# CS-417 INTRODUCTION TO ROBOTICS AND INTELLIGENT SYSTEMS

## Ultrasonic Sensing and Mapping

Ioannis Rekleits

# Sonar sensing

"The sponge"

sonar timeline

**0**

a "chirp" is emitted into the environment

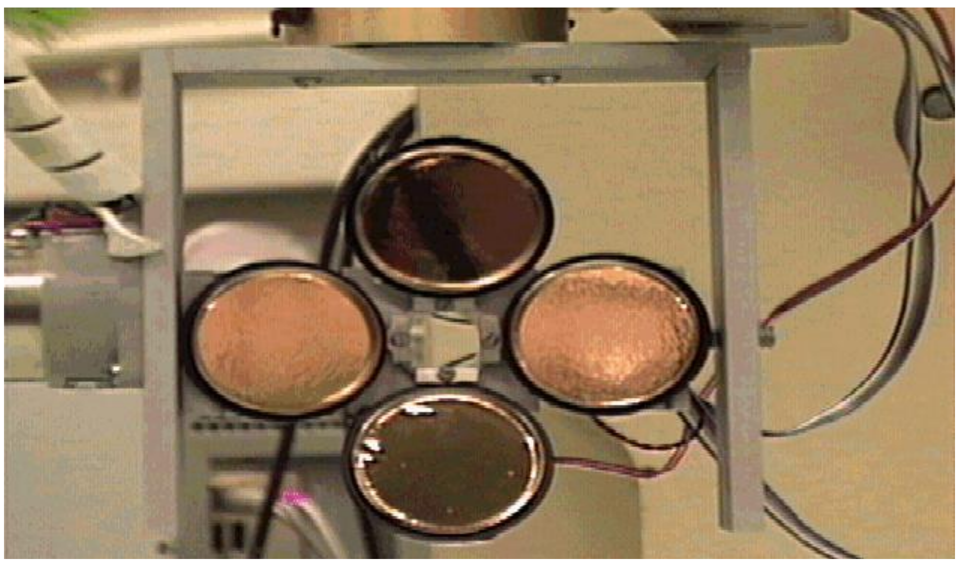
**75μs**

typically when reverberations from the initial chirp have stopped

the transducer goes into "receiving" mode and awaits a signal...

**.5s**

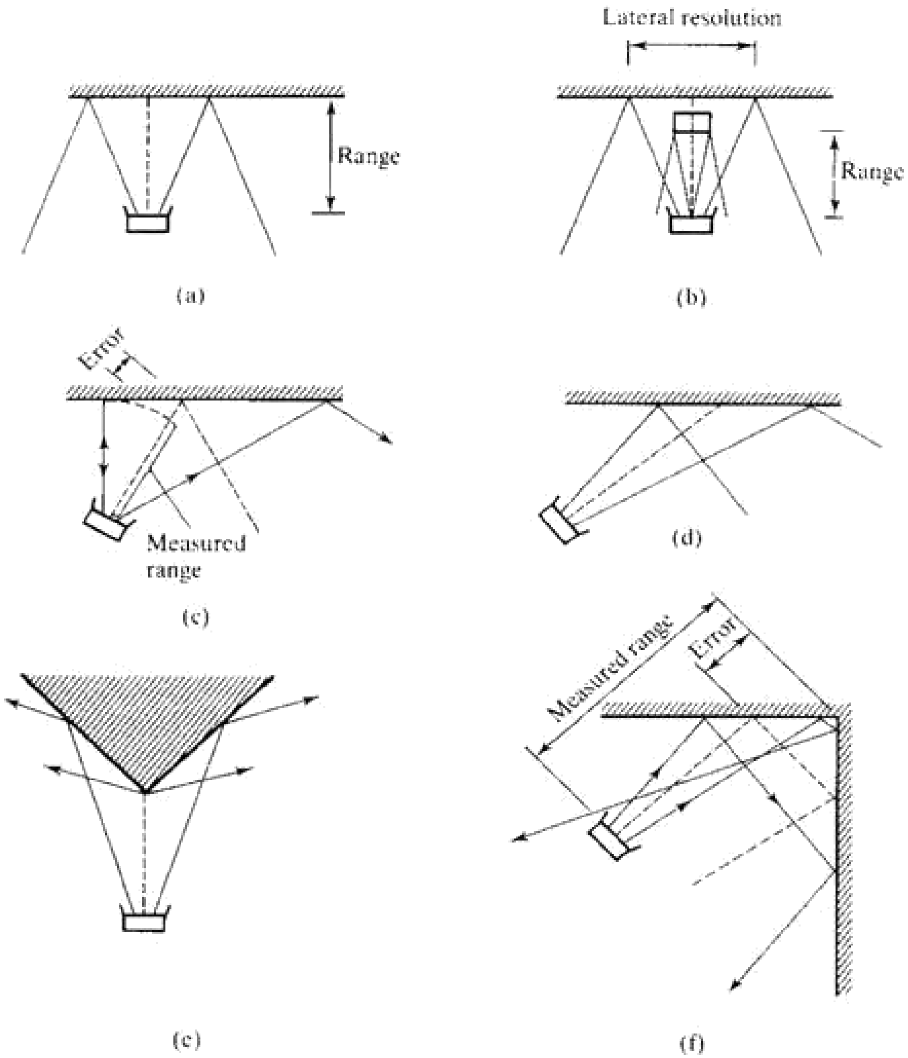
after a short time, the signal will be too weak to be detected



Polaroid sonar emitter/receivers

Why is sonar sensing limited to between ~12 in. and ~25 feet ?

# Sonar effects




(a) Sonar providing an accurate range measurement

(b-c) Lateral resolution is not very precise; the closest object in the beam's cone provides the response

(d) Specular reflections cause walls to disappear

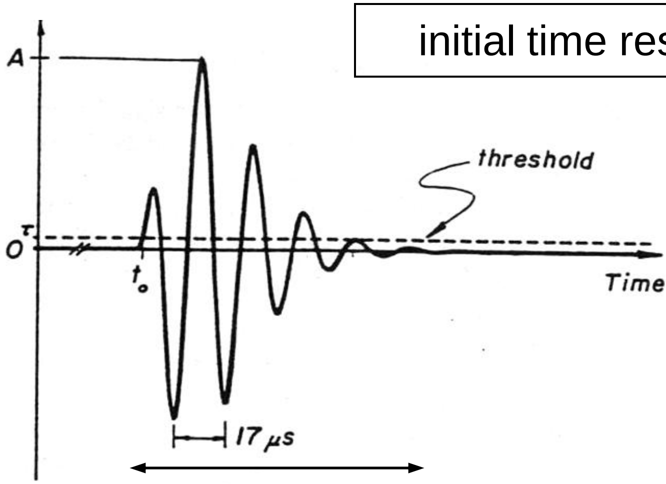
(e) Open corners produce a weak spherical wavefront

(f) Closed corners measure to the corner itself because of multiple reflections --> sonar ray tracing

resolution: time / space

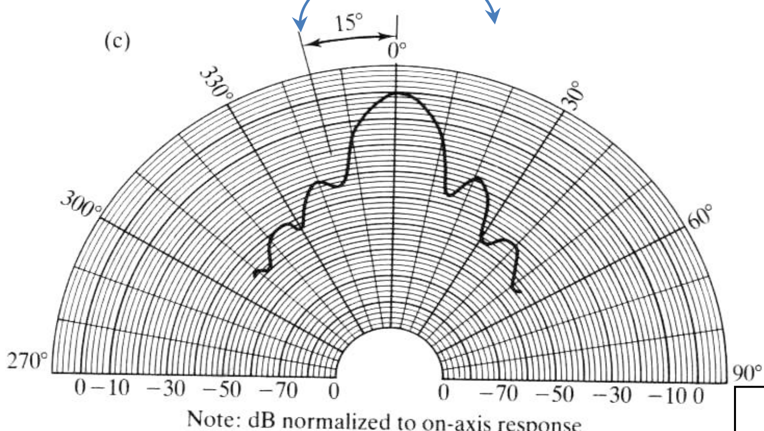
# Sonar modeling

initial time response



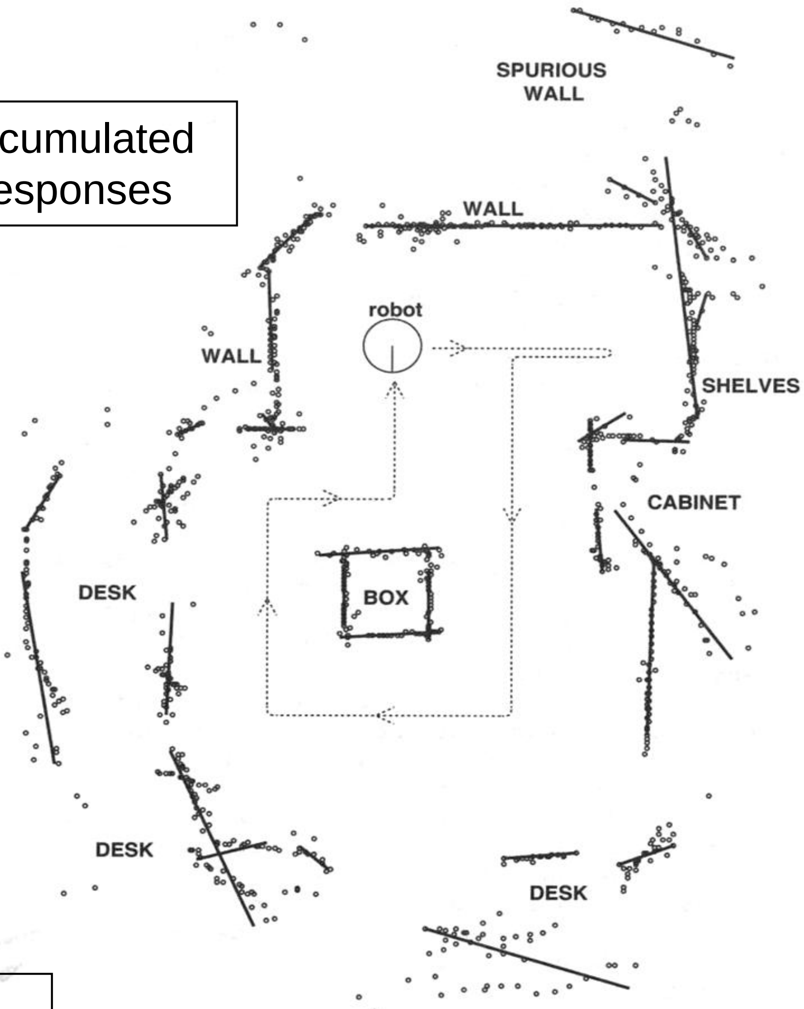

blanking time

cone width




spatial response

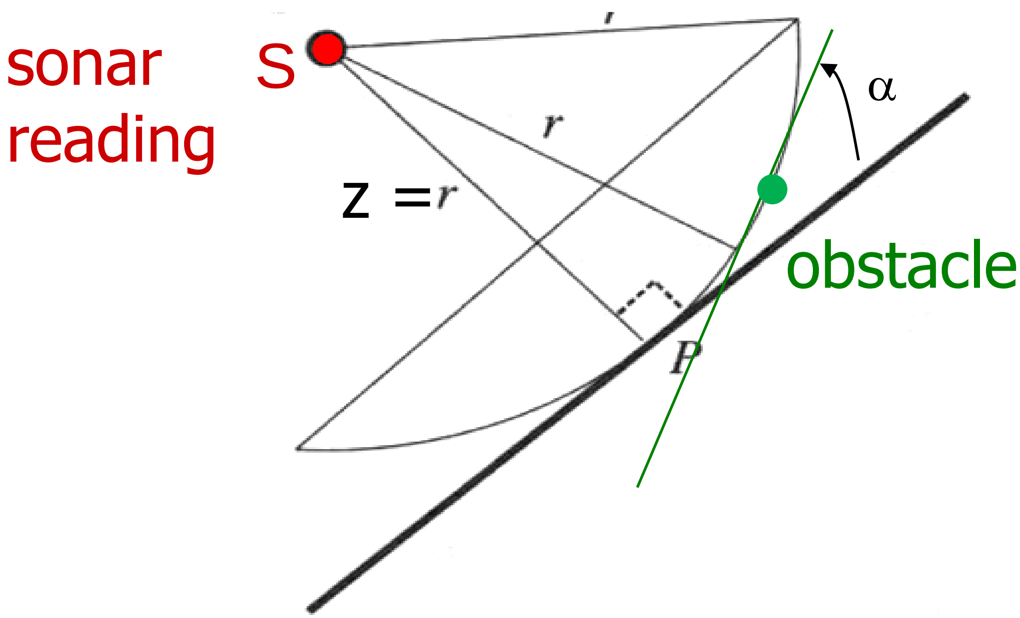
accumulated responses



# Sonar Modeling

response model (Kuc)

$$h_R(t, z, a, \alpha) = \frac{2c\cos\alpha}{\pi a \sin\alpha}\sqrt{1 - \frac{c^2(t - 2z/c)^2}{a^2 \sin^2 \alpha}}$$




- Models the response, $h_R$, with:

  c = speed of sound

  a = diameter of sonar element

  t = time

  z = orthogonal distance

  $\alpha$ = angle of environment surface

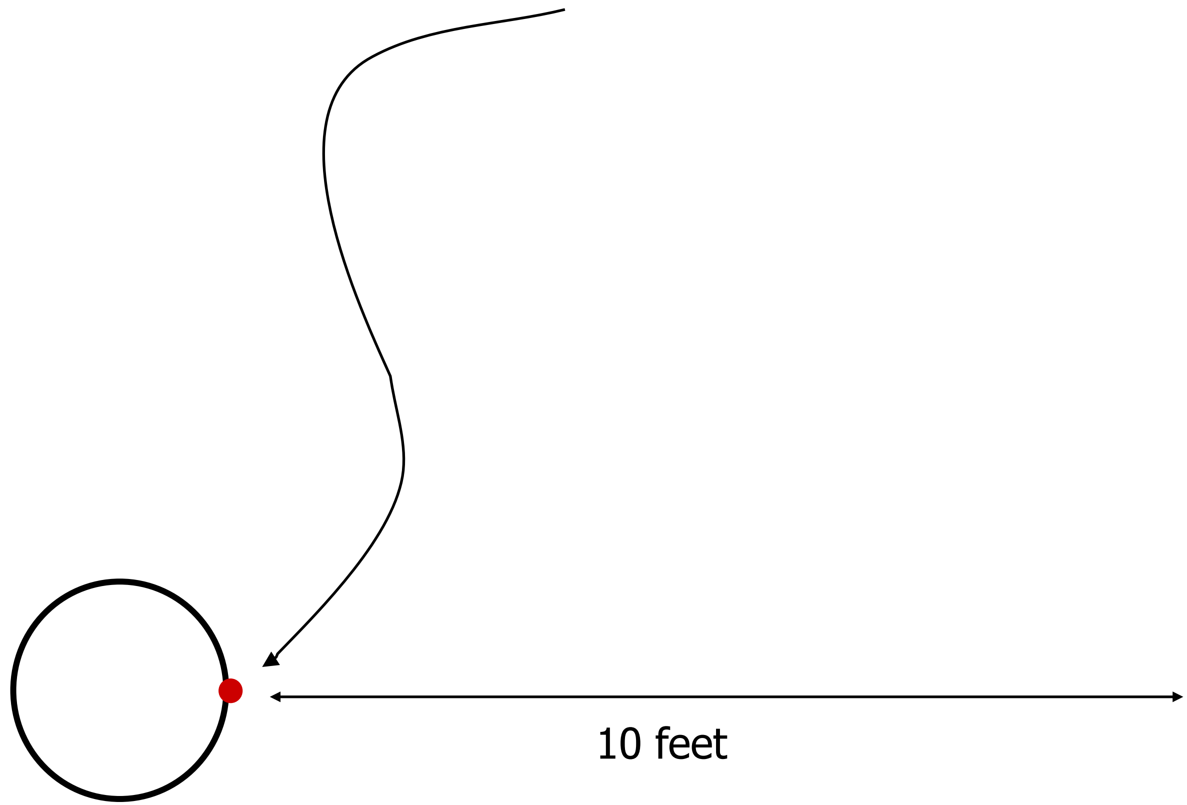
- Then, add noise to the model to obtain a probability: p( S | o )

chance that the sonar reading is S,

given an obstacle at location o

# Using sonar to create maps

What should we conclude if this sonar reads 10 feet?



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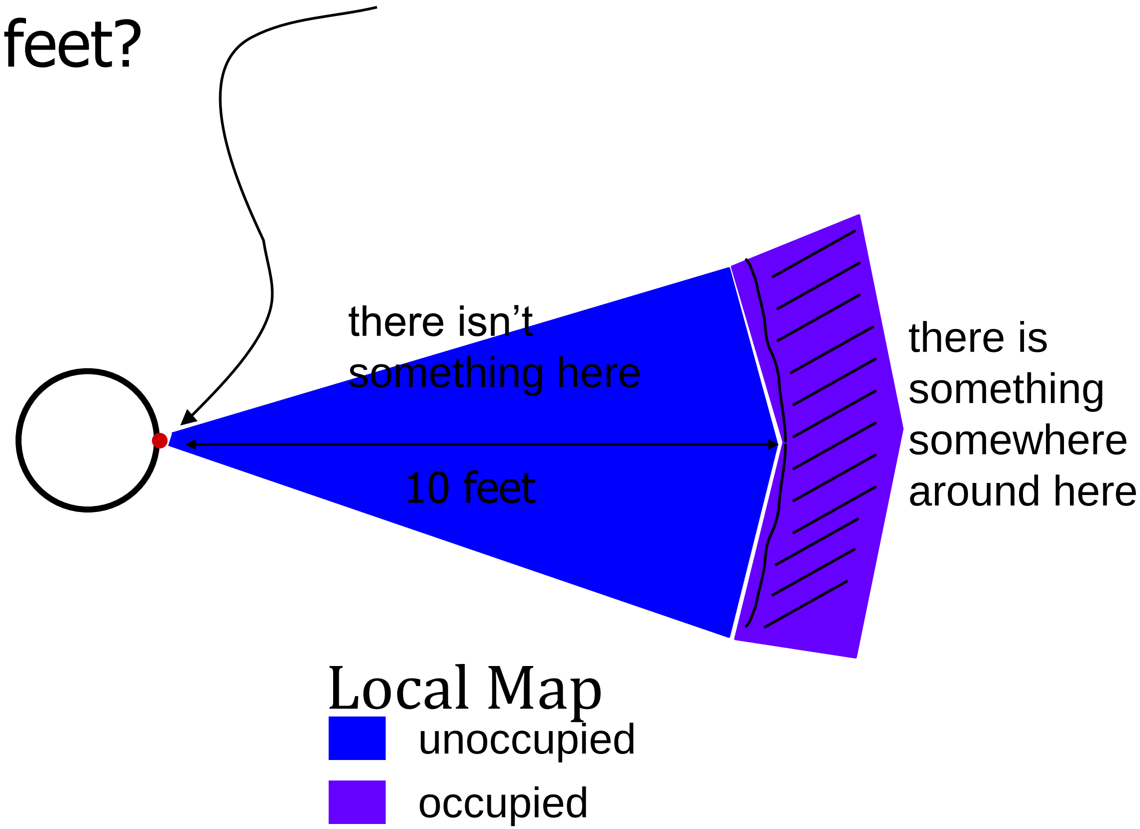

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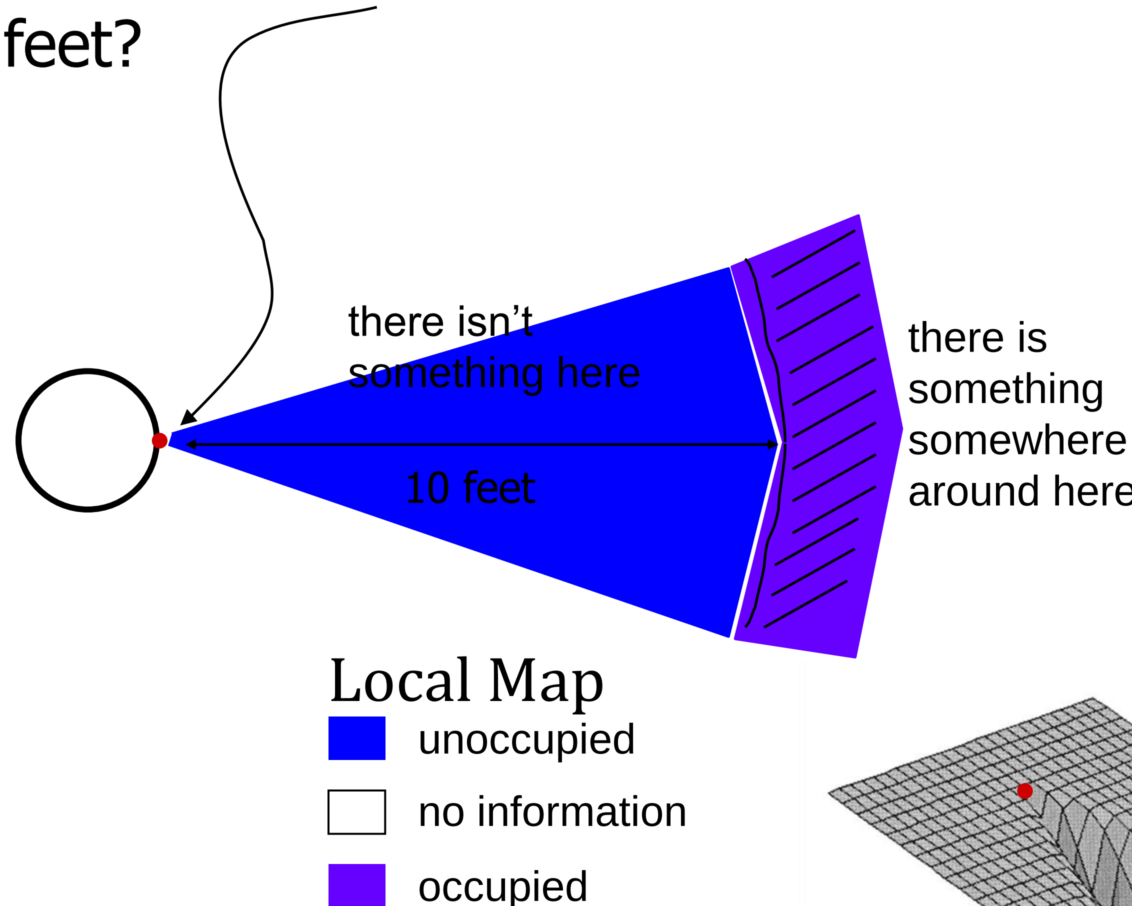
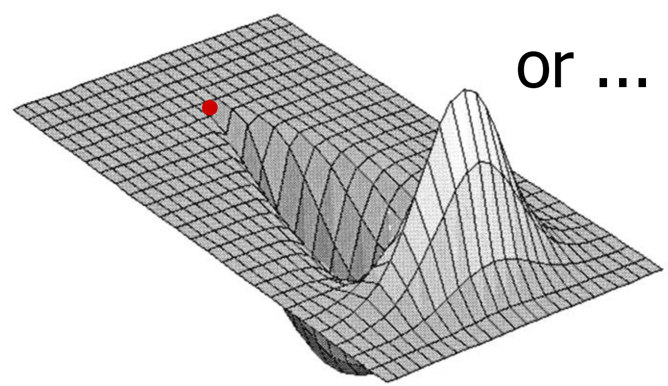



or ...



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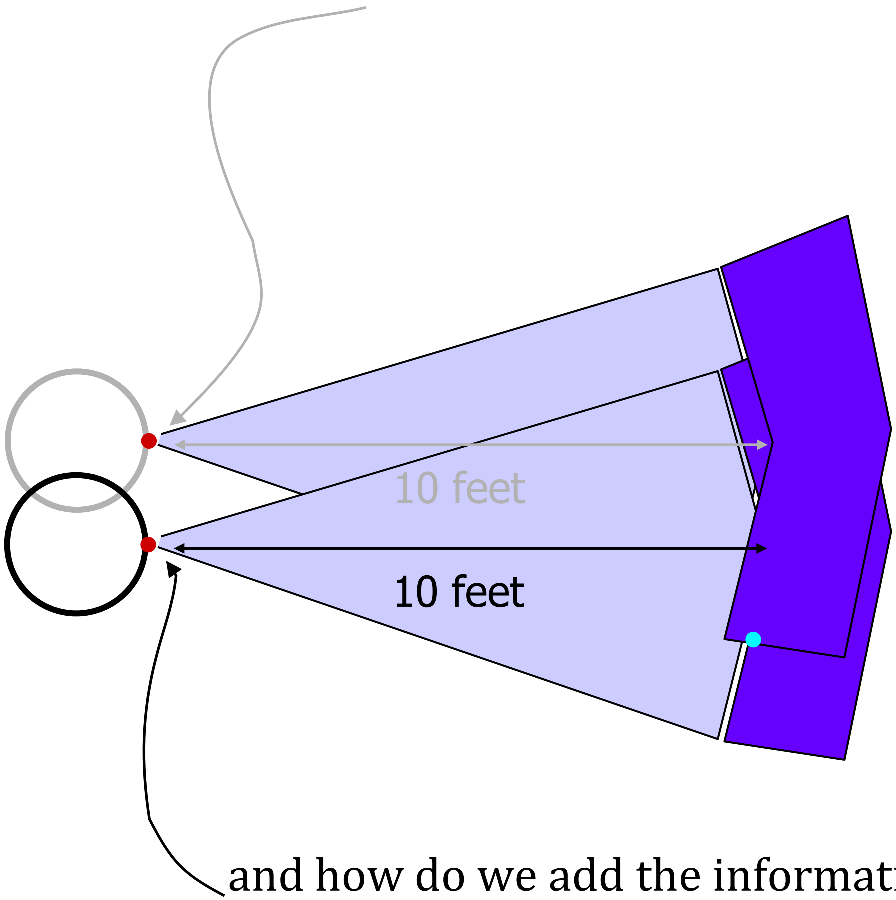
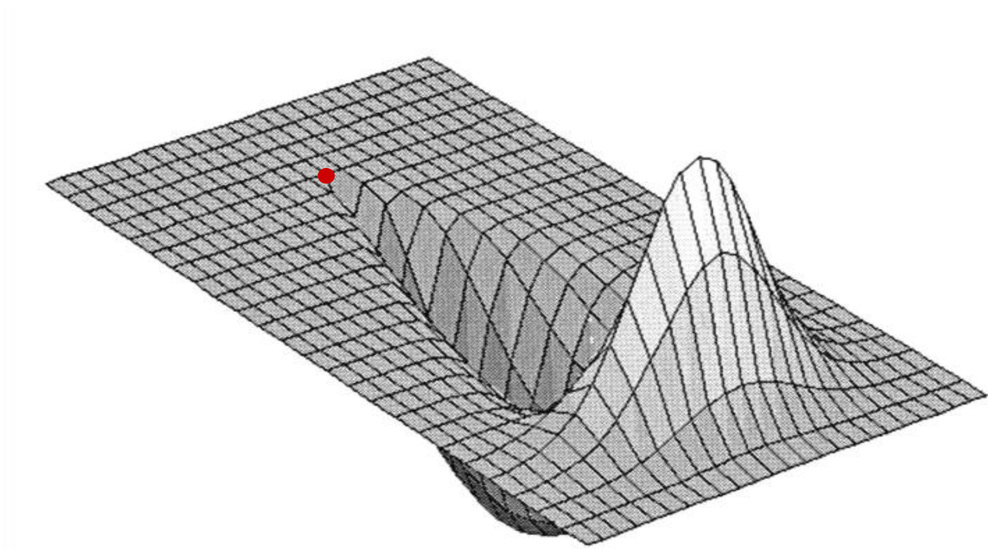



and how do we add the information that the next sonar reading (as the robot moves) reads 10 feet, too?

# Combining sensor readings

- The key to making accurate maps is combining lots of data.
- But combining these numbers means we have to know what they are !



what is in each cell of this sonar model / map ?

What should our map contain ?

- small cells
- each represents a bit of the robot's environment
- larger values => obstacle
- smaller values => free

# What is it a map of?

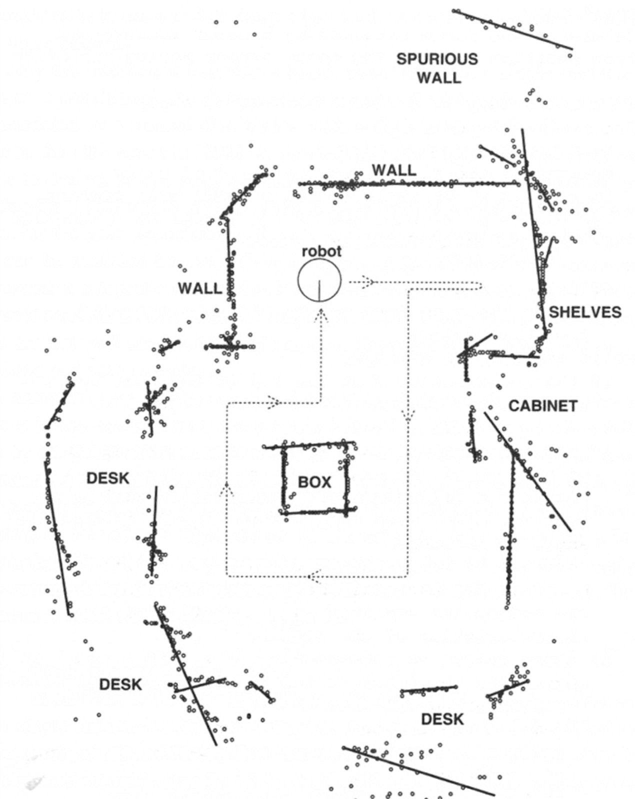
Several answers to this question have been tried:

It's a map of occupied cells.

pre '83

$o_{xy}$ ← cell (x,y) is occupied

$\overline{o}_{xy}$ ← cell (x,y) is unoccupied




Each cell is either occupied or unoccupied -- this was the approach taken by the Stanford Cart.

What information **should** this map contain, given that it is created with sonar ?

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---

'83 - '88 It's a map of probabilities: $p(\, o \mid S_{1..i} \,)$ ← The certainty that a cell is **occupied**, given the sensor readings $S_1, S_2, \ldots, S_i$

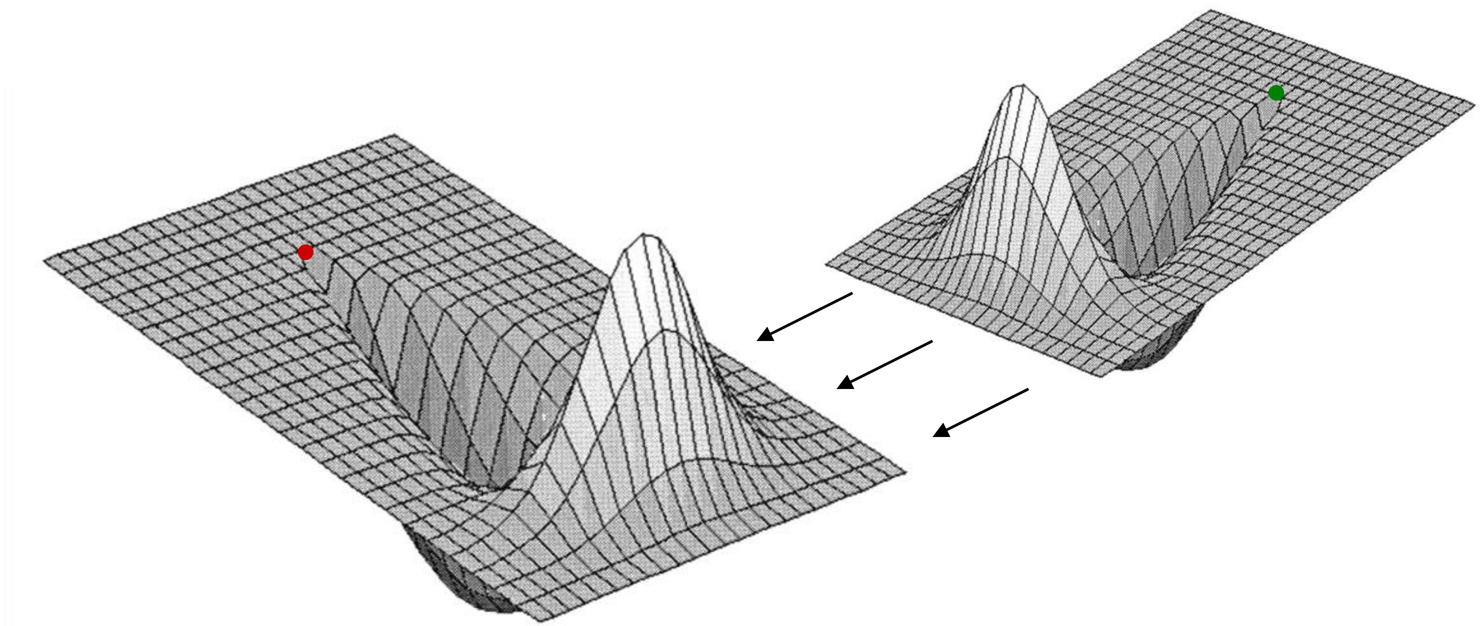
$p(\, o \,\underline{|}\, S_{1..i} \,)$ ← The certainty that a cell is **unoccupied**, given the sensor readings $S_1, S_2, \ldots, S_i$

- maintaining related values separately?
- initialize all certainty values to zero
- contradictory information will lead to both values near 1
- combining them takes some work...

# A Geometric (non-probabilistic) Approach

## Arc-Carving

# Combining probabilities



How to combine two sets of probabilities into a single map ?

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---

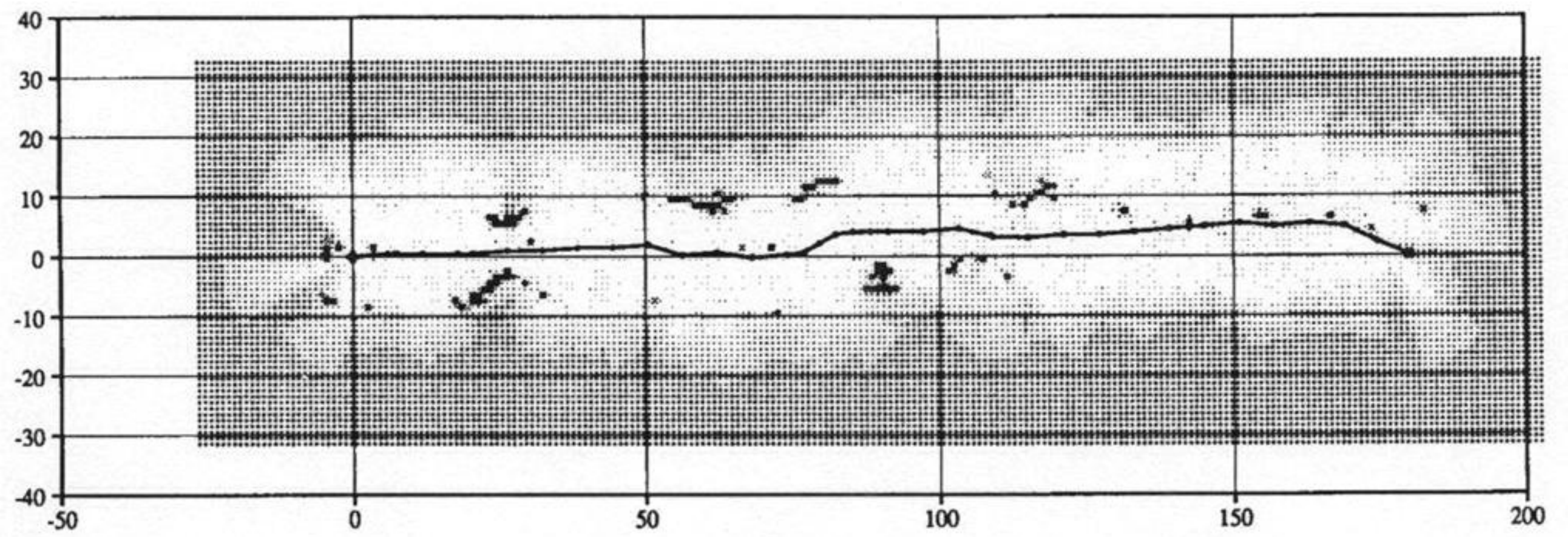
★ It's a map of *odds*. The odds of an event are expressed *relative to the complement* of that event.

The odds that a cell is **occupied**, given the sensor readings $S_1, S_2, \ldots, S_i$ →

$$odds(o \mid S_{1..i}) = \frac{p(o \mid S_{1..i})}{p(\overline{o} \mid S_{1..i})}$$

probabilities ↙

# An example map




units: feet

Evidence grid of a tree-lined outdoor path

lighter areas: *lower* odds of obstacles being present

darker areas: *higher* odds of obstacles being present

how to combine them?

# Conditional probability

Some intuition...

$$p(\,o \mid S\,) = \frac{\text{The probability of event } \mathbf{o}\text{, given event } \mathbf{S}\,.}{\text{The probability that a certain cell } \mathbf{o} \text{ is occupied, given that the robot sees the sensor reading } \mathbf{S}\,.}$$

$$p(\,S \mid o\,) = \frac{\text{The probability of event } \mathbf{S}\text{, given event } \mathbf{o}\,.}{\text{The probability that the robot sees the sensor reading } \mathbf{S}\text{, given that a certain cell } \mathbf{o} \text{ is occupied.}}$$

- What is really meant by conditional probability ?
- How are these two probabilities related?

# Bayes Rule

- Conditional probabilities

$$p( o \wedge S ) = p( o \mid S ) \, p( S )$$

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$$p(o \mid S) = \frac{p(o \mid S)\, p(o)}{p(S)}$$

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Bayes rule

- So, what does this say about odds( o | $S_2 \wedge S_1$ ) ?

Can we update easily ?

# Combining evidence

So, how do we combine evidence to create a map?

What we want --

$$\text{odds}( o \mid S_2 \wedge S_1)$$

the new value of a cell in the map after the sonar reading $S_2$

---

What we know --

$$\text{odds}( o \mid S_1)$$

the old value of a cell in the map (before sonar reading $S_2$)

$$p( S_i \mid o ) \;\; \& \;\; p( S_i \mid \overline{o} )$$

the probabilities that a certain obstacle causes the sonar reading $S_i$

# Combining evidence

$$odds(o \mid S_2 \wedge S_1) = \frac{p(o \mid S_2 \wedge S_1)}{p(\overline{o} \mid S_2 \wedge S_1)}$$

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definition of odds

$$= \frac{p(S_2 \wedge S_1 \mid o)\, p(\bar{o})}{p(S_2 \wedge S_1 \mid \bar{o})\, p(o)}$$

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Bayes' rule (+)

$$= \frac{p(S_2 \mid o)\, p(S_1 \mid o)\, p(\bar{o})}{p(S_2 \mid \bar{o})\, p(S_1 \mid \bar{o})\, p(o)}$$

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conditional independence of $S_1$ and $S_2$

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conditional independence of $S_1$ and $S_2$

$$= \frac{p(S_2 \mid o)}{p(S_2 \mid \bar{o})} \frac{p(o \mid S_1)}{p(\bar{o} \mid S_1)}$$

Bayes' rule (+)

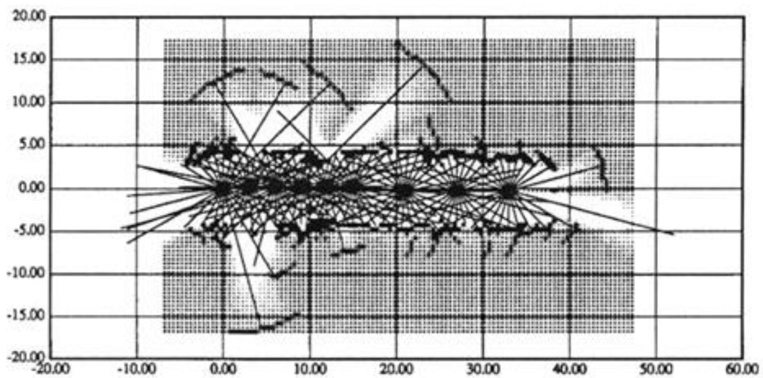
precomputed values

previous odds

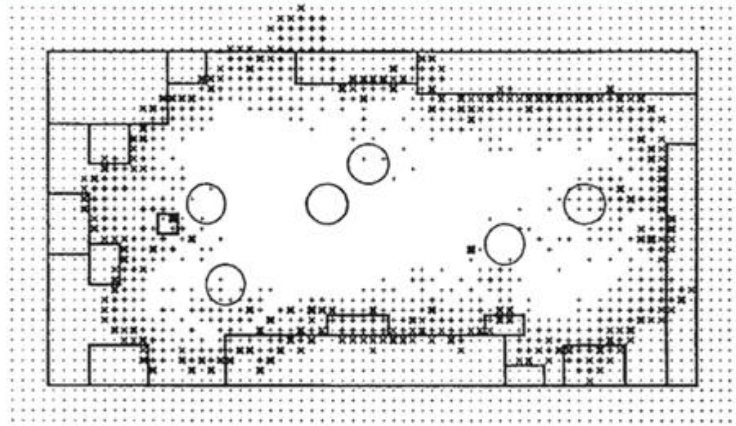
the sensor model

Update step = multiplying the previous odds by a precomputed weight.

# Evidence grids

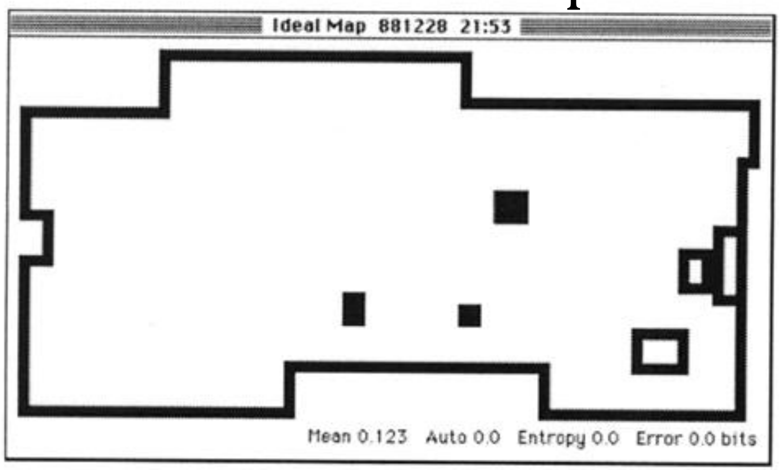


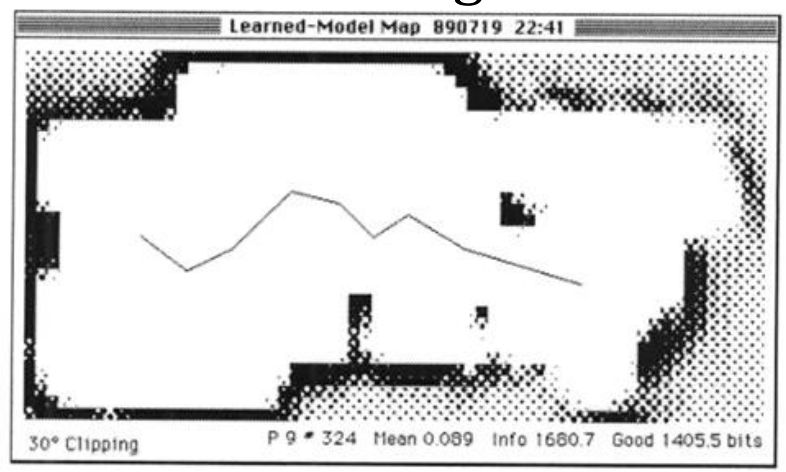

hallway with some open doors



lab space

known map and estimated evidence grid



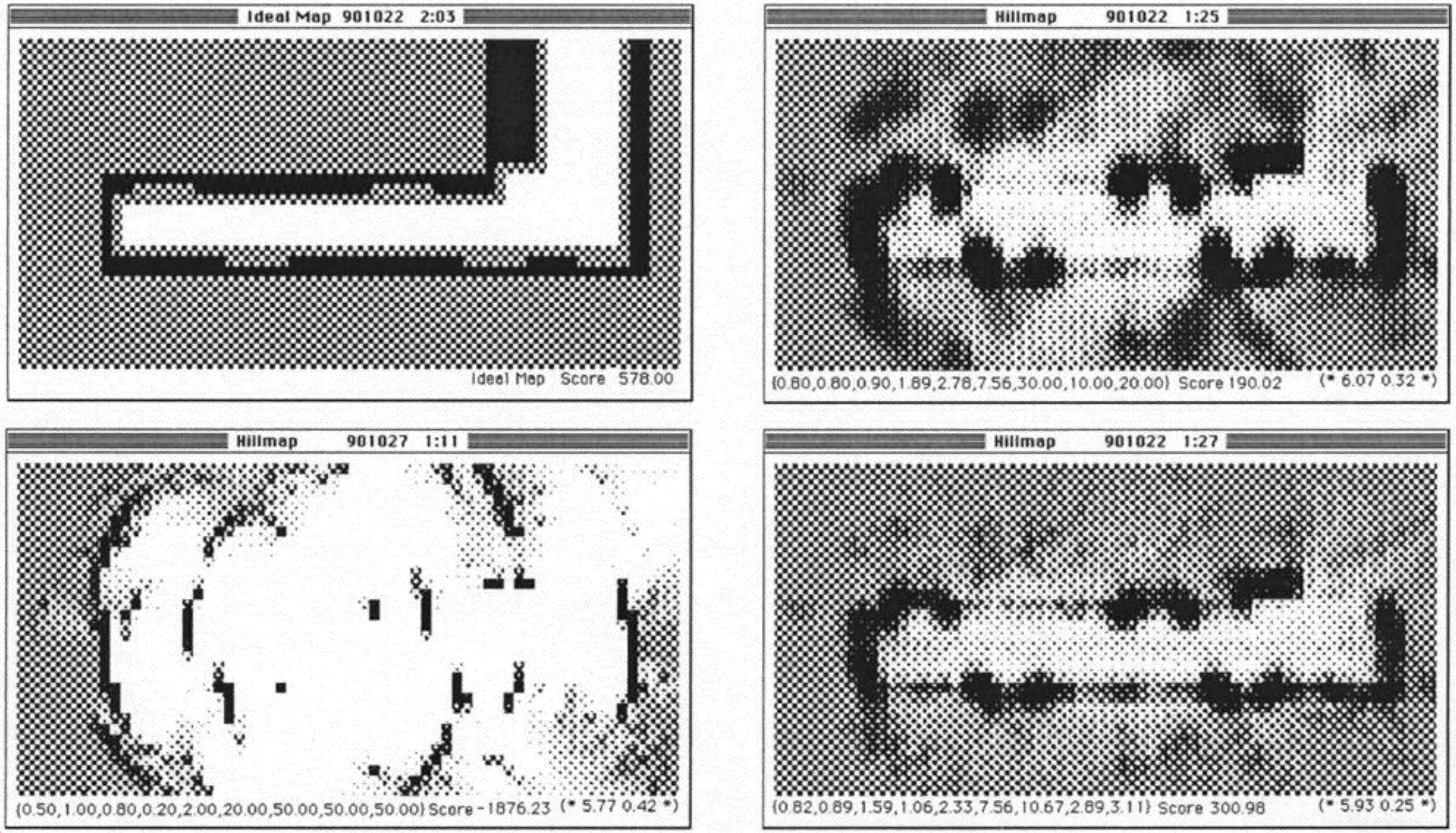




CMU -- Hans Moravec

# Learning the Sensor Model

The sonar model depends dramatically on the environment -- we'd like to *learn* an appropriate sensor model

**rather than hire Roman Kuc to develop another one...**



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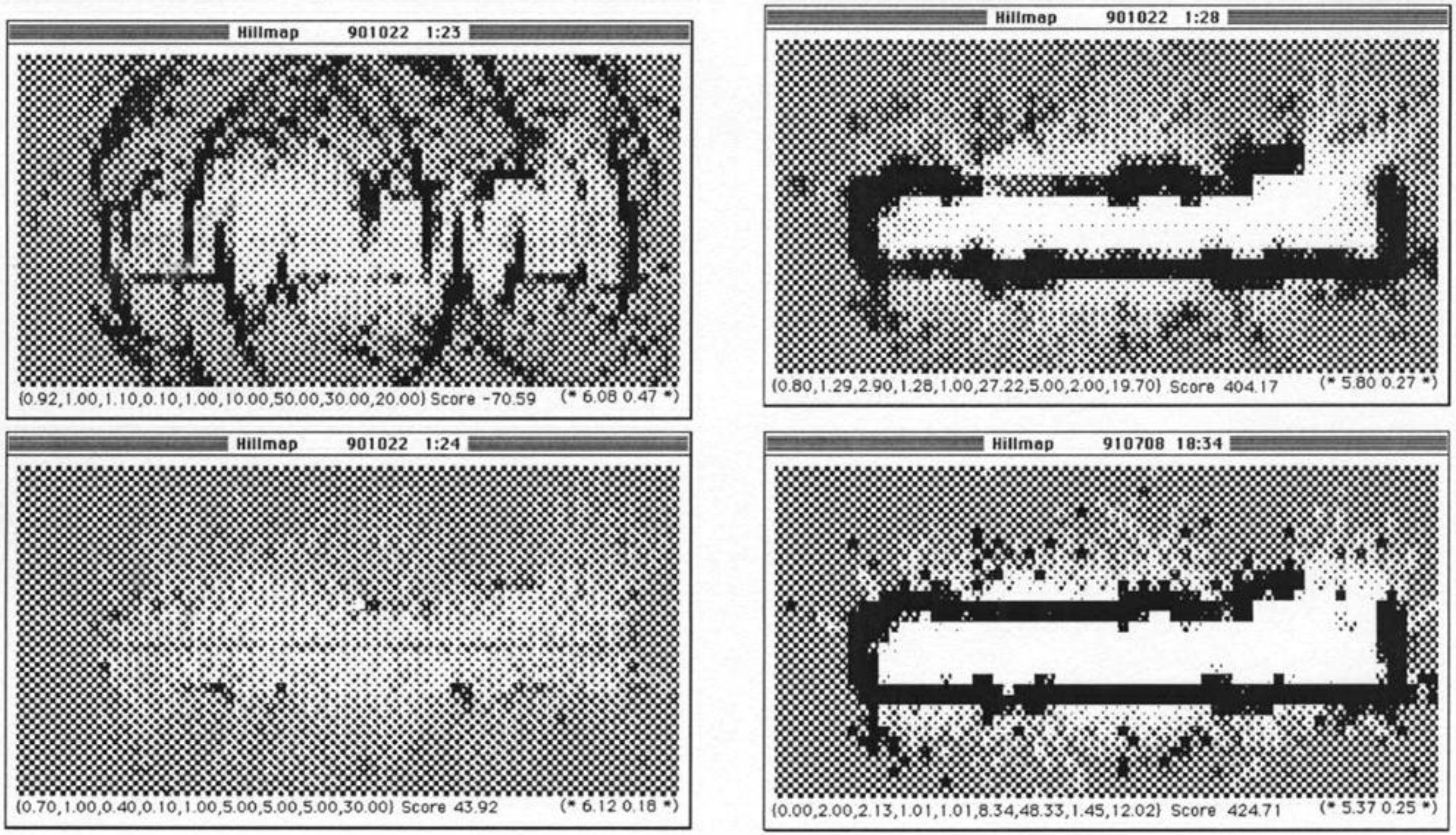

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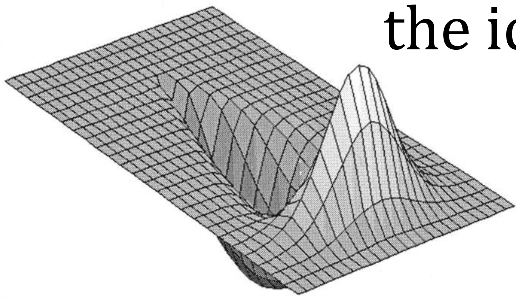
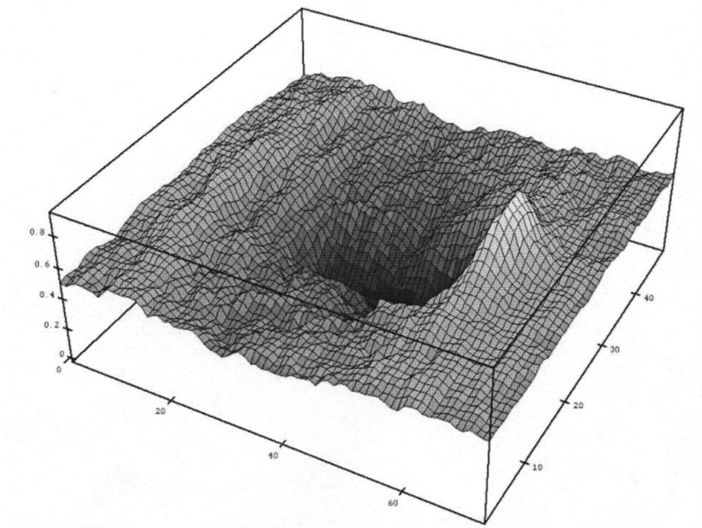
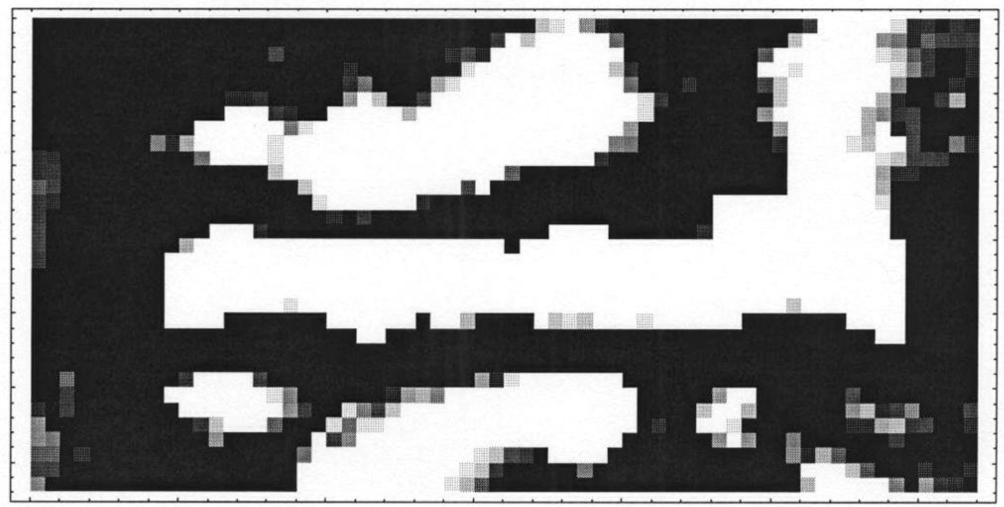


the idealized model




part of the learned model



the mapping results of a model that had an even better match score (against the ideal map)

# Sensor fusion

Incorporating data from other sensors -- e.g., IR rangefinders and stereo vision...

(1) create another sensor model

(2) update along with the sonar