MECH 541 Kinematic Synthesis

Project # 1

Assigned: September 1st, 2009 Due: October 4, 2009

Synthesis of an Ackermann Steering Linkage for Automotive Design

The purpose of this project is to produce a planar four-bar linkage that will approximate the *Ackermann steering conditions*[1] using a least-square approach. As illustrated in Fig. 1, the Ackermann steering condition (ASC) states that:

> *The axes of the front wheels of a terrestrial vehicle must intersect the axis of the rear axle.*

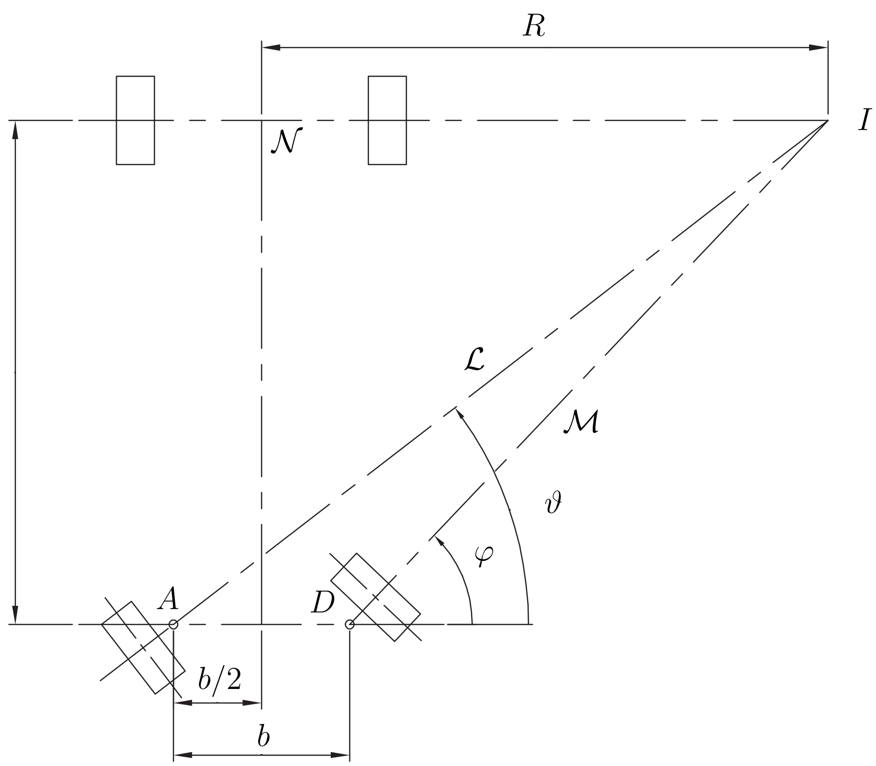

Figure 1: An illustration of the Ackermann steering condition

With the notation of Fig. 1, the Ackermann condition takes the form

$$\sin(\varphi - \vartheta) - \rho \sin\vartheta \sin\varphi = 0, \quad \rho = \frac{b}{a} \tag{1}$$

[1]Ackermann is sometimes spelled as "Ackerman."

which relates the two wheel steering angles by means of an implicit nonlinear function. The two foregoing angles are regarded as the input and output of a four-bar linkage, as illustrated in Fig. 2, which are known[2] to satisfy the *Freudenstein input-output equation*:

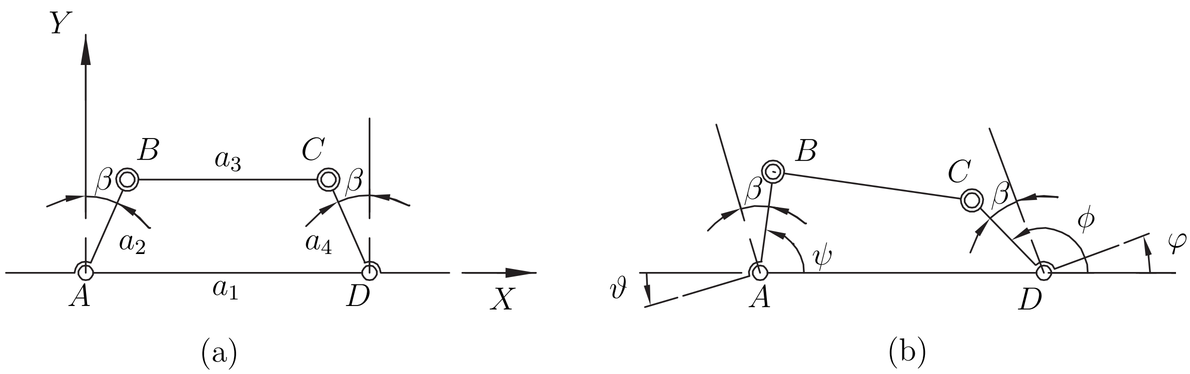

Figure 2: A planar four-bar linkage used to realize the Ackermann condition under: (a) a straight-course configuration and (b) negotiating a curve

$$k_1 + k_2 \cos\phi - k_3 \cos\psi = \cos(\phi - \psi) \tag{2a}$$

with the *Freudenstein parameters* $\{ k_i \}_1^3$ defined below:

$$k_1 \equiv \frac{a_1^2 + a_2^2 - a_3^2 + a_4^2}{2a_2 a_4}, \quad k_2 \equiv \frac{a_1}{a_2}, \quad k_3 = \frac{a_1}{a_4} \tag{2b}$$

in which $\{ a_i \}_1^4$ denote the four link lengths, defined in Fig. 2. Moreover, the relations between angles ψ and ϑ as well as angles ϕ and φ, for $i = 1, 2$, are derived from Fig. 2b, namely,

$$\psi = \pi/2 - (\beta - \vartheta), \quad \phi = \pi/2 + (\beta + \varphi) \tag{3}$$

Prescribe *judiciously* a set of $m > 3$ input-output pairs $\{ \psi_i, \phi_i \}_1^m$ within a certain steering range, in terms of the steering capabilities required by your task, and that you must include in your report. Next, write the Freudenstein equation m times, once for each pair of input-output values, thereby obtaining m equations linear in $\{ k_i \}_1^3$, namely,

$$\mathbf{Sk} = \mathbf{b} \tag{4}$$

where $\mathbf{S}$ is the $m\times 3$ *synthesis matrix*; $\mathbf{k}$ is the 3-dimensional vector of unknown Freudenstein parameters; and $\mathbf{b}$ is an m-dimensional vector of known components, i.e.,

$$\mathbf{S} \equiv \begin{bmatrix} 1 & \cos\phi_1 & -\cos\psi_1 \\ 1 & \cos\phi_2 & -\cos\psi_2 \\ \vdots & \vdots & \vdots \\ 1 & \cos\phi_m & -\cos\psi_m \end{bmatrix}, \quad \mathbf{k} \equiv \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}, \quad \mathbf{b} \equiv \begin{bmatrix} \cos(\phi_1 - \psi_1) \\ \cos(\phi_2 - \psi_2) \\ \vdots \\ \cos(\phi_m - \psi_m) \end{bmatrix} \tag{5}$$

[2]The input-output function of four-bar linkages is intensively studied in Ch. 2 of the Lecture Notes.

Now, as the number of unknowns is only three, and the number of equations m is greater than three, an overdetermined system of equations is obtained, that cannot be satisfied by any vector $\mathbf{k}$. What you should do is, then, look for the vector $\mathbf{k}$ that provides the *best approximation* to the m equations in the least-square sense.

Choose a large enough number of input-outpairs that will give you the best possible fit. You may have to try various values of m, between, say, five and 20. In each case, record the rms value of the components of the *design-error vector* $\mathbf{e}$, defined as

$$\mathbf{e} \equiv \mathbf{S}\mathbf{k} - \mathbf{b} \tag{6}$$

Project requirements:

1. A piece of code that will accept input-output pairs of values, in degrees, and return the least-square approximation $\mathbf{k}_0$ to the m synthesis equations, along with the rms value of the components of the design error vector;
2. the linkage dimensions of the steering mechanism of a truck with one single rear axle, used to pull a trailer;
3. the linkage dimensions of a very long limousine, for which ρ is substantially smaller than for item 2;
4. numerical results for various values of m, the results including the condition number of the synthesis matrix; and
5. comments on your results.

Remark: *The link lengths computed may turn out to be too big to allow for a practical implementation of the linkage in a truck or a limousine. Nevertheless, the four-bar linkage admits an* focal equivalent *multi-loop linkage of more appropriate dimensions, as proposed by Yao and Angeles (2000).*

The best results should be illustrated with—preferably CAD—sketches. Your report should be written according with the Report Guidelines posted on the course website.

Reference:

Yao, J. and Angeles, J., 2000, "The kinematic synthesis of steering mechanisms," *Transactions of the CSME*, Vol. 24, Nos. 3 & 4, pp. 453–475.