If the chain is composed of l links and p kinematic pairs, then its dof f is given by the difference between its total dof before coupling and the sum of its restrictions, i.e.,

$$f = d_m(l - 1) - \sum_{i=1}^{p} r_i \tag{2.12}$$

The above relation can be termed a *generalized Chebyshev-Grübler-Kutzbach* (CGK) formula in that it generalizes the concept involved in parameter d_m above. Conventional CGK formulas usually consider that d_m can attain one of two possible values, 3 for planar and spherical chains and 6 for spatial chains. In the generalized formula, d_m can attain any of the values 2, 3, 4, or 6. Moreover, rather than considering only three subgroups of displacements, we consider all 12 described above, none of which is of dimension five.

As an example of the application of the above formula, we consider the *vise mechanism*, displayed in Fig. 2.14. In that figure, we distinguish three links and three LKPs. The links are the frame 1, the crank 2 and the slider 3, which define three bonds, namely,

$$\mathcal{L}(1,2) = \mathcal{R}(\mathcal{A}), \quad \mathcal{L}(2,3) = \mathcal{H}(\mathcal{A}), \quad \mathcal{L}(3,1) = \mathcal{P}(\mathbf{a})$$

in which $\mathcal{A}$ is the common axis of the R and the H pairs, while $\mathbf{a}$ is the unit vector parallel to $\mathcal{A}$. In this case, it is apparent that all three bonds lie in the $\mathcal{C}$ subgroup, and hence, $d_m = 2$. Moreover, if we number the three joints in the order R, H, P, and notice that the dimension d_i associated with each of the three joints is unity, then $r_i = 1$, for $i = 1, 2, 3$. Application of the generalized CGK formula (2.12) yields

$$f = 2(3 - 1) - 3 \times 1 = 4 - 3 = 1$$

which is indeed the correct value of the vise dof.

2.7.2 Exceptional Chains

The Sarrus mechanism of Figs. 2.12 and 2.13 is an example of an exceptional chain. Indeed, all its links undergo motions of either one of two planar subgroups, $\mathcal{F}(\mathbf{u})$ and $\mathcal{F}(\mathbf{v})$. Moreover, the product of these two subgroups does not yield the group $\mathcal{D}$—notice that the linkage has two sets of R pairs, each parallel to a distinct unit vector, $\mathbf{u}$ or $\mathbf{v}$. The dof of this mechanism can still be found, but not with the aid of the CGK formula of eq.(2.12), for all its kinematic bonds do not belong to the same subgroup of $\mathcal{D}$. This dof is found, rather, as the dimension of the intersection of the two foregoing subgroups, i.e.,

$$f = \dim[\mathcal{F}(\mathbf{u}) \cap \mathcal{F}(\mathbf{v})] = \dim[\mathcal{P}(\mathbf{u} \times \mathbf{v})] = 1$$

Another example of exceptional chain is the familiar slider-crank mechanism of internal combustion engines and compressors, as shown in Fig. 2.15. It is customary to represent this mechanism as a planar RRRP mechanism. However, a close look at the coupling of

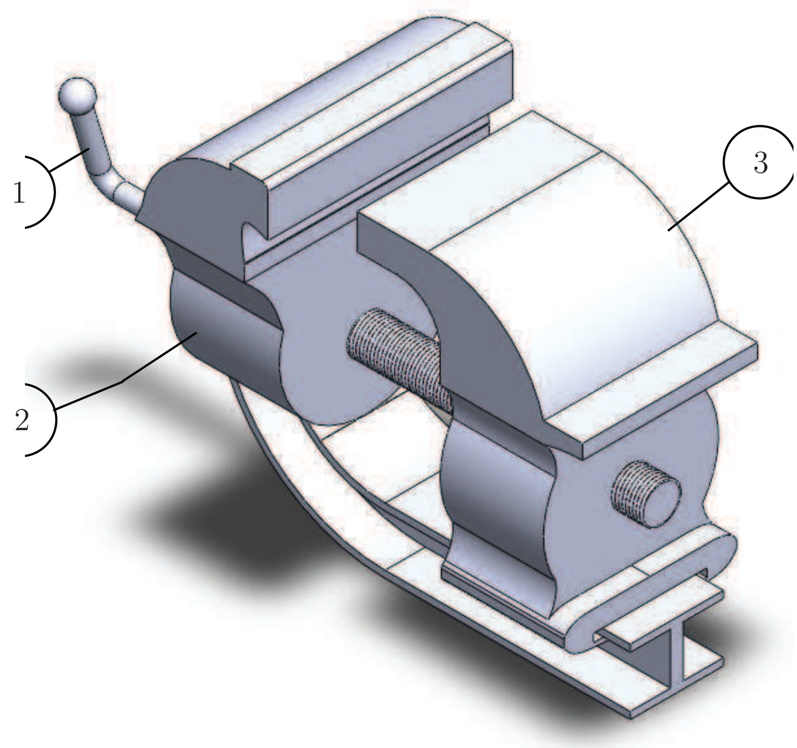

Figure 2.14: The well-known vise mechanism

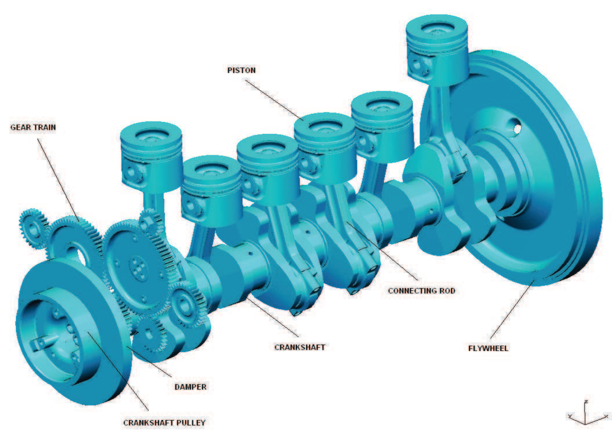

Figure 2.15: The slider-crank mechanism as a key component of an internal combustion engine: a power-generation system with six cylinders in line (courtesy of MMM International Motores, Campinas, Brazil)