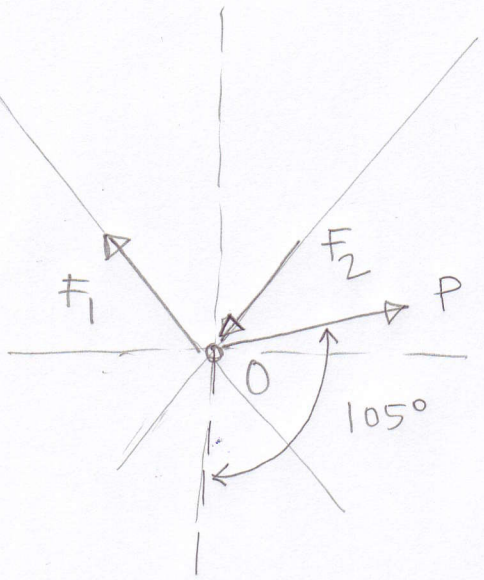
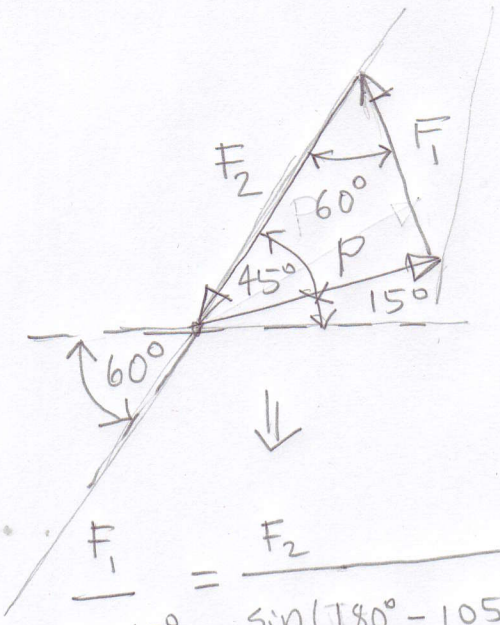
Project 1





$$\frac{F_1}{\sin 45^\circ} = \frac{F_2}{\sin(180^\circ - 105^\circ)}, \quad 180^\circ - 105^\circ = 75^\circ$$

$$\Rightarrow \frac{F_2}{F_1} = \frac{\sin 75^\circ}{\sin 45^\circ} = \frac{\sin(45^\circ + 30^\circ)}{\sin 45^\circ}$$

$$\sin(45^\circ + 30^\circ) = \underbrace{\sin 45^\circ}_{\frac{\sqrt{2}}{2}} \underbrace{\cos 30^\circ}_{\frac{\sqrt{3}}{2}} + \underbrace{\cos 45^\circ}_{\frac{\sqrt{2}}{2}} \underbrace{\sin 30^\circ}_{\frac{1}{2}} = \frac{\sqrt{2}(1+\sqrt{3})}{4}$$

$$\Rightarrow \frac{F_2}{F_1} = \frac{\sqrt{2}(1+\sqrt{3})}{4\sqrt{2}/2} = \frac{1+\sqrt{3}}{2} \approx 1.3660$$

$$\sigma_i \equiv \frac{F_i}{A_i} \Rightarrow \sigma_1 = \frac{F_1}{A_1}, \quad \sigma_2 = \frac{F_2}{A_2} = \frac{\frac{1+\sqrt{3}}{2} F_1}{A - A_1}$$

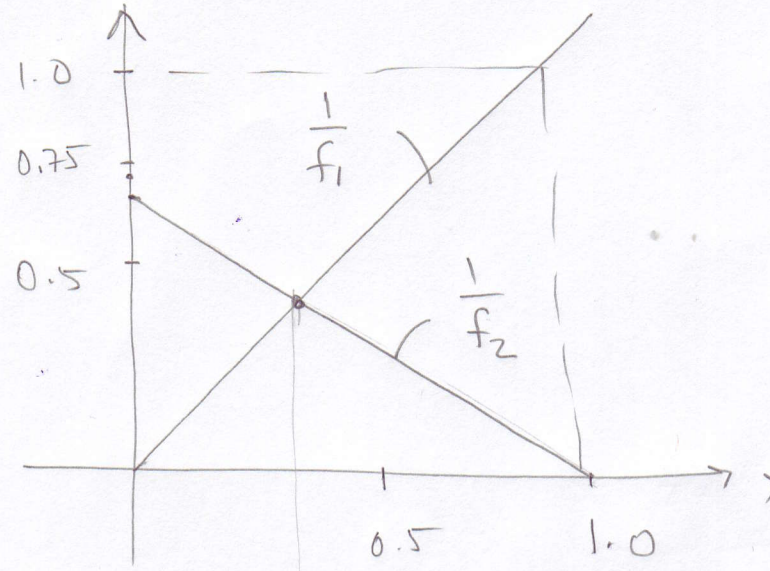
Let $A_1 = Ax \Rightarrow A - A_1 = A(1-x)$

$$\Rightarrow \sigma_1 = \frac{F_1}{A}\frac{1}{x}, \quad \sigma_2 = \frac{1+\sqrt{3}}{2}\frac{F_1}{A}\frac{1}{1-x} \Rightarrow \sigma_1 \sim \frac{1}{x}, \quad \sigma_2 \sim \frac{1+\sqrt{3}}{2}\frac{1}{1-x}$$

Hence, problem can be formulated as

$$\min_x f(x), \quad f(x) \equiv \max\left\{\underbrace{\frac{1}{x}}_{\equiv f_1}, \underbrace{\frac{1+\sqrt{3}}{2}\frac{1}{1-x}}_{\equiv f_2}\right\}, \quad 0 \le x \le 1$$

Plotting f_1 & f_2 for $0 \le x \le 1$ because f_1 & f_2 become unbounded at $x=0$ & $x=1$, respectively. Their reciprocals $\frac{1}{f_1}$ & $\frac{1}{f_2}$, however, are bounded in the same interval. Plot its reciprocals:



$$\frac{1}{f_1} = x$$

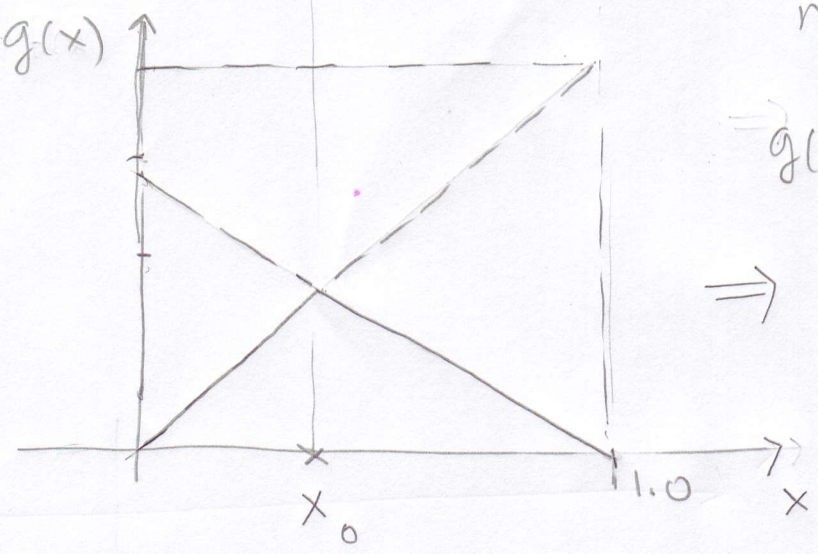
$$\frac{1}{f_2} = \frac{2}{1+\sqrt{3}}(1-x)$$

$$\approx 0.7321(1-x)$$

Now the problem changes to

$$\max_x g(x),$$

$$g(x) = \min\left\{\frac{1}{f_1(x)}, \frac{1}{f_2(x)}\right\}$$



$\Rightarrow$ maximum located at $x_0 = 1 - \frac{\sqrt{3}}{3} = 0.4226$