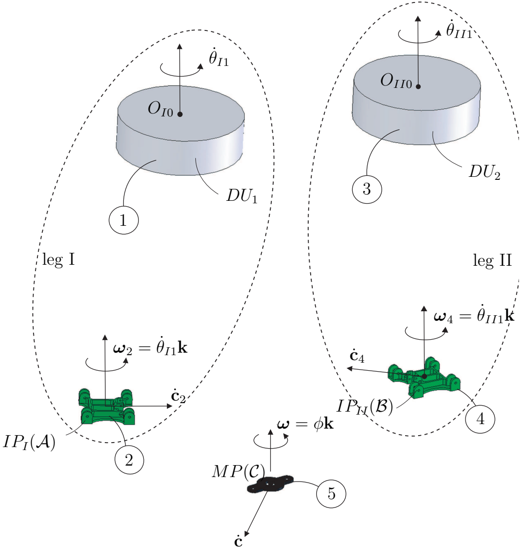
Displacement along Z



Orientation of the end-effector



SMG Model



SMG
$\Updownarrow$
system of five rigid bodies coupled by **massless joints** undergoing Schönflies displacements

Equations of motion

Natural Orthogonal Complement:

$$\mathbf{I}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} = \boldsymbol{\tau} + \boldsymbol{\gamma} - \boldsymbol{\delta} \tag{3}$$

- $\mathbf{I}$: 4×4 inertia matrix
- $\mathbf{C}$: 4×4 matrix of Coriolis and centrifugal forces
- $\boldsymbol{\theta}$: the four-dimensional vector of actuated joint variables
- $\boldsymbol{\tau}$: the four-dimensional vector of actuated joint torques
- $\boldsymbol{\gamma}$: the four-dimensional vector of gravity forces
- $\boldsymbol{\delta}$, the four-dimensional vector of dissipative forces, which are neglected

Equations of motion

- Inertia matrix:

$$\mathbf{I} = \mathbf{I}_{\mathcal{A}} + \mathbf{I}_{\mathcal{B}} + \mathbf{I}_{\mathcal{C}} \tag{4}$$

$$\mathbf{I}_i = \mathbf{T}_i^T \mathbf{M}_i \mathbf{T}_i \, , \; i = \mathcal{A}, \mathcal{B}, \mathcal{C} \tag{5}$$

$\mathcal{A}$ and $\mathcal{B}$ being the two elbows and $\mathcal{C}$ the end-plate of the manipulator.

- Matrix of Coriolis and centrifugal forces:

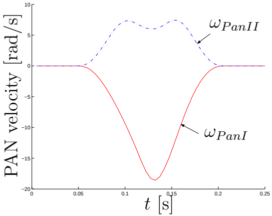
$$\mathbf{C}(\theta, \dot{\theta})\dot{\theta} = \mathbf{T}_{\mathcal{A}}^T \mathbf{M}_{\mathcal{A}} \dot{\mathbf{T}}_{\mathcal{A}} \dot{\theta} + \mathbf{T}_{\mathcal{B}}^T \mathbf{M}_{\mathcal{B}} \dot{\mathbf{T}}_{\mathcal{B}} \dot{\theta} + \mathbf{T}_{\mathcal{C}}^T \mathbf{M}_{\mathcal{C}} \dot{\mathbf{T}}_{\mathcal{C}} \dot{\theta} \tag{6}$$

- Gravity forces:

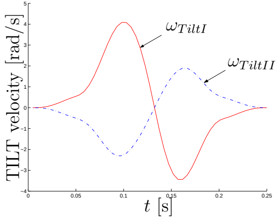
$$\gamma = \sum_{i=\mathcal{A}, \mathcal{B}, \mathcal{C}} \mathbf{T}_i^T \mathbf{w}_i^G \tag{7}$$

$$\mathbf{w}_i^G = \begin{bmatrix} \mathbf{0}_3 \\ m_i \, g \end{bmatrix}, \quad i = \mathcal{A}, \, \mathcal{B}, \, \mathcal{C}$$

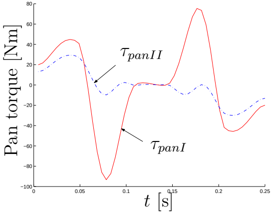
Pan and Tilt: Velocities and Torques



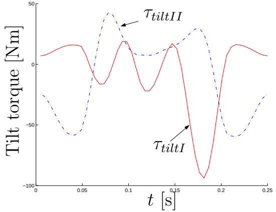
(a) PAN velocities



(b) TILT velocities

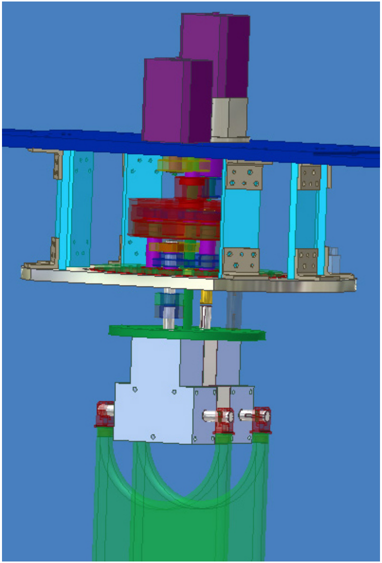


(c) PAN torques



(d) TILT torques

Drive Units



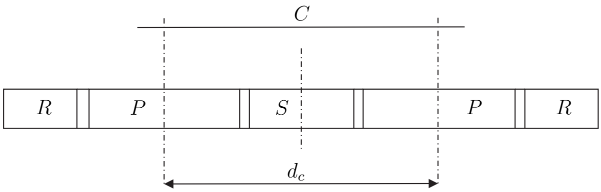
Kinematics

$$\boldsymbol{\omega}_{in} = \mathbf{J}_D^{-1}\boldsymbol{\omega}_{out} \tag{8}$$

$$\boldsymbol{\omega}_{in} = \begin{bmatrix} \omega_A \\ \omega_B \end{bmatrix}, \quad \boldsymbol{\omega}_{out} = \begin{bmatrix} \omega_p \\ \omega_t \end{bmatrix}$$

$$\mathbf{J}_D^{-1} = -r \begin{bmatrix} 1 & -r_{6,5} \\ 1 & \dfrac{r_{6,5}}{1 + 2r_{6,5}} \end{bmatrix}$$

Drive Units



Dynamics

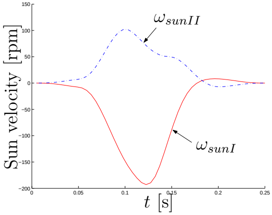
$$T = \frac{1}{2}I_S\omega_S^2 + \frac{1}{2}I_R\omega_R^2 + \frac{1}{2}I_C\omega_C^2 + \frac{3}{2}I_P\omega_P^2 + \frac{3}{2}m_P\left(\frac{d_C}{2}\omega_C\right)^2 \quad (9)$$

- Lagrange equations:

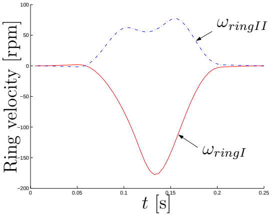
$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = \boldsymbol{\tau}_{SR} - \mathbf{J}_D^T\boldsymbol{\tau}_{CP}, \quad i = 1,\, 2 \quad (10a)$$

$$\boldsymbol{\tau}_{SR} = \mathbf{I}_D\ddot{\mathbf{q}} + \mathbf{J}_D^T\boldsymbol{\tau}_{CP} \quad (10b)$$

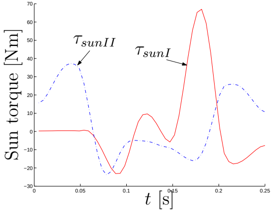
Sun and Ring: Velocities and Torques



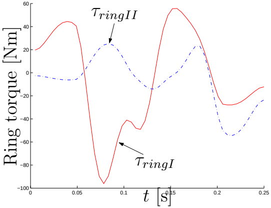
(a) SUN Velocities



(b) RING Velocities



(c) SUN Torques



(d) RING Torques

Motor and Gearhead specifications

Table: Peak requirements

$\overline{\omega}_S$ [rpm]	$\overline{\tau}_S$ [Nm]	$\overline{\omega}_R$ [rpm]	$\overline{\tau}_R$ [Nm]
192	72	179	84

Table: Motor and Gearhead specifications

Gear head ratio	$\overline{\omega}_M$ [rpm]	$\overline{\tau}_M$ [Nm]
20	5000	7.1

Conclusions

- A novel parallel robot generator of Schönflies motions was introduced
- Link-dimensioning was based on kinetostatic conditions
- Kinematics modelled using 6×4 Jacobian matrices for robustness against formulation singularities
- Motor and gearhead selection were based on dynamics model
- Dynamics model was based on a MBS of five rigid links coupled by massless joints
- Dynamics model is a 4-dimensional systems of second-order ODEs
- Currently undergoing tests and adjustments to reach a 500 ms (or shorter) cycle time

Conclusions

- A novel parallel robot generator of Schönflies motions was introduced
- Link-dimensioning was based on kinetostatic conditions
- Kinematics modelled using 6×4 Jacobian matrices for robustness against formulation singularities
- Motor and gearhead selection were based on dynamics model
- Dynamics model was based on a MBS of five rigid links coupled by massless joints
- Dynamics model is a 4-dimensional systems of second-order ODEs
- Currently undergoing tests and adjustments to reach a 500 ms (or shorter) cycle time

Conclusions

- A novel parallel robot generator of Schönflies motions was introduced
- Link-dimensioning was based on kinetostatic conditions
- Kinematics modelled using 6×4 Jacobian matrices for robustness against formulation singularities
- Motor and gearhead selection were based on dynamics model
- Dynamics model was based on a MBS of five rigid links coupled by massless joints
- Dynamics model is a 4-dimensional systems of second-order ODEs
- Currently undergoing tests and adjustments to reach a 500 ms (or shorter) cycle time

Conclusions

- A novel parallel robot generator of Schönflies motions was introduced
- Link-dimensioning was based on kinetostatic conditions
- Kinematics modelled using 6×4 Jacobian matrices for robustness against formulation singularities
- Motor and gearhead selection were based on dynamics model
- Dynamics model was based on a MBS of five rigid links coupled by massless joints
- Dynamics model is a 4-dimensional systems of second-order ODEs
- Currently undergoing tests and adjustments to reach a 500 ms (or shorter) cycle time

Conclusions

- A novel parallel robot generator of Schönflies motions was introduced
- Link-dimensioning was based on kinetostatic conditions
- Kinematics modelled using 6×4 Jacobian matrices for robustness against formulation singularities
- Motor and gearhead selection were based on dynamics model
- Dynamics model was based on a MBS of five rigid links coupled by massless joints
- Dynamics model is a 4-dimensional systems of second-order ODEs
- Currently undergoing tests and adjustments to reach a 500 ms (or shorter) cycle time

Conclusions

- A novel parallel robot generator of Schönflies motions was introduced
- Link-dimensioning was based on kinetostatic conditions
- Kinematics modelled using 6×4 Jacobian matrices for robustness against formulation singularities
- Motor and gearhead selection were based on dynamics model
- Dynamics model was based on a MBS of five rigid links coupled by massless joints
- Dynamics model is a 4-dimensional systems of second-order ODEs
- Currently undergoing tests and adjustments to reach a 500 ms (or shorter) cycle time

Conclusions

- A novel parallel robot generator of Schönflies motions was introduced
- Link-dimensioning was based on kinetostatic conditions
- Kinematics modelled using 6×4 Jacobian matrices for robustness against formulation singularities
- Motor and gearhead selection were based on dynamics model
- Dynamics model was based on a MBS of five rigid links coupled by massless joints
- Dynamics model is a 4-dimensional systems of second-order ODEs
- Currently undergoing tests and adjustments to reach a 500 ms (or shorter) cycle time