McGill

Book Number /Cahier d'examen n° ____ de /of ____

EXAMINATION BOOK/CAHIER D'EXAMEN

Last Name/*Nom* : CLASS TEST 2

First Name/*Prénom* : SOLUTION

McGill ID/*N° de matricule McGill* : _ _ _ _ _ _ _ _ _

Date of Exam/*Date de l'examen* : 2010/11/08 (yyyy/mm/dd) *(année/mois/jour)*

Subject and Course Code/*Sujet et code du cours* : MECH 577 Optimum Design

Section/*Section* : ______ Room/*Salle* : ______ Row and Seat Number/*Rangée et siège* : ______

INSTRUCTIONS:

1. Fill in the above carefully.
2. Write your answers on the right-hand side of the exam book. Use the left-hand side for rough work and calculations.
3. Do not write in the margins.
4. If a page is accidentally left blank, write "P.T.O." on it.
5. Do not tear pages from the exam book.
6. At the time of the examination, you must not have in your possession any cellphones, books, calculators, dictionaries, notes or any other extraneous material unless otherwise indicated on the Exam Paper Cover instructions.
7. Put additional books inside the first book when submitting your exam.
8. **This book cannot be taken from the examination room.**

DIRECTIVES :

1. Remplissez soigneusement la section ci-dessus.
2. Écrivez vos réponses dans la section de droite du cahier d'examen. Utilisez la section de gauche pour l'ébauche et le calcul.
3. N'écrivez pas dans les marges.
4. Si vous avez laissé involontairement une page blanche, veuillez y inscrire « voir page suivante ».
5. Aucune page du cahier d'examen ne doit être retirée.
6. Durant l'examen, vous ne pouvez avoir en votre possession de cellulaire, livre, calculatrice, dictionnaire, note ou tout matériel superflu à moins d'indication contraire dans les directives indiquées sur la couverture de l'examen.
7. Veuillez insérer les cahiers additionnels à l'intérieur du premier cahier au moment de remettre votre examen.
8. **Le présent cahier doit demeurer dans la salle d'examen.**

For Examiner's Use Only Section réservée à l'examinateur
1.
2.
3.
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12.
13.
14.
15.

1(a) $(\vec{s}^1)^T \underset{\sim}{A} \vec{s}^2 = 0$, $\vec{s}^2 = [x_2\ y_2\ 0]$, $\|\vec{s}^2\| = 1$

$$\Rightarrow \underbrace{[1\ 1\ 1]\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 9\end{bmatrix}\begin{bmatrix}x_2\\ y_2\\ 0\end{bmatrix}}_{\begin{bmatrix}x_2\\ 4y_2\\ 0\end{bmatrix}} = 0 \Rightarrow x_2 + 4y_2 = 0 \Rightarrow y_2 = -\frac{1}{4}x_2$$

$$\Rightarrow x_2^2 + \frac{1}{16}x_2^2 = 1 \Rightarrow \frac{17}{16}x_2^2 = 1 \Rightarrow x_2 = \pm 4\frac{\sqrt{17}}{17}. \text{Take } +$$

$$\Rightarrow \vec{s}^2 = \frac{\sqrt{17}}{17}[4\ -1\ 0]^T$$

$\underrightarrow{\text{Ans.}}$

(b) $(\vec{s}^1)^T \underset{\sim}{A} \vec{s}^3 = 0$, $\vec{s}^3 = [x_3\ y_3\ z_3]^T$

$$\Rightarrow \underbrace{[1\ 1\ 1]\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 9\end{bmatrix}\begin{bmatrix}x_3\\ y_3\\ z_3\end{bmatrix}}_{\begin{bmatrix}x_3\\ 4y_3\\ 9z_3\end{bmatrix}} = 0 \Rightarrow x_3 + 4y_3 + 9z_3 = 0 \qquad (1)$$

$$(\vec{s}^2)^T \underset{\sim}{A} \vec{s}^3 = 0 \Rightarrow [4\ -1\ 0]\begin{bmatrix}x_3\\ 4y_3\\ 9z_3\end{bmatrix} = 0 \Rightarrow 4x_3 - 4y_3 = 0 \qquad (2)$$

$$(1) + (2) \Rightarrow 5x_3 + 9z_3 = 0 \qquad (3)$$

$$(2) \Rightarrow y_3 = x_3\ ;\ (3) \Rightarrow z_3 = -\frac{5}{9}x_3 \qquad (4)$$

$$x_3^2 + y_3^2 + z_3^2 = 1 \Rightarrow x_3^2 + x_3^2 + \frac{25}{81}x_3^2 = 1 \Rightarrow \frac{187}{81}x_3^2 = 1$$

$$\Rightarrow x_3 = \pm 9\frac{\sqrt{187}}{187} \quad \underset{(\text{with } +)}{(5) \text{ into } (2)} \Rightarrow y_3 = x_3 = 9\frac{\sqrt{187}}{187}$$

$$(5) \text{ into } (4) \Rightarrow z_3 = -\frac{5}{9}9\frac{\sqrt{187}}{187} = -\frac{5}{187}\sqrt{187}$$

$$\Rightarrow \vec{s}^3 = \frac{\sqrt{187}}{187}[9\ 9\ -5]^T$$

$\underrightarrow{\text{Ans}}$

2(a) $\underset{\sim}{H}_c = \nabla\nabla f + \dfrac{\partial(\underset{\sim}{J}^T\vec{\lambda})}{\partial\vec{x}}$, $\underset{\sim}{J}^T\vec{\lambda} = \underbrace{\lambda\nabla h}_{} = \nabla(\lambda h)$

because we have one single constraint

$$\Rightarrow \underset{\sim}{H}_c = \nabla\nabla f + \nabla\nabla(\lambda h) \equiv \nabla\nabla f + \lambda\nabla\nabla h$$

∇f from $\partial F/\partial\theta_i$, $i=1,2$, taking term indep of λ: (in p. 118)

$$\Rightarrow \nabla f = \begin{bmatrix} 3\sin\theta_1 \\ \sin\theta_2 \end{bmatrix}, \quad \nabla h = \begin{bmatrix} \cos\theta_1 \\ \cos\theta_2 \end{bmatrix}$$

$$\Rightarrow \nabla\nabla f = \begin{bmatrix} 3\cos\theta_1 & 0 \\ 0 & \cos\theta_2 \end{bmatrix}, \quad \nabla\nabla h = \begin{bmatrix} -\sin\theta_1 & 0 \\ 0 & -\sin\theta_2 \end{bmatrix}$$

$$\Rightarrow \underset{\sim}{H}_c = \begin{bmatrix} 3\cos\theta_1 - \lambda\sin\theta_1 & 0 \\ 0 & \cos\theta_2 - \lambda\sin\theta_2 \end{bmatrix}$$ Ans.

(b) $\|\nabla h\| = \sqrt{\cos^2\theta_1 + \cos^2\theta_2}$

$\vec{l} = \dfrac{1}{\|\nabla h\|}\underset{\sim}{E}_2\nabla h$, $\underset{\sim}{E}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ from eq. (3.15c)

$$\Rightarrow \vec{l} = \frac{\sqrt{\cos^2\theta_1 + \cos^2\theta_2}}{\cos^2\theta_1 + \cos^2\theta_2}\begin{bmatrix} -\cos\theta_2 \\ \cos\theta_1 \end{bmatrix}$$ Ans.

(c) NC: $\nabla f\big|_P + \lambda\nabla h\big|_P = 0$

$$\nabla f\big|_P = \begin{bmatrix} 3\sin 2.697 \\ \sin 2.181 \end{bmatrix} = \begin{bmatrix} 3\sin 154.5^\circ \\ \sin 125.0^\circ \end{bmatrix} = \begin{bmatrix} 1.2915 \\ 0.8192 \end{bmatrix}$$

$$\nabla h\big|_P = \begin{bmatrix} \cos 154.5^\circ \\ \cos 125.0^\circ \end{bmatrix} = \begin{bmatrix} -0.9026 \\ -0.5736 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.9026 \\ 0.5736 \end{bmatrix} \lambda = \begin{bmatrix} 1.2915 \\ 0.8192 \end{bmatrix} \begin{matrix} \Rightarrow \lambda = 1.4309 & (i) \\ \Rightarrow \lambda = 1.4282 & (ii) \end{matrix}$$

Two foregoing eqns should yield one and the same numerical value of λ. Small difference due to roundoff error, which can be filtered by taking mean value:

$$\bar{\lambda} = 1.4295 \quad \underline{\underline{\text{Ans.}}}$$

Alternatively, λ can be computed with left Moore-Penrose generalized inverse:

$$\underset{\sim}{M} \lambda = \vec{n} \Rightarrow \lambda = (\underset{\sim}{M}^T \underset{\sim}{M})^{-1} \underset{\sim}{M}^T \vec{n}$$

$$\underset{\sim}{M}^T \underset{\sim}{M} = \begin{bmatrix} 0.9026 & 0.5736 \end{bmatrix} \begin{bmatrix} 0.9026 \\ 0.5736 \end{bmatrix} = [1.1437]$$

$$\Rightarrow (\underset{\sim}{M}^T \underset{\sim}{M})^{-1} = \left[\frac{1}{1.1437} \right] = [0.8744]$$

$$\underset{\sim}{M}^T \vec{n} = \begin{bmatrix} 0.9026 & 0.5736 \end{bmatrix} \begin{bmatrix} 1.2915 \\ 0.8192 \end{bmatrix} = [1.6356]$$

$$\Rightarrow \lambda = [0.8744][1.6356] = 1.4302$$

a value $\neq \bar{\lambda}$, but in-between (i) & (ii).

d) $\underset{\sim}{H}_u = \vec{l}^T \underset{\sim}{H}_c \vec{l}$

We need the numerical value of $\underset{\sim}{H}_c$, with λ found above:

$$\underset{\sim}{H}_c = \begin{bmatrix} 3\cos 154.5° - 1.4295 \sin 154.5° & 0 \\ 0 & \cos 125° - 1.4295 \sin 125° \end{bmatrix}$$

$$= \begin{bmatrix} -3.3232 & 0 \\ 0 & -1.7446 \end{bmatrix}$$

We also need the numerical value of $\vec{l}$:

$$\sqrt{\cos^2\theta_1 + \cos^2\theta_2} = \sqrt{(-0.9026)^2 + (-0.5736)^2}$$
$$= 1.0694$$

$$\Rightarrow \vec{l} = \frac{1}{1.0694}\begin{bmatrix} -\cos 125^\circ \\ \cos 154.5^\circ \end{bmatrix} = \frac{1}{1.0694}\begin{bmatrix} 0.5736 \\ -0.9026 \end{bmatrix}$$
$$= \begin{bmatrix} 0.5364 \\ -0.8440 \end{bmatrix}$$

$$\Rightarrow \underset{\sim}{H}_u = \begin{bmatrix} 0.5364 & -0.8440 \end{bmatrix} \underbrace{\begin{bmatrix} -3.3232 & 0 \\ 0 & -1.7446 \end{bmatrix}\begin{bmatrix} 0.5364 \\ -0.8440 \end{bmatrix}}_{\begin{bmatrix} -1.7826 \\ 1.4724 \end{bmatrix}}$$

$$= -2.1979$$

$\Rightarrow$ $\underset{\sim}{H}_u$ negative definite $\Rightarrow$ P is a local maximum.

Ans.