# MECH577 Optimum Design

### Project # 2: Approximate Synthesis of a Four-Bar Motion-Generator

**Project Marks:** Project 1 is worth 20%; Project 2 is worth 35%, with 15% for Part I and 20% for Part II.

Assigned: September 26, 2002 Due: October 31, 2002

The objective of this project is to synthesize a four-bar linkage intended to guide a rigid body, rigidly attached to its coulper link, through $m+1$ poses—positions and orientations–in the plane, where $m > 4$. It is known (Norton, 2001) that a four-bar linkage allows the *exact* guidance of its coupler link through up to five poses, by properly selecting the location of the centres of its four joints. When more than five poses are specified, a *positioning error* is incurred. In this case, an *approximate synthesis* approach is followed, whereby the designer aims not at meeting exactly any of the given poses, but rather at allowing for positioning errors at all of these, while attempting to minimize the overall positioning error—i.e., the error at all prescribed poses—in the least-square sense.

The use of the ODA package is highly recommended. This is a library of C routines that is available at

`http://www.cim.mcgill.ca/~chinpunt/research.html`

The project is divided into two parts. For both, reference is made to the problem solved in (Yao and Angeles, 2000). In this paper, the synthesis procedure is based on a method by which the linkage is divided into two pairs of adjacent links coupled one to the other by a revolute joint, one of these being coupled to the fixed link by an anchored, second revolute joint. This pair of links and joints is termed a *dyad*. The synthesis method produces one dyad at a time, which thus leads to an optimization problem in four design variables, the two Cartesian coordinates of each of the two dyad joints. Application of the *normality conditions* is plausible in this paper because of the elimination approach used, to be outlined below. This approach yields a system of four polynomial equations in four unkowns.

The problem of least-square approximate synthesis is solved in (Yao and Angeles, 2000) using *dialytic elimination*, by which two of the four variables are eliminated, thereby leading to a system of two bivariate polynomial equations, each defining a contour in the plane of the two variables. All solutions to the normality conditions, i.e., all stationary points, are thus found by inspection, upon plotting the two contours. Inspection gives only a rough estimate of the solutions; a refinement of these estimates is possible by application of the Newton-Raphson method.

In this project, an alternative, numerical approach is to be followed, by means of the routines available in the ODA package.

## Part I: Unconstrained Optimization

Formulate the optimum-design job as an unconstrained nonlinear least-square problem. To this end, use various randomly selected initial guesses comprised in a square of side 10 u centred at the origin of the $x$-$y$ plane, with "u" indicating the units of length in which the data are given.

Then, verify the first- and the second-order normality conditions to properly identify the local minima. Give an estimate of the overall linkage positioning error of the optimum linkage.

### Part II: Constrained Optimization

Obtain the optimum linkage producing a least-square positioning error, but now imposing the condition that the linkage visit *exactly* the intermediate ($j = 14$) and the last ($j = 30$) poses.

## References


Norton, R.L., 2001, *Design of Machinery*, Second Edition, McGraw-Hill, New York.

Yao, J. and Angeles, J., 2000, "Computation of all optimum dyads in the approximate synthesis of planar linkages for rigid-body guidance," *Mechanism and Machine Theory*, Vol. 35, pp. 1065–1078.