MECH 577 101018

Rao's Example 6.6 (encore). For quick reference, the objective function is

$$f(\vec{x}) = 6x_1^2 + 2x_2^2 - 6x_1x_2 - x_1 - 2x_2 \to \min_{\vec{x}}, \quad \vec{x} = [x_1, x_2]^T$$

Rao: "If $\vec{s}_1 = [1, 2]^T$ denotes a search direction, find a direction $\vec{s}_2$ which is [$\underset{\sim}{A}$-conjugate] to $\vec{s}_1$"

Rao's solution:

$$\underset{\sim}{A} = \begin{bmatrix} 12 & -6 \\ -6 & 4 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} -1 \\ -2 \end{bmatrix} \quad \text{(already obtained, 101004)}$$

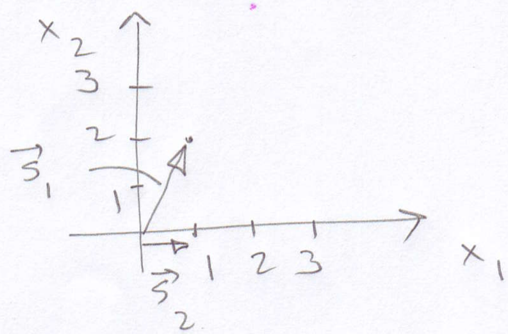
Let $\vec{s}_2 = [s_{21} \quad s_{22}]^T$. Thus, for $\underset{\sim}{A}$-conjugacy,

$$\vec{s}_1^T \underset{\sim}{A} \vec{s}_2 = \underbrace{[1 \quad 2]\begin{bmatrix} 12 & -6 \\ -6 & 4 \end{bmatrix}}_{[0 \quad 2]} \begin{bmatrix} s_{21} \\ s_{22} \end{bmatrix} = 0 \Rightarrow s_{22} = 0$$

$$\Downarrow$$

$$s_{21} = \text{arbitrary}$$

Rao chooses $s_{21} = 0 \Rightarrow \vec{s}_2 = [1, 0]^T$ (*)



In our approach, $\vec{s}_1 \to \vec{\sigma}_1 = \underset{\sim}{B}\vec{s}_1 = \begin{bmatrix} 2\sqrt{3} & -\sqrt{3} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

i.e., $\vec{\sigma}_1 = [2\sqrt{3} - 2\sqrt{3}, \ 2]^T = [0, 2]^T$, $\vec{\sigma}_2 \perp \vec{\sigma}_1$

$$\Rightarrow [0 \quad 2]\begin{bmatrix} \sigma_{21} \\ \sigma_{22} \end{bmatrix} = 0 \Rightarrow 2\sigma_{22} = 0 \Rightarrow \sigma_{21} = \text{arb}$$

If $\vec{\sigma}_2 = [x, 0]^T$, then $\vec{\sigma}_1^T \vec{\sigma}_2 = 0x = 0$ identically

$$\vec{s}_2 = \underset{\sim}{B}^{-1}\vec{\sigma}_2 = \frac{1}{6}\begin{bmatrix} \sqrt{3} & 3 \\ 0 & 6 \end{bmatrix}\begin{bmatrix} x \\ 0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} \sqrt{3}x \\ 0 \end{bmatrix} = \frac{\sqrt{3}}{6}\begin{bmatrix} x \\ 0 \end{bmatrix}$$

To match (*), $x = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$