MECH541 Kinematic Synthesis

Project # 3: Synthesis of a Straight-Line Tracer for the Handling of Heavy Loads

Assigned: November 6th, 2003   Due: December 3rd, 2003 at 5:00 p.m. in Room MD461

The handling of heavy loads is frequent in operations such as the loading and unloading of containers in cargo ships. In these applications, a heavy load, weighing various metric tons, is transported between two stations a few meters apart in a horizontal plane. If the transportation between stations is done with variations in the vertical position, then extra load is imposed on the motors, to cope with the variations in potential energy due to the variations in height.

As a means to cope with height variations, an expert mechanism designer has proposed a four-bar linkage whose coupler link is capable of tracing a segment of a curve that approximates a straight line with high accuracy. This mechanism is displayed in Fig. 1 at an unspecified scale. Here, notice that the coupler curve of an arbitrary planar four-bar linkage is known to be a sextic, i.e., a curve described by an implicit function (Hartenberg and Denavit, 1964) of the form

$$F(x,\, y) = 0$$

in which $F(x,\, y)$ is a linear combination of terms of the form $x^i y^j$, in which $i + j \le 6$, and $i,\, j = 0, 1, \ldots, 6$. Such a curve intersects a straight line in at most six points, that can be real or complex. For this reason, it is impossible to trace a straight line with a point of the coupler link of a planar four-bar linkage. For many years, mechanism designers were striving to produce linkages capable of generating a straight-line trajectory, until the English kinematician Samuel Roberts (1876)[1] derived the above-mentioned sextic, thereby showing that the task at hand was futile.

Moreover, the coupler point traverses the foregoing segment with a fairly constant velocity, when the input link turns at a constant rate throughout a 180° turn.

The purpose of this project is to try and beat the performance of the linkage of the expert designer using methods of linkage synthesis and computational design. To this end, you are asked to

(a) Analyze the error of the given linkage in the segment where it is intended to approximate a line. More specifically, plot the percentage of error between the coupler curve and line L_1 within the two extreme positions of point C, shown in Fig. 1. We shall call ℓ the length of the L_1-segment comprised between the two projections of those positions onto line L_1. By measuring the percentage in terms of ℓ, calculate the rms value of this error;

(b) Now plot the two components of the "velocity" error vector, one along and one normal to L_1. Here, the "velocity" error vector has units of length, if the input angle ψ plays the role of time. Moreover, these plots should show percentage values, while normalizing the actual components using ℓ as well. Then, compute the rms value for the velocity error vector percentagewise.

[1] In the times of Roberts, kinematicians would not count the fixed link when referring to a four-bar linkage.

(c) Repeat item (c) for the "acceleration" error vector defined similar to the velocity error vector.

(d) Devise a scheme that will allow you to produce alternative four-bar linkages that will visit exactly five points of line L_1 within the desired segment.

(e) By properly choosing the five visited points, try to produce smaller errors when approximating line L_1. **Note: The only variables you have at your disposal to reduce the position, velocity and acceleration errors are the ordinates of the five points over line L_1.** You can distribute these points in many possible ways, e.g., uniformly, symmetrically, following a *Chebyshev distribution* (Henrici, 1964; Dahlquist,1974) , etc.

References:

Dahlquist, G. and Björck, Å., 1974, *Numerical Methods*, Prentice-Hall, Inc., Englewood Cliffs.

Denavit, J. and Hartenberg, R.S., 1964. *Kinematic Synthesis of Linkages*, McGraw-Hill Book Company, New York.

Henrici, P., 1964. *Elements of Numerical Analysis*, John Wiley & Sons, Inc. , New York.

Roberts, S., 1876, "On three-bar motion in plane space," *Proc. London Math. Society*, Vol. 7

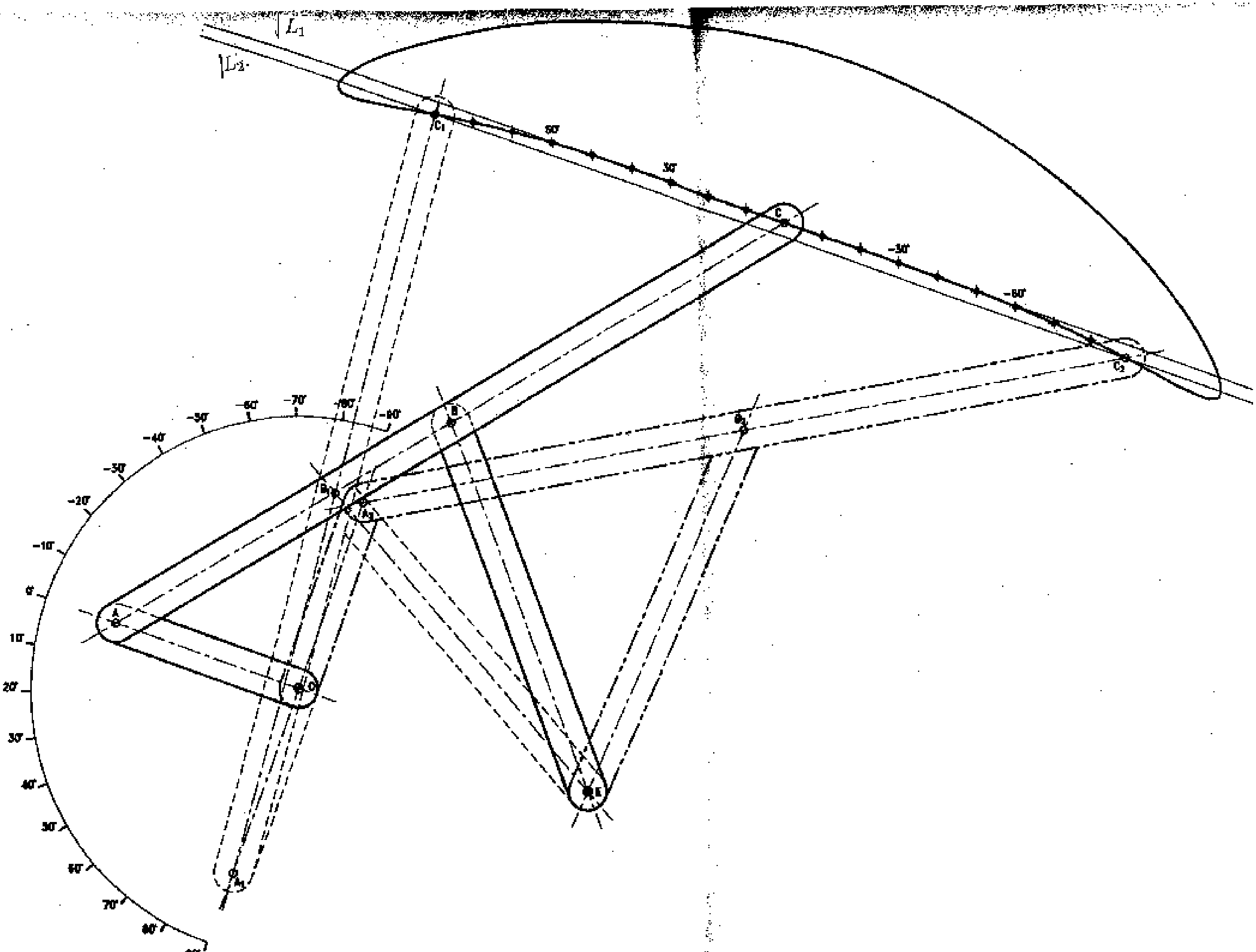

Fig. 1: Proposed straight-line tracer with uniform speed transmission