2.6 Kinematic Bonds

Displacement subgroups can be combined to produce new sets of displacements that may or may not be displacement subgroups themselves. To combine subgroups, we resort to the group operations of product ($\bullet$) and intersection ($\cap$).

Now we introduce the concept of *kinematic bond*, which is a generalization of kinematic pair, as first proposed by Hervé (1978), who called this concept *liaison cinématique* in French. This concept has been termed *kinematic liaison* (Angeles, 1982) or *mechanical connection* (Hervé, 1999) in English. Since "liaison" in English is usually applied to human relations, the term "bond" seems more appropriate, and is thus adopted here.

We illustrate the concept with an example: Let us assume three links, numbered from 1 to 3, and coupled by two kinematic pairs generating the two subgroups $\mathcal{G}_1$ and $\mathcal{G}_2$, where these two subgroups are instanced by specific displacement subgroups below. We then have

$$\begin{aligned}
\mathcal{G}_1 \bullet \mathcal{G}_2 &= \mathcal{R}(\mathcal{A}) \bullet \mathcal{P}(\mathbf{e}) = \mathcal{C}(\mathcal{A}), \quad \text{for } \mathbf{e} \parallel \mathcal{A} && \text{(2.10a)}\\
\mathcal{G}_1 \bullet \mathcal{G}_2 &= \mathcal{R}(\mathcal{A}) \bullet \mathcal{T}_2(\mathbf{u}, \mathbf{v}) = \mathcal{F}(\mathbf{e}), \quad \text{for } \mathbf{e},\ \mathcal{A} \perp \mathbf{u},\ \mathbf{v} && \text{(2.10b)}\\
\mathcal{G}_1 \bullet \mathcal{G}_2 &= \mathcal{R}(\mathcal{A}) \bullet \mathcal{R}(\mathcal{B}) = \mathcal{L}(1,3) && \text{(2.10c)}\\
\mathcal{G}_1 \cap \mathcal{G}_2 &= \mathcal{R}(\mathcal{A}) \cap \mathcal{C}(\mathcal{A}) = \mathcal{R}(\mathcal{A}) && \text{(2.10d)}\\
\mathcal{G}_1 \cap \mathcal{G}_2 &= \mathcal{R}(\mathcal{A}) \cap \mathcal{S}(O) = \mathcal{R}(\mathcal{A}), \quad \text{for } O \in \mathcal{A} && \text{(2.10e)}\\
\mathcal{G}_1 \cap \mathcal{G}_2 &= \mathcal{R}(\mathcal{A}) \cap \mathcal{P}(\mathbf{e}) = \mathcal{I}, \quad \text{for any } \mathcal{A},\ \mathbf{e} && \text{(2.10f)}
\end{aligned}$$

All of the above examples, except for the third one, amount to a displacement subgroup. This is why no subgroup symbol is attached to that set. Instead, we have used the symbol $\mathcal{L}(1,3)$ to denote the kinematic bond between the first and third links of the chain. In general, a kinematic bond between links i and n of a kinematic chain, when no ambiguity is possible, is denoted by $\mathcal{L}(i,n)$. When the chain connecting these two links is not unique, such as in a closed chain, where these two links can be regarded as connected by two possible *paths*, a subscript will be used, e.g., $\mathcal{L}_I(i,j)$, $\mathcal{L}_{II}(i,j)$, etc. A kinematic bond is, thus, a set of displacements, as stemming from a binary operation of displacement subgroups, although the bond itself need not be a subgroup. Obviously, the 12 subgroups described above are themselves special cases of kinematic bonds.

The kinematic bond between links i and n can be conceptualized as the product of the various subgroups associated with the kinematic pairs between the ith and the nth links. To keep the discussion general enough, we shall denote the subgroup associated with the kinematic pair coupling links i and $i+1$ as $\mathcal{L}(i, i+1)$, with a similar notation for all other kinematic-pair subgroups. Thus,

$$\mathcal{L}(i,n) = \mathcal{L}(i,i+1) \bullet \mathcal{L}(i+1,i+2) \bullet \cdots \bullet \mathcal{L}(n-1,n) \tag{2.11}$$

For example, in a six-axis serial manipulator, we can set $i = 1$, $n = 7$, all six kinematic pairs in-between being revolutes of skew axes $\mathcal{R}(\mathcal{A}_1)$, $\mathcal{R}(\mathcal{A}_2)$, $\ldots$, $\mathcal{R}(\mathcal{A}_6)$. Then,

$$\mathcal{L}(1, 7) = \mathcal{D}$$

That is, the manipulator is a generator of the general six-dimensional group of rigid-body displacements $\mathcal{D}$.

As an example of group-intersection, let us consider the *Sarrus mechanism*, depicted in Fig. 2.12.

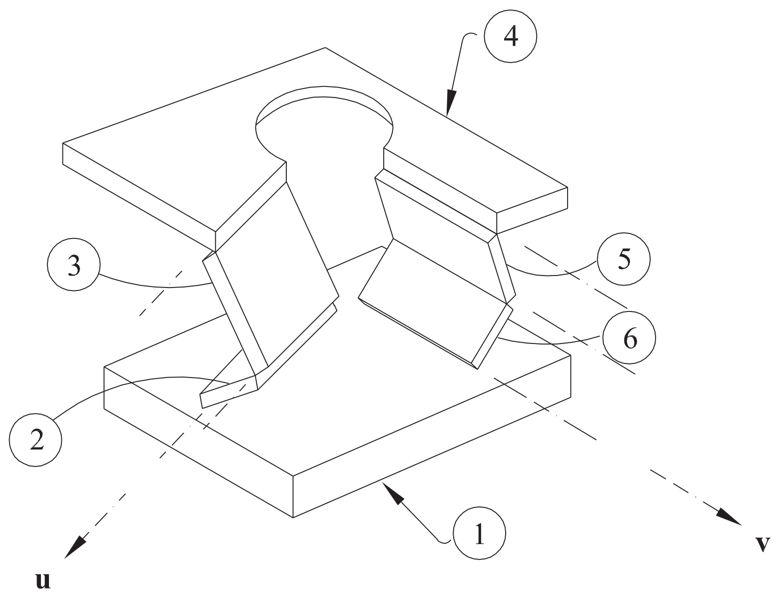

Figure 2.12: The Sarrus mechanism

A less common realization of the Sarrus mechanism is depicted in Fig. 2.13. This is a $\Pi\Pi\Pi\Pi\Pi\Pi$ closed kinematic chain, modelled as a *compliant mechanism*, which bears a *monolithic* structure, made of a polymer. The R joints of the mechanism are realized by removing material at the joint locations, so as to render these areas much more compliant than the other areas. The mechanism is designed so as to serve as a uniaxial accelerometer.

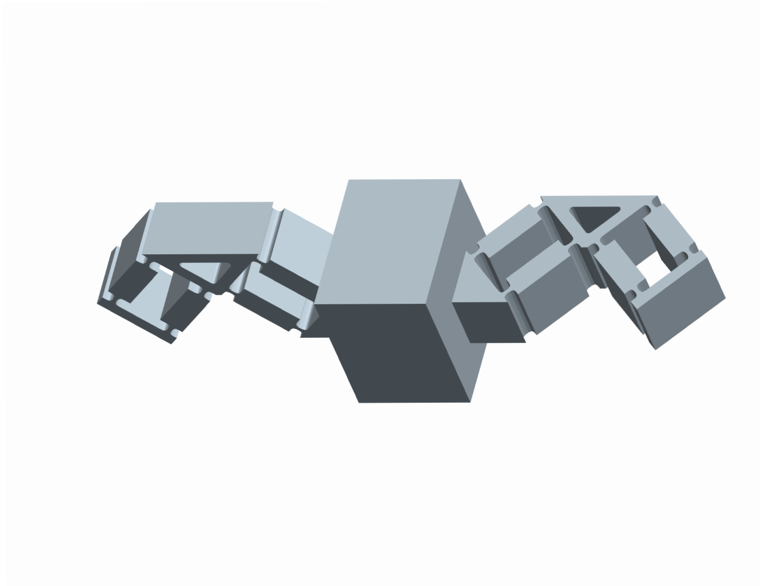

Figure 2.13: An alternative realization of the Sarrus mechanism

In the Sarrus mechanism of Fig. 2.12, we have six links, coupled by six revolute pairs. Moreover, the revolute pairs occur in two triplets, each on one leg of the mechanism. The axes of the three revolute pairs of each leg are parallel to each other. The bond $\mathcal{L}(1,4)$, apparently, is not unique, for it can be defined by traversing any of the two legs. Let the leg of links 1, 2, 3 and 4, coupled by revolutes of axes parallel to the unit vector $\mathbf{u}$, be labelled I; the other leg, of links 4, 5, 6 and 1, coupled by revolutes of axes parallel to the unit vector $\mathbf{v}$, is labelled II. It is apparent that, upon traversing leg I, we obtain

$$\mathcal{L}_I(1,4) = \mathcal{F}(\mathbf{u})$$

Moreover, upon traversing leg II,

$$\mathcal{L}_{II}(1,4) = \mathcal{F}(\mathbf{v})$$

That is, leg I is a generator of the planar subgroup $\mathcal{F}$ of plane normal to vector $\mathbf{u}$, while leg II is that of the subgroup $\mathcal{F}$ of plane normal to vector $\mathbf{v}$. Therefore, the intersection $\mathcal{L}_I(1,4) \cap \mathcal{L}_{II}(1,4)$ is the set of displacements common to the two $\mathcal{F}$-subgroups, namely, the prismatic subgroup of translations in the direction $\mathbf{w} = \mathbf{v} \times \mathbf{u}$, i.e.,

$$\mathcal{L}_I(1,4) \cap \mathcal{L}_{II}(1,4) = \mathcal{P}(\mathbf{w})$$

The Sarrus mechanism is thus a revolute realization of the prismatic joint.