MECH 577 Optimum Design

Project # 2: Unconstrained Optimization
The Maximum Reach of a RRR Orthogonal Robot

Assigned: September 23, 2008 | Due: October 23, 2008

Reference is made to the robot illustrated in Fig. 5.1 of the Lecture Notes. The purpose of this project is to find the maximum reach of the robot using an *unconstrained-optimization* approach. Moreover, two techniques introduced in Ch. 4 are to be used, the Fletcher-Reeves method and the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method.

Notice that the same problem is solved in Ch. 5 using a *constrained-optimization* approach. The difference between the two approaches lies in the choice of *design variables*. In this project, the design variables are the joint angles, which are assumed to be free to attain any real value—in practice, all robots are provided with joint limits. In Ch. 5 the design variables are the Cartesian coordinates of the *operation point* C of the robot, the search being conducted along a planar curve that guarantees that point C lies on the boundary of the robot workspace.

The objective function is defined as 1/2 the square of the *nondimensional distance* of point C from the Z_1-axis, which turns out to be independent of θ_1, as should have been expected[1]. The problem thus entails only two design variables, the joint angles θ_2 and θ_3, namely,

$$f = 2 + c_2c_3 + \frac{1}{2}c_2^2c_3^2 + c_2s_2 + c_2^2c_3 + c_2 + s_2 + s_3 - \frac{1}{2}c_3^2 + c_2s_2c_3$$

where c_i and s_i denote $\cos\theta_i$ and $\sin\theta_i$, respectively. Moreover,the two-dimensional gradient of f has components

$$\begin{aligned}
(\nabla f)_1 &\equiv \frac{\partial f}{\partial \theta_2} = -s_2c_3 - c_2s_2c_3^2 - s_2^2 + c_2^2 - 2c_2s_2c_3 - s_2 + c_2 - s_2^2c_3 + c_2^2c_3 \\
(\nabla f)_2 &\equiv \frac{\partial f}{\partial \theta_3} = -c_2s_3 - c_2^2c_3s_3 - c_2^2s_3 + c_3 + c_3s_3 - c_2s_2s_3
\end{aligned}$$

Details of the foregoing derivations are available in

`RRRortho080920.mw`

[1] The distance is obtained in nondimensional form upon dividing it by the length a depicted in Fig. 5.1.