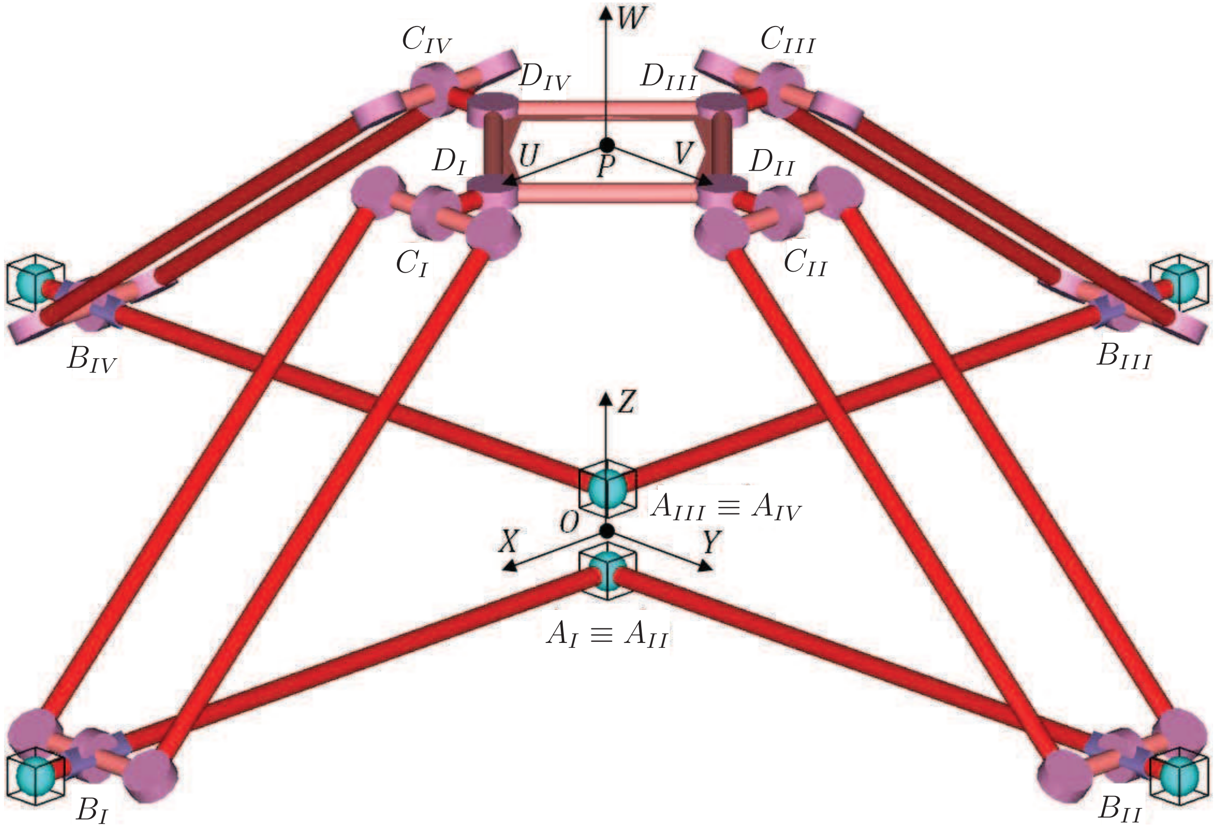

Figure 2.16: The Schönflies-motion generator developed at the University of the Basque Country, in Bilbao, Spain

3. the subset of displacements $\mathcal{D}_\Pi(\mathbf{n}_J)$ associated with the Π-joint, characterized by translations along circles of radius $\overline{B_J C_J}$ lying in the plane of the Jth parallelogram, of normal $\mathbf{n}_J$;

4. the rotation subgroup $\mathcal{R}(C_J, \mathbf{j})$, of axis of rotation passing through point C_J and parallel either to $\mathbf{j}$, for $J = I, III$ or to $\mathbf{i}$ for $J = II, IV$;

5. the rotation subgroup $\mathcal{R}(D_J, \mathbf{k})$ of axis of rotation passing through D_J and parallel to $\mathbf{k}$.

Therefore,

$$\mathcal{L}_J = \underbrace{\mathcal{P}(\mathbf{i}) \bullet \mathcal{R}(B_J, \mathbf{j}) \bullet \mathcal{D}_\Pi(\mathbf{n}_J) \bullet \mathcal{R}(C_J, \mathbf{j})}_{\mathcal{X}(\mathbf{j})} \bullet \mathcal{R}(D_J, \mathbf{k}) = \mathcal{X}(\mathbf{j}) \bullet \mathcal{R}(D_J, \mathbf{k}), \quad J = I, III$$

Likewise,

$$\mathcal{L}_J = \mathcal{X}(\mathbf{i}) \bullet \mathcal{R}(D_J, \mathbf{k}), \quad J = II, IV$$

Notice that none of the four bonds derived above is a subgroup of $\mathcal{D}$, which disqualifies the multiloop kinematic chain from being trivial. However, notice also that

$$\mathcal{X}(\mathbf{j}) \bullet \mathcal{R}(D_J, \mathbf{k}) = \mathcal{X}(\mathbf{k}) \bullet \mathcal{R}(C_J, \mathbf{j}), \quad J = I, III$$

and

$$\mathcal{X}(\mathbf{i}) \bullet \mathcal{R}(D_J, \mathbf{k}) = \mathcal{X}(\mathbf{k}) \bullet \mathcal{R}(C_J, \mathbf{i}), \quad J = II, IV$$

Therefore,

$$\mathcal{L}_J \cap \mathcal{L}_K = \mathcal{X}(\mathbf{k}), \quad J,\, K = I, \ldots IV, \quad J \neq K$$

thereby proving that, indeed, the intersection of all limb bonds is a subgroup of $\mathcal{D}$, namely, the Schönflies subgroup $\mathcal{X}(\mathbf{k})$. The dof f of the robot at hand is, thus,

$$f = \dim[\mathcal{X}(\mathbf{k})] = 4$$

and, according to Hervé's classification, the multiloop chain can be considered exceptional.

While the generalized CGK formula is more broadly applicable and less error-prone than its conventional counterpart, it is not error-free. Indeed, let us consider the HHHRRH closed chain of Fig. 2.17, first proposed by Hervé (1978). The four H pairs of this figure have distinct pitches.

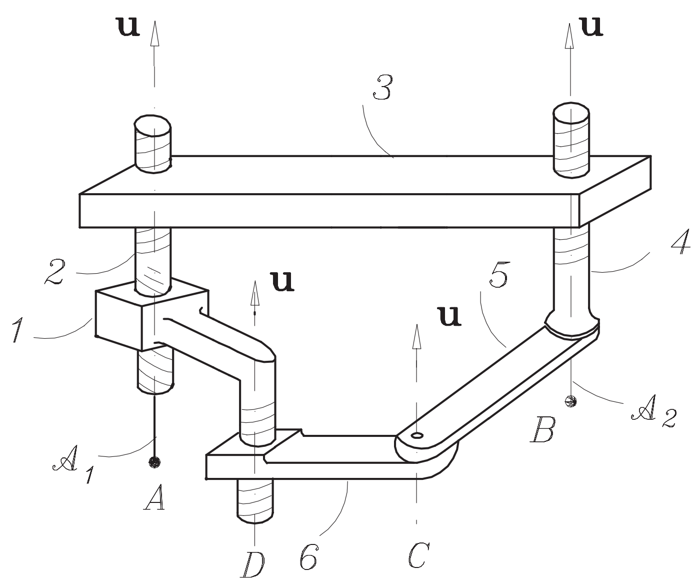

Figure 2.17: The HHHRRH mechanism

It is apparent that all links move in parallel planes, and that these planes also translate along their common normal direction. The displacement subgroup containing all possible kinematic bonds of the mechanism under study, of minimum dimension, is thus the Schönflies subgroup $\mathcal{X}(\mathbf{u})$, and hence, $d_m = 4$. Since we have six links and six joints, each of restriction $r_i = d_m - f_i$, for $f_i = 1$ and $i = 1, \ldots, 6$, the dof of the mechanism is obtained from the CGK formula as

$$f = 4(6-1) - 6 \times 3 = 2$$

However, the above result is wrong, for it predicts a too large dof. Indeed, the mechanism has one idle dof, as can be readily shown by means of a bond analysis: Let us