MECH 577 Optimum Design

Open Book Class Test 3 **December 3rd, 2010 11:35–12:55**

Only calculators approved by the Faculty of Engineering are allowed.

1. A construction company needs to mount a pulley, intended to support a steel cable to lift heavy weights, at a point $P(x,\, y,\, z)$ of an axis $\mathcal{L}$, the fixed end of the cable being located at a point designated as the origin O of of a coordinate frame. In order for the cable to reach the pulley throat axially, it is necessary that the segment $\overline{OP}$ be normal to $\mathcal{L}$, and hence, of *minimum length*. The company has hired you to find the coordinates of P. Furthermore, $\mathcal{L}$ is given by the intersection of two planes, Π_1 and Π_2, namely,

$$\begin{aligned} \Pi_1:\quad x+y+z-1 &= 0 \\ \Pi_2:\quad x-y+z+1 &= 0 \end{aligned}$$

or, in array form, $\mathbf{Cp}=\mathbf{d}$, with $\mathbf{C}$, $\mathbf{d}$ and $\mathbf{p}$ given as

$$\mathbf{C}=\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}, \quad \mathbf{d}=\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{p}=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

As well, in order to ensure the correctness of your result, the company requires the necessary computations using the two methods described below:

(a) (20%) The right Moore-Penrose generalized inverse and

(b) (40%) Householder reflections. To this end, you have assigned the job to a young engineer, who has found two Householder reflections, $\mathbf{H}_1$ and $\mathbf{H}_2$, that will render $\mathbf{C}^T$ in upper-triangular form. The engineer has provided you with the product $\mathbf{H}=\mathbf{H}_2\mathbf{H}_1$, given as

$$\mathbf{H}=\begin{bmatrix} -\sqrt{3}/3 & -\sqrt{3}/3 & -\sqrt{3}/3 \\ -\sqrt{6}/6 & \sqrt{6}/3 & -\sqrt{6}/6 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 \end{bmatrix}$$

2. You have been hired as a consultant to test *SmartOpt*, an intelligent optimization software package. Moreover, your client wants you to test the package with the problem of minimizing the Rosenbrock (a.k.a. *banana*) function

$$f(\mathbf{x})=100(x_2-x_1^2)^2+(1-x_1)^2$$

subject to the inequality constraint

$$g(\mathbf{x})\equiv x_2-x_1+\frac{1}{2}\le 0$$

which is illustrated in Fig. 1. *SmartOpt* returns the putative solution, to six decimal places,

$$\mathbf{x}_{\rm opt}=\begin{bmatrix} .509800 & 0.009800 \end{bmatrix}^T$$

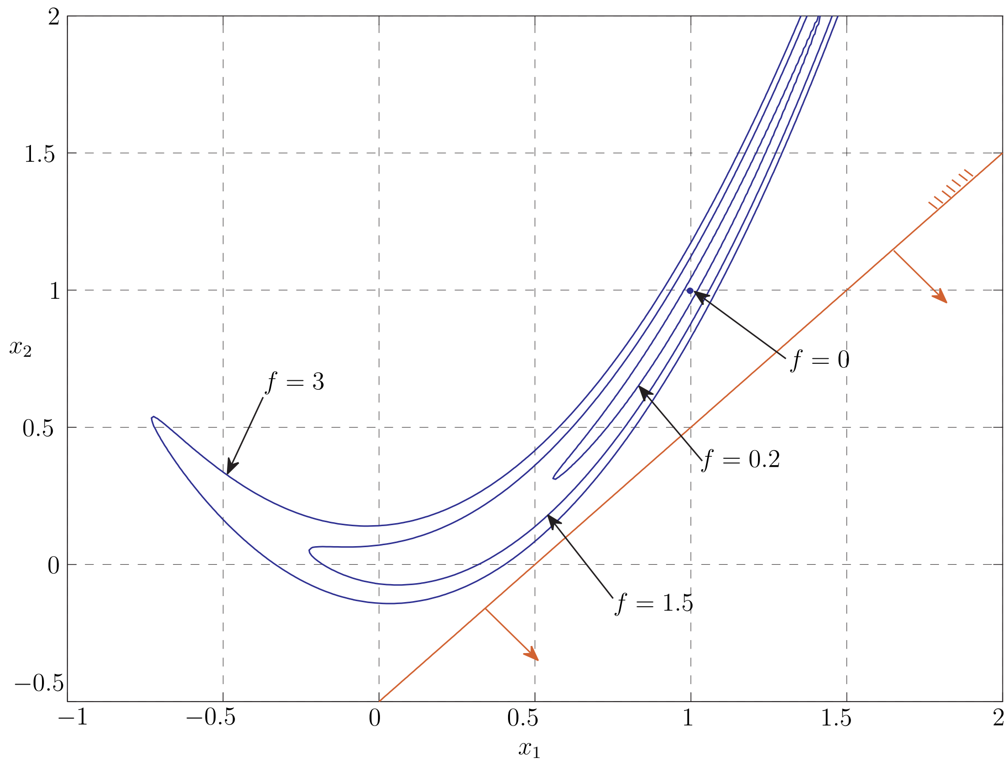

Figure 1: Minimization of the Rosenbrock (a.k.a. banana) function under an inequality constraint

(a) (25%) Show that $\mathbf{x}_{\text{opt}}$ is a feasible stationary point. In doing this, **explain your rationale**. *Hint: the gradient of $f(\boldsymbol{x})$ is given below, as obtained with computer algebra:*

$$\nabla f = \begin{bmatrix} -400(x_2 - x_1^2)x_1 - 2 + 2x_1 \\ 200x_2 - 200x_1^2 \end{bmatrix}$$

(b) (15%) Decide on the nature of stationary point that $\mathbf{x}_{\text{opt}}$ is, i.e., maximu, minimum or saddle point. **Explain your rationale**. *Hint: the Hessian of $f(\boldsymbol{x})$, as obtained with computer algebra as well, is given below:*

$$\nabla\nabla f = \begin{bmatrix} 1200x_1^2 - 400x_2 + 2 & -400x_1 \\ -400x_1 & 200 \end{bmatrix}$$