# Comments on "Development and Validation of a Geometry-Based Contact Force Model"

**A Ph.D. thesis by Lianzhen Luo, McGill University**

**Submitted by J. Angeles, May 10, 2010**

My main criticism to the original version lied in Ch. 3; more specifically, I pointed out flaws in both the formulation and the solution method of the problem stated in Section 3.4. The author addressed some of my concerns, but there is still work to be done. Nevertheless, this work should be doable prior to the final submission of the thesis.

## I. Contents

Some outstanding questions:

1. Why was the $x$-$y$ plane chosen for projection and not an arbitrary plane?

2. The opening phrase of 3.4.1.1 needs rephrasing. It is incomprehensible.

3. Equations (3.8 & 3.9) are thrown in without a justification. It seems to me that the former is a restatement of Stokes' Theorem, while the latter of Gauss's Divergence Theorem, but the author doesn't state the origin of these relations. Even worse, these theorems, or any theorem for that matter, carries hypotheses that limit their application domain. Are these hypotheses satisfied by the conditions of the problem under study? No word is said about the smoothness of the outward normal $\mathbf{n}_i$.

4. The author displays an eq.(3.16) that requires that matrix $\mathbf{W}$ be singular. Is this always the case? Otherwise, the equation cannot admit a non-trivial solution!

5. Apparently, eq.(3.16) calls for a non-zero vector $\mathbf{n}$ that lies in the nullspace of $\mathbf{W}$. Finding a basis for the nullspace of a rank-deficient matrix, or of a singular matrix as the case may be, does not require the solution of an eigenvalue problem. This problem is, by its nature, iterative, and hence, computationally costly. The author suggests using the Jacobi method to solve the problem, but this isn't needed. I directed Ms. Luo to (Golub and Van Loan, 1996) for an account on how to compute a basis for the nullspace in question directly—as opposed to iteratively—using QR-decomposition. However, the author did not follow my advice.

6. As a matter of fact, eq.(3.16) is nothing but the *normal equations* (NE) of a minimization problem. As numerical analysts (Golub and Van Loan, 1996) claim, using the NE to solve least-square problems numerically leads to a loss of precision in the data by virtue of the product of a matrix by its transpose. Normal equations are to be avoided, as I told the author. My suggestion was to write the equation of the plane sought for each of the $n > 3$ points, namely,

$$ax_i + by_i + cz_i - d = 0, \quad i = 1, \ldots, n \tag{1}$$

Next, divide both sides of the above equation by $\sqrt{a^2+b^2+c^2}$, which leads to

$$\lambda x_i + \mu y_i + \nu z_i - \delta = 0, \quad i = 1, \ldots, n \tag{2}$$

where $\lambda$, $\mu$ and $\nu$ are the normalized values of $a$, $b$ and $c$, respectively; they are thus the direction cosines of the normal to the plane, and hence, obey

$$\lambda^2 + \mu^2 + \nu^2 = 1 \tag{3}$$

while $\delta = d/\sqrt{a^2+b^2+c^2}$. Now the $n > 3$ equations 2 can be cast in the form

$$\mathbf{P}\mathbf{x} = \mathbf{0}_n \tag{4}$$

where $\mathbf{P}$ and $\mathbf{x}$ are given below

$$\mathbf{P} = \begin{bmatrix} \mathbf{p}_1^T & -1 \\ \mathbf{p}_2^T & -1 \\ \vdots & \\ \mathbf{p}_n^T & -1 \end{bmatrix}, \ \mathbf{p}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} \lambda \\ \mu \\ \nu \\ \delta \end{bmatrix} \tag{5}$$

the problem thus reducing to finding a non-zero vector $\mathbf{x}$ that lies in the nullspace of $\mathbf{P}$. This problem is treated by (Golub and Van Loan, 1996). It is straightforward and doesn't require iterations. It is based, as stated above, on QR-decomposition.

## II. Presentation

The thesis, and the author's "Response" itself, still contain some language flaws, which must be cleared before the final submission.

## Reference

Golub, G.H. and Van Loan, C.F., 1996, *Matrix Computations*, third edition, The Johns Hopkins University Press, Baltimore.