

Computer Vision

COMP417 – Introduction to
Robotics and Intelligent Systems

Why vision?

- Passive (emits nothing).
 - Discreet.
 - Energy efficient.
- Intuitive.
- Powerful (works well for us, right?)
- Long and short range.
- Fast.

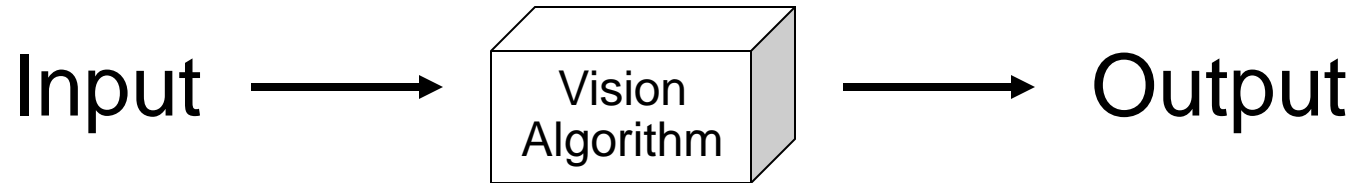
So, what's the problem?

- How hard is vision? Why do we think is do-able?

Problems:

- Slow.
- Data-heavy.
- Impossible.
- Mixes up many factors.

The “Vision Problem”



The “Vision Problem”



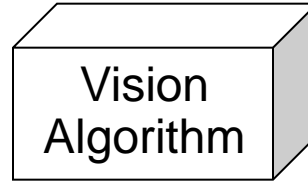
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254 228 164 209 235 144 188 144 161 192 230 250 253 255 252
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047 109 107 118 107 115 110 120 120 124 120 128 124 132 131
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254 219 170 247 253 104 121 057 042 047 243 235 233 252 254
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222 224 063 046 065 080 101 090 054 104 109 123 052 216 218
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173 175 039 127 038 024 053 054 050 051 036 038 022 063 126
170 160 058 020 027 027 045 074 057 035 124 036 016 072 124
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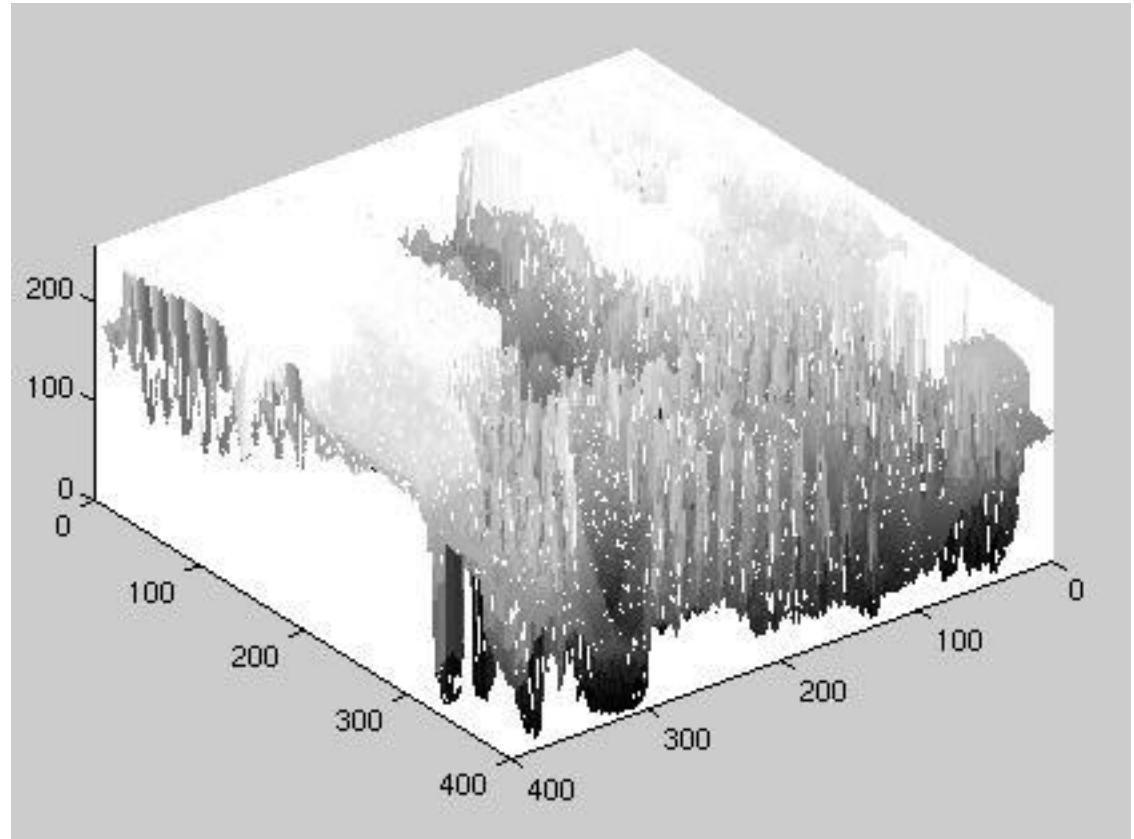
Looking at vision

Input



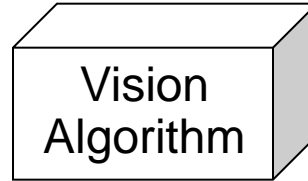
Output

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248 248 169 227 252 111 066 118 061 021 252 251 255 255 142
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236 216 178 208 230 156 077 062 110 088 244 249 230 220 221
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230 223 185 185 112 079 008 124 158 125 119 119 232 225 232
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224 216 041 102 090 162 079 111 118 164 083 170 065 221 219
215 222 046 111 077 075 060 046 069 032 179 068 157 224 226
219 216 092 045 074 143 013 171 159 072 087 065 143 217 222
222 224 070 041 074 131 085 150 112 140 139 154 055 231 218
226 232 118 109 041 165 130 105 097 175 078 081 067 064 174
253 254 079 072 116 089 020 068 103 074 031 130 106 052 161
047 034 090 045 145 027 135 109 082 082 048 113 087 061 157
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184 181 066 019 043 038 046 083 057 050 145 048 035 087 158
138 074 030 082 030 038 076 041 141 046 045 040 009 063 149
135 016 057 071 035 025 040 062 030 084 130 043 059 113 151
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Looking at vision

Input → Vision Algorithm → Output

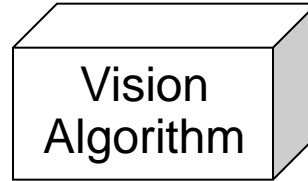


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135 016 057 071 035 025 040 062 030 084 130 043 059 113 151
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Looking at vision

Input → Vision Algorithm → Output



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135 016 057 071 035 025 040 062 030 084 130 043 059 113 151
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The “Vision Problem”

Input



Output

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020 067 073 058 055 076 069 050 074 064 065 066 066 059 023
047 109 107 118 107 115 110 120 120 124 120 128 124 132 131
047 125 130 130 122 121 117 142 131 133 134 141 149 144 135
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015 009 010 010 012 011 014 009 008 007 007 005 005 008 002
014 007 008 011 007 012 010 009 007 008 007 005 005 007 003
020 011 015 019 013 017 017 013 019 013 012 013 011 009 005
020 067 073 058 055 076 069 050 074 064 065 066 066 059 023
025 161 174 172 167 049 200 193 112 028 120 169 173 177 173
```



What does a robot need ?

doesn't need a full interpretation of available images

“This is Prof. X in his office offering me a can of spam.”

does need information about what to do...

“Run Away!!”

reactive

- avoiding obstacles (or predators)
- pursuing objects
- localizing itself
- mapping
- finding targets
- reasoning about the world ...

environmental interactions

deliberative

What does a robot need ?

What a camera does to the 3d world...



squeezes away one dimension

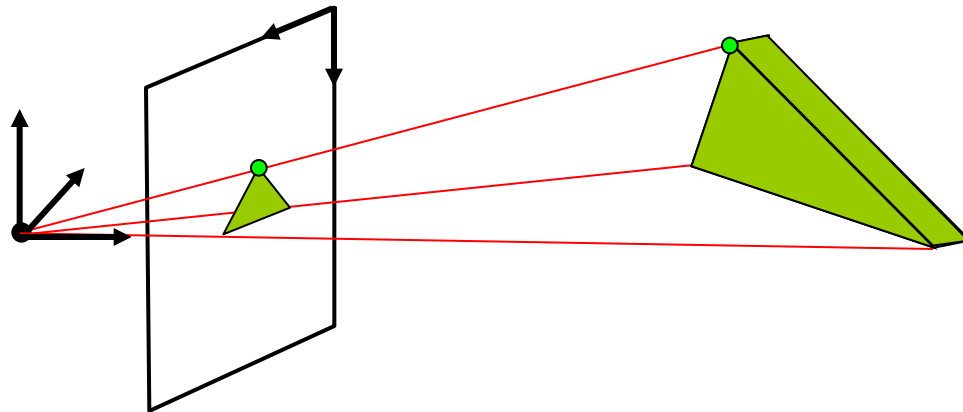
Ill-posed

- In trying to extract 3d structure from 2d images, vision is an *ill-posed* problem.



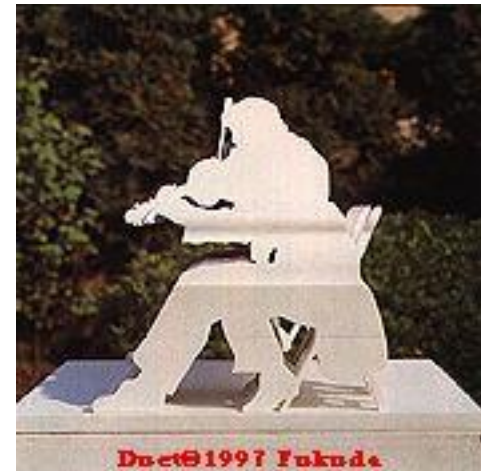
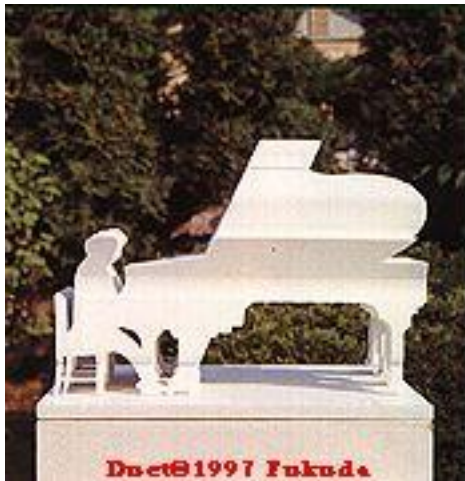
The vision problem in general...

- In trying to extract 3d structure from 2d images, vision is an *ill-posed* problem.
- Basically, there are too many possible worlds that might (in theory) give rise to a particular image



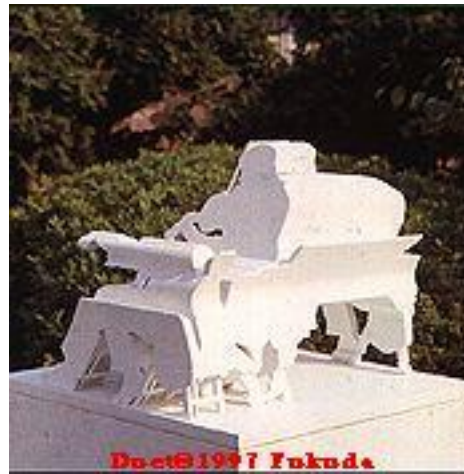
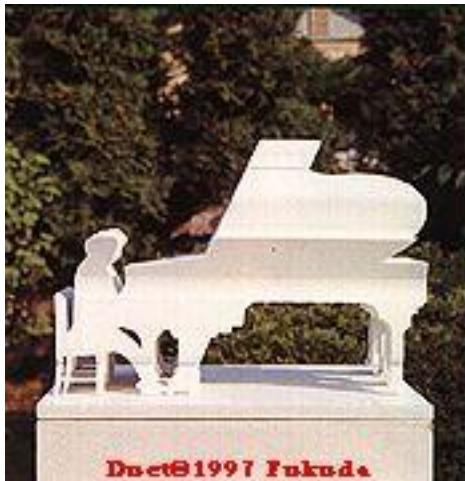
Ill-posed

- In trying to extract 3d structure from 2d images, vision is an *ill-posed* problem.



Ill-posed

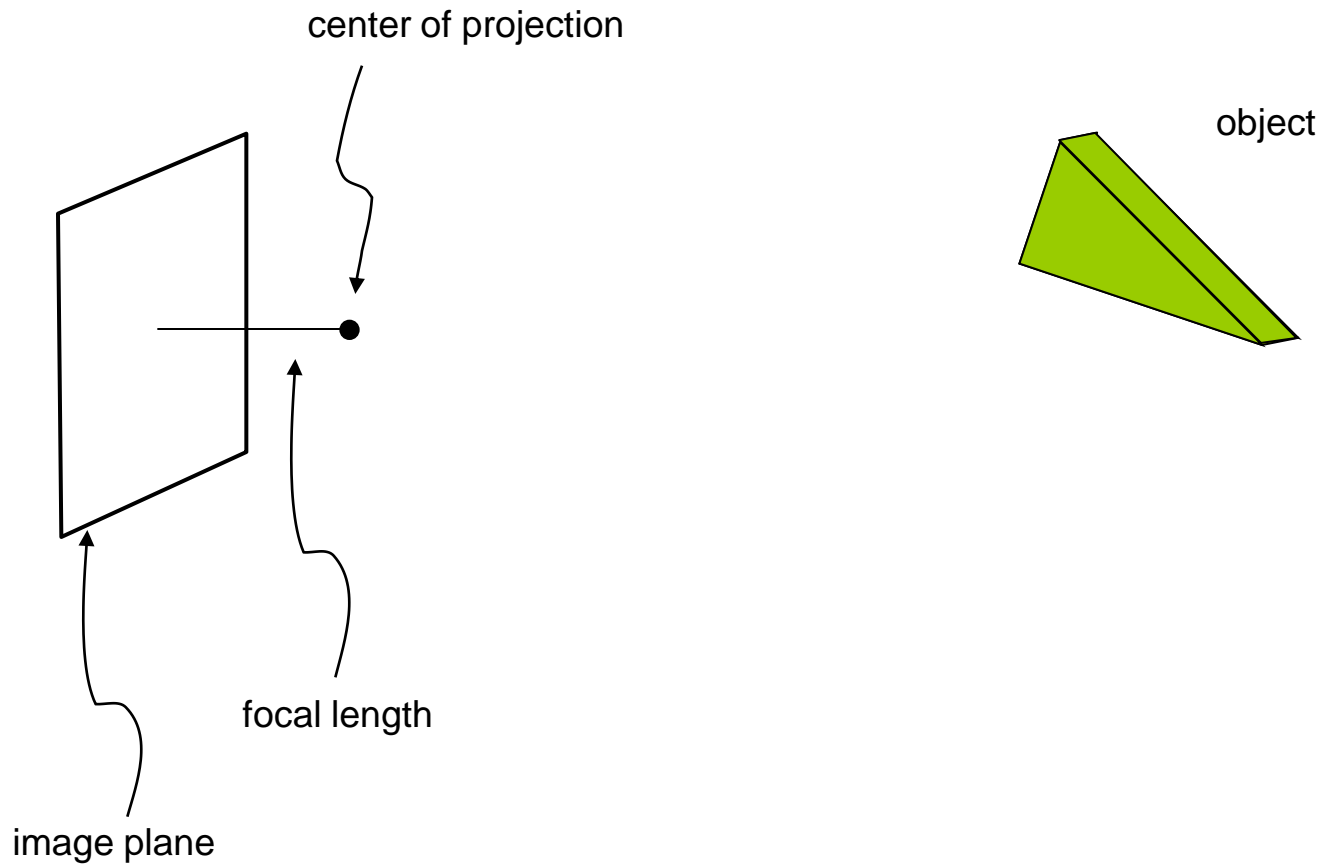
- In trying to extract 3d structure from 2d images, vision is an *ill-posed* problem.



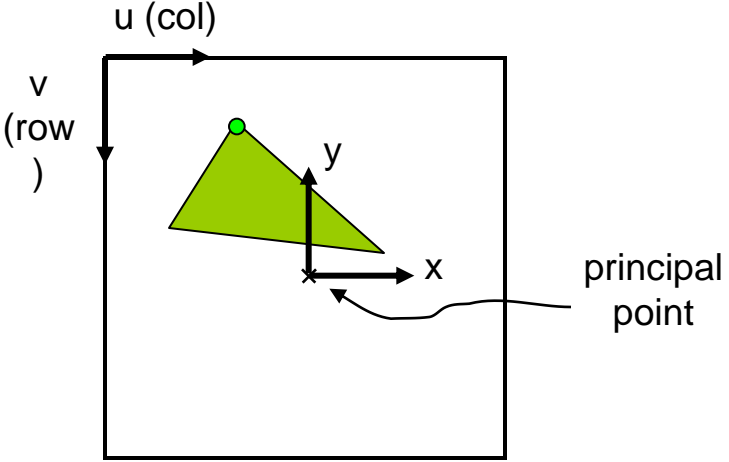
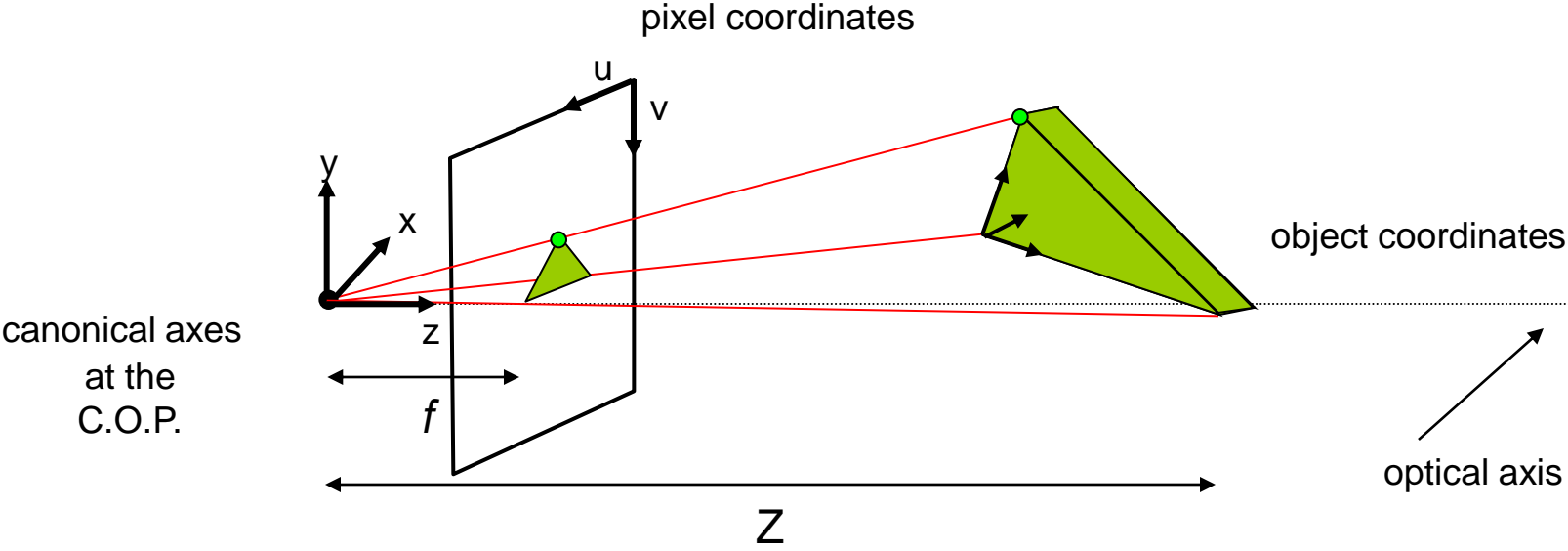
- An image isn't enough to disambiguate the many possible 3d worlds that could have produced it.

Camera Geometry

3d \rightarrow 2d transformation: perspective projection

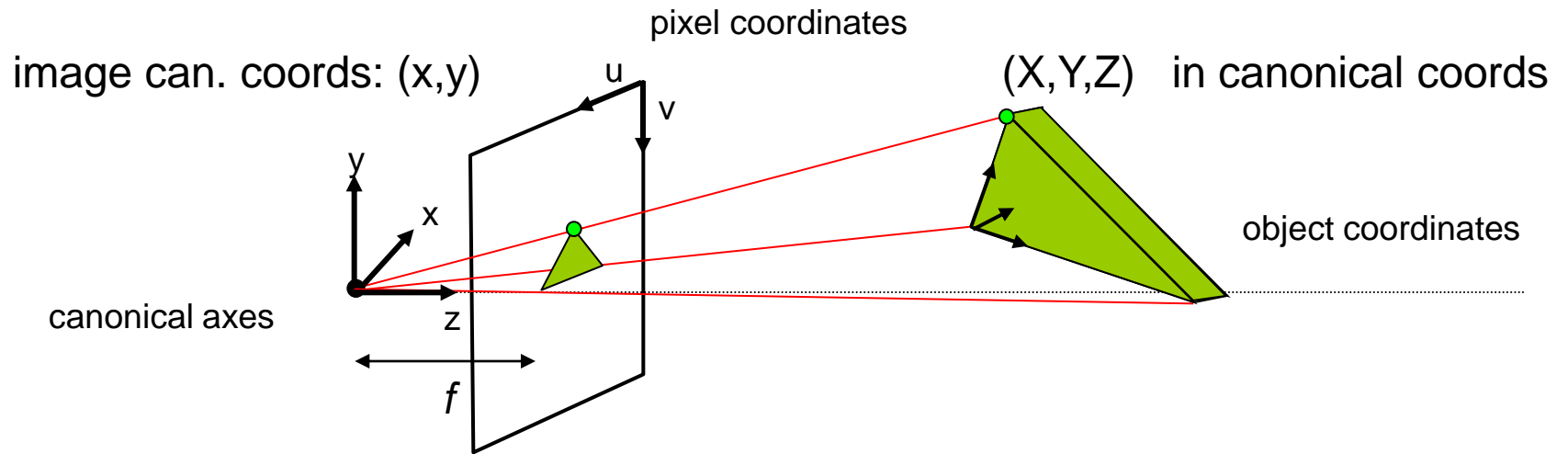


Coordinate Systems

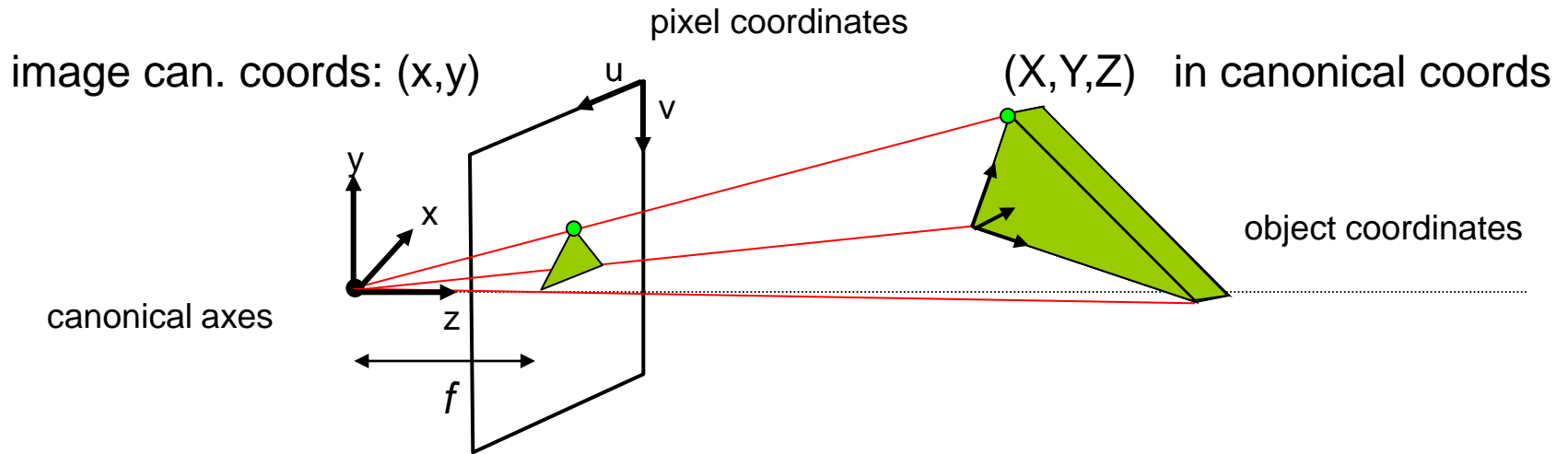


Add coordinate systems in order to describe feature points...

Coordinate Systems



From 3d to 2d



$$x = \frac{fX}{Z} \quad y = \frac{fY}{Z}$$

a nonlinear transformation

goal: to recover information about (X,Y,Z) from (x,y)

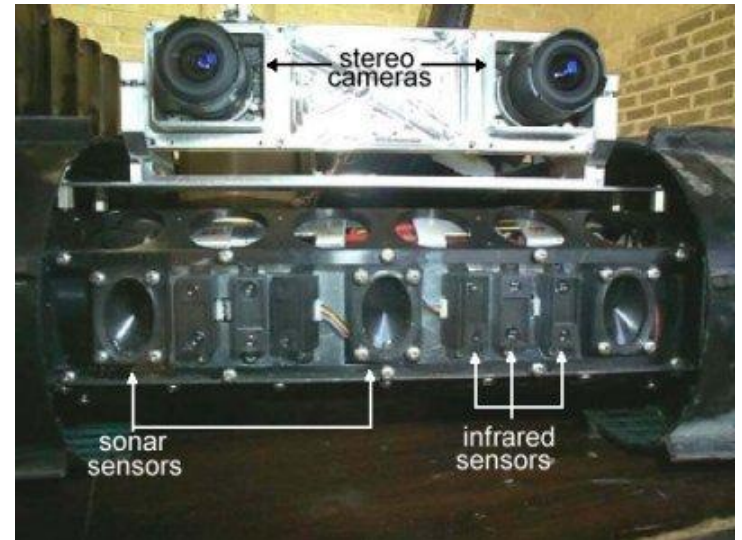
A Vision “solution”

- If interpreting a single image is difficult...



What about more ?!

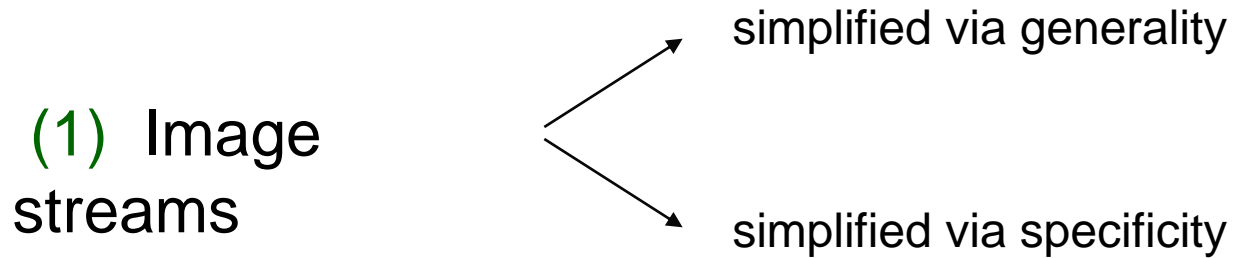
multiple cameras



multiple times

Robot vision sampler

A brief overview of robotic vision processing...



(2) Stereo vision → (or beyond...)

(3) Incorporating vision within robot control

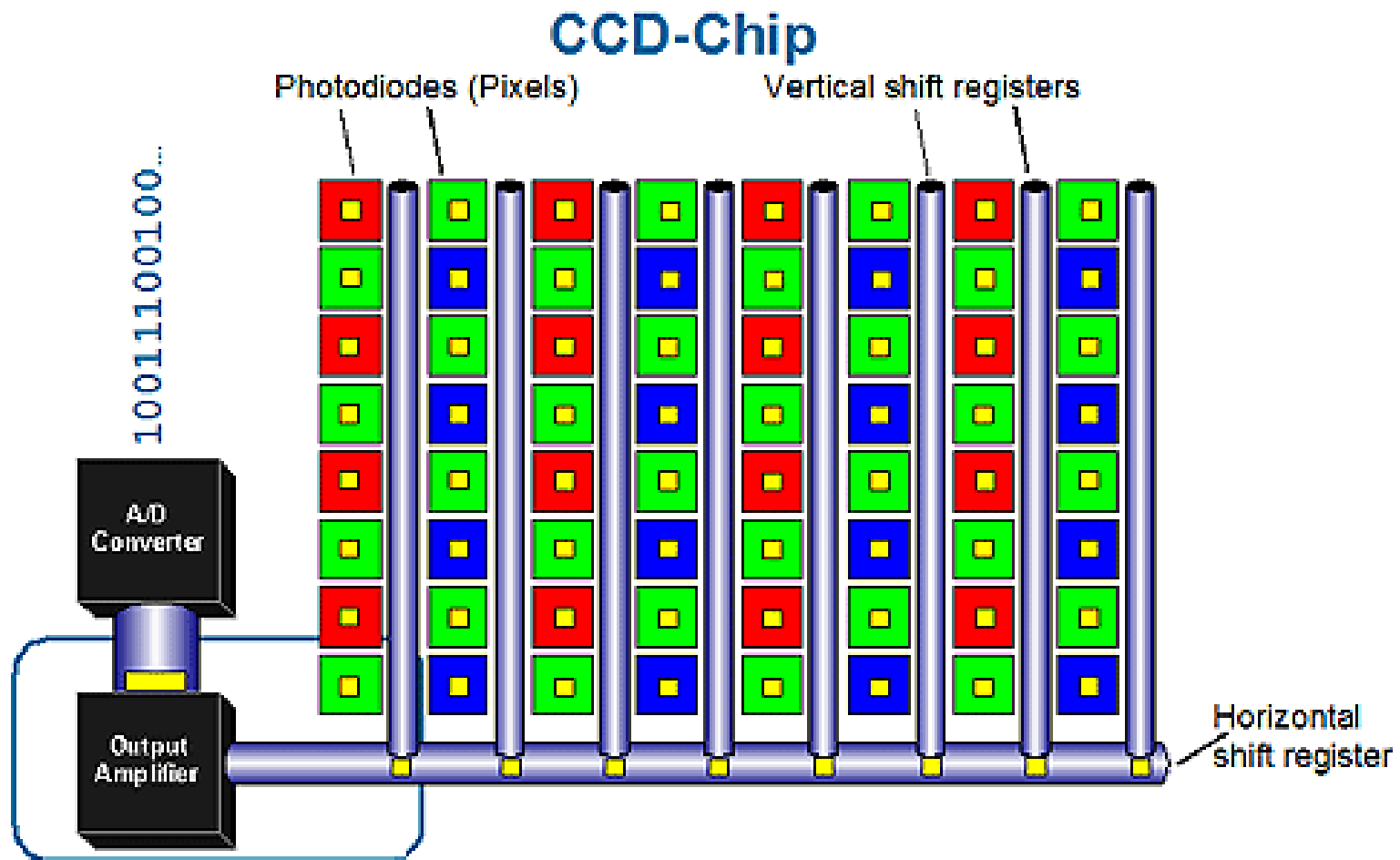
↓
3d reconstruction

↓
Visual “servoing”

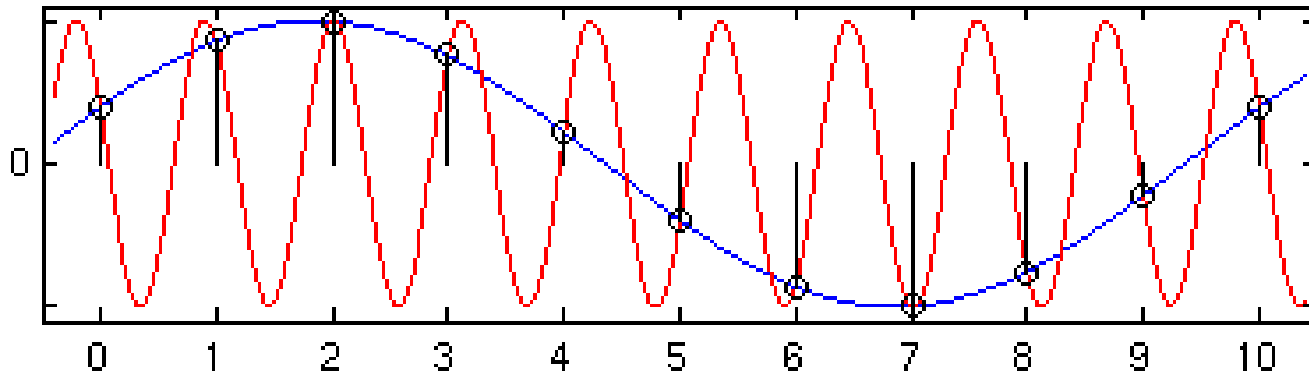
Details

- Images are not actually continuous.
- The sampling (and hardware) issues lead to a few other minor problems.

CCD (Charge-Coupled Device)

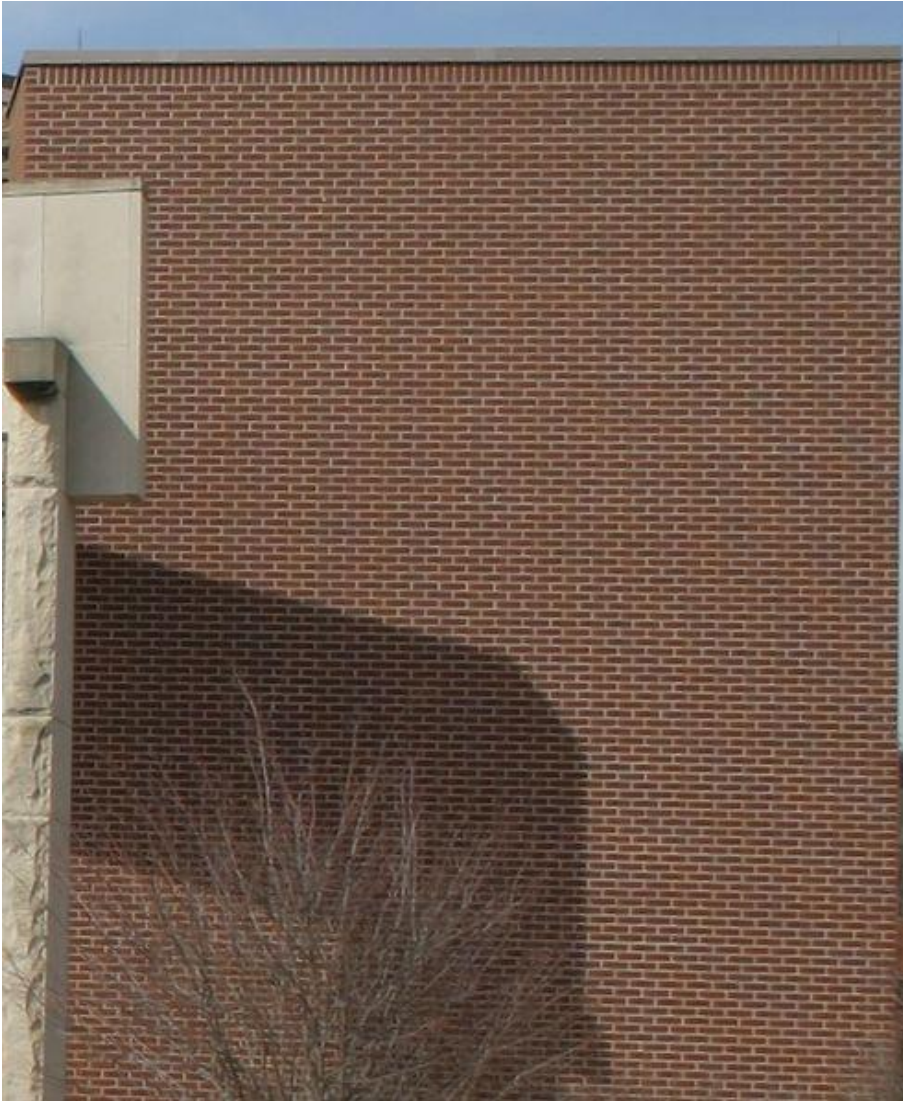


Aliasing.



- To avoid: $f_{\text{sampling}} > 2F_{\text{max}}$
 - Nyquist Rate

Aliasing: Moiré Patterns



Key problems

- Recognition:
 - What is that thing in the picture?
 - What are all the things in the image?
- Scene interpretation
 - Describe the image?
- Scene “reconstruction”:
 - What is the 3-dimensional layout of the scene?
 - What are the physical parameters that gave rise to the image?
 - What is a description of the scene?

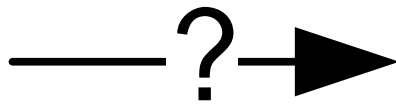
Notion of an “inverse problem.”

Correspondence Problem

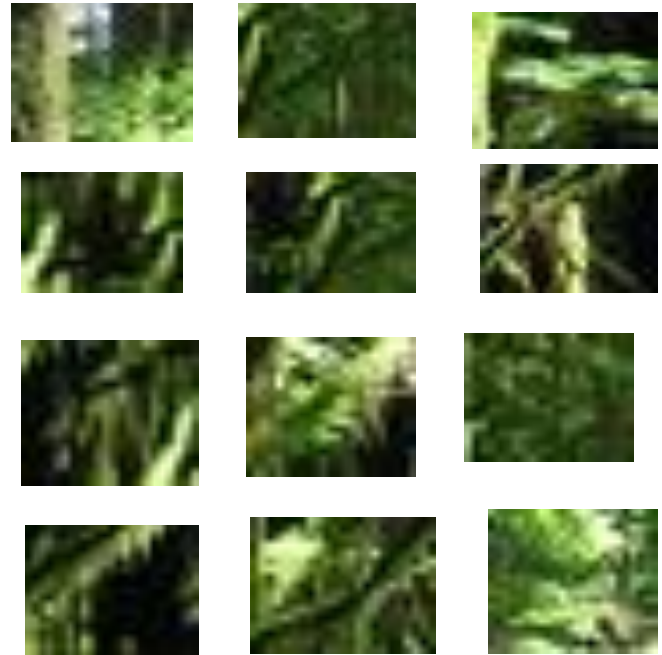


Correspondence

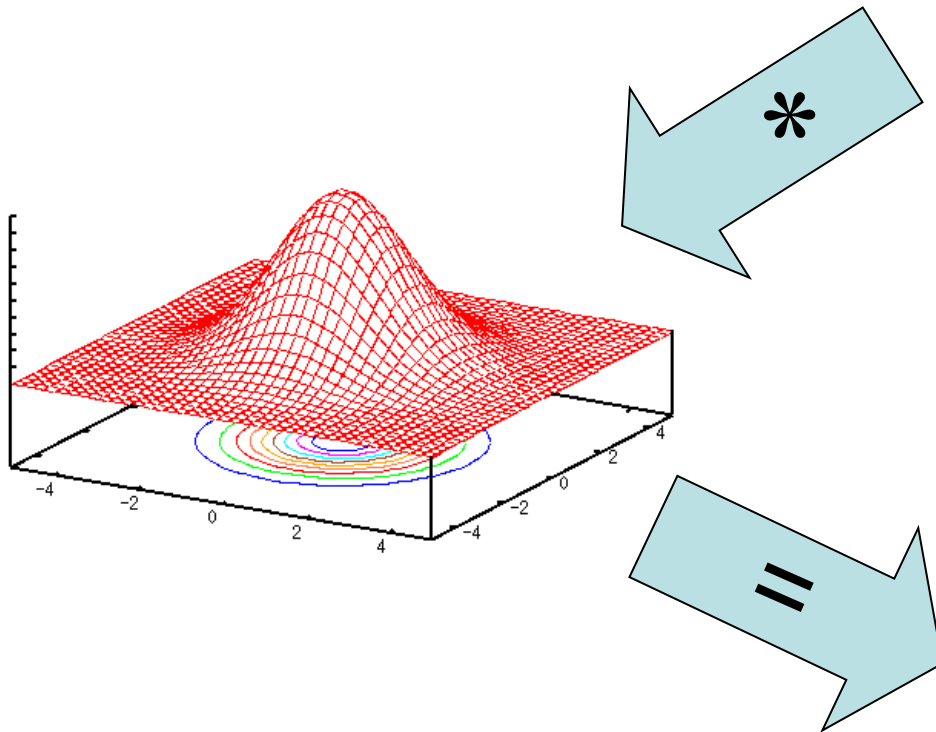
From I_1



From I_2



Gaussian Blur



Gaussian Blur and Noise

$\sigma = 4.0$ pix



$\sigma = 8.0$ pix



$\sigma = 12.0$ pix



$\sigma = 4.0$ pix



$\sigma = 8.0$ pix



$\sigma = 12.0$ pix



Gaussian Blur and Noise

$\sigma = 4.0$ pix



$\sigma = 8.0$ pix



$\sigma = 12.0$ pix



$\sigma = 4.0$ pix



$\sigma = 8.0$ pix



$\sigma = 12.0$ pix



Gaussian Blur, Noise, Sobel

$\sigma = 0.0$ pix



$\sigma = 4.0$ pix



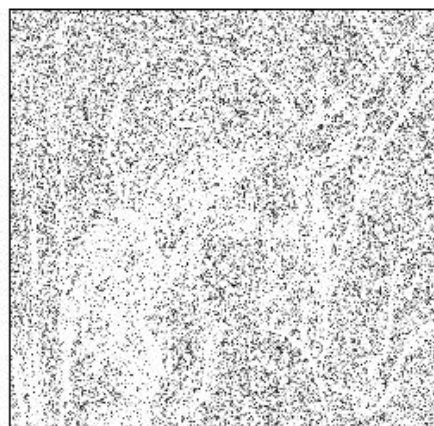
$\sigma = 8.0$ pix



$\sigma = 0.0$ pix



$\sigma = 4.0$ pix



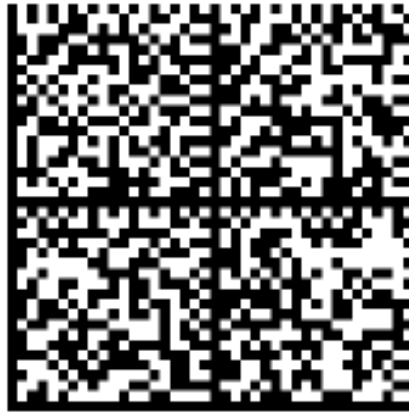
$\sigma = 8.0$ pix



Fiduciary Markers/Fiducial



(a) MaxiCode



(b) DataMatrixSymbol



(c) ARToolkit

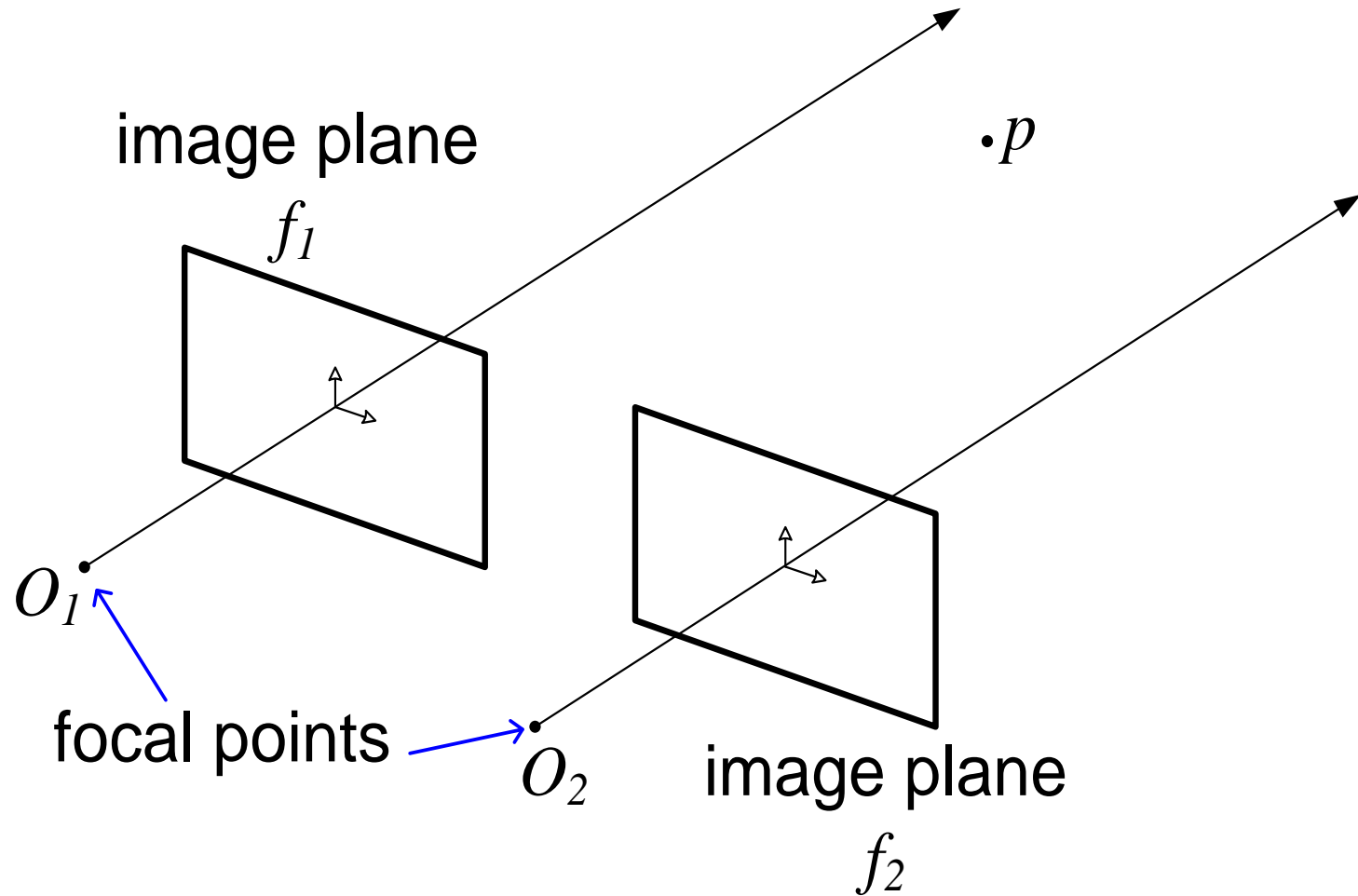


(d) ARTag

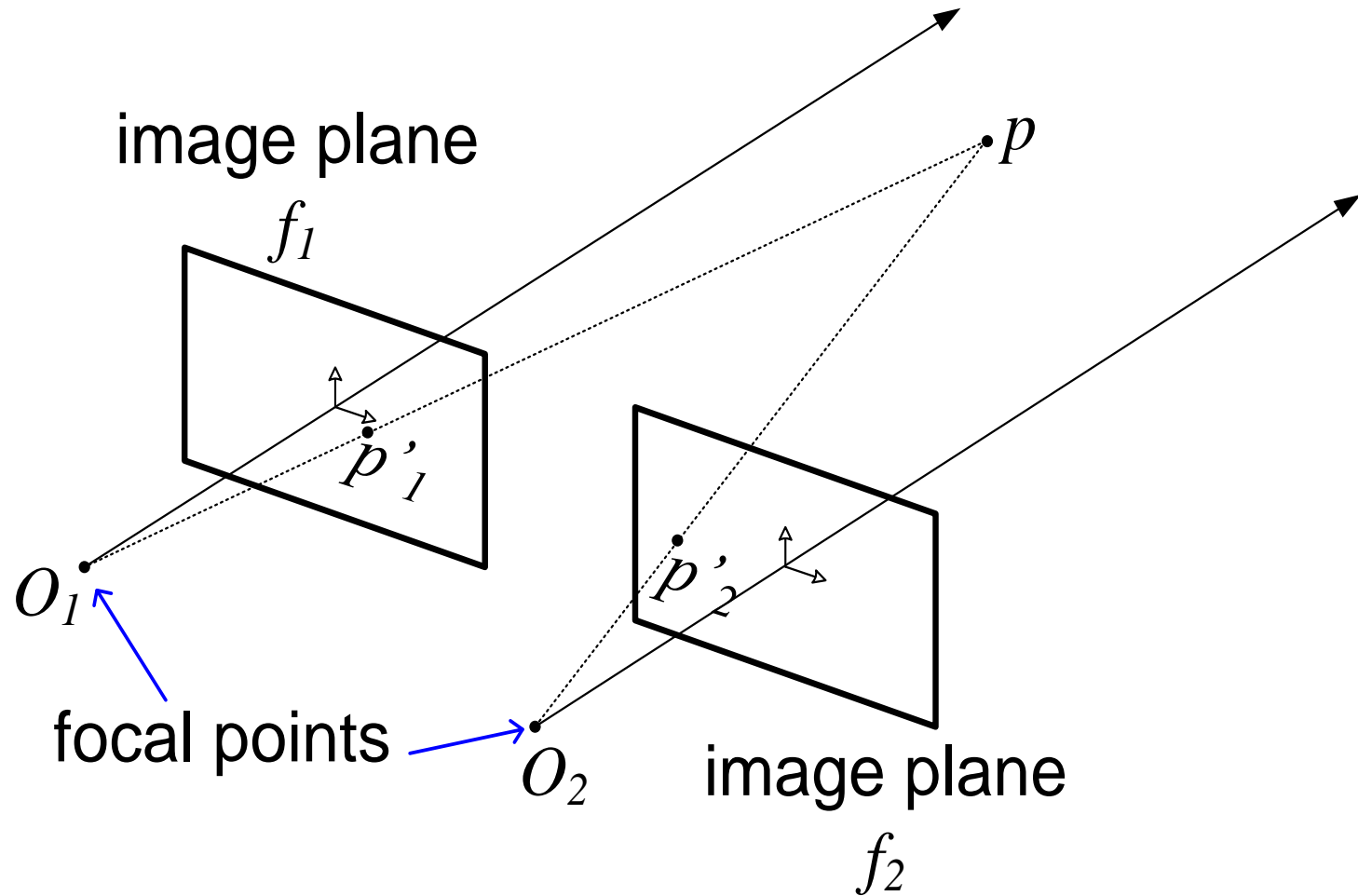


Fourier Tag

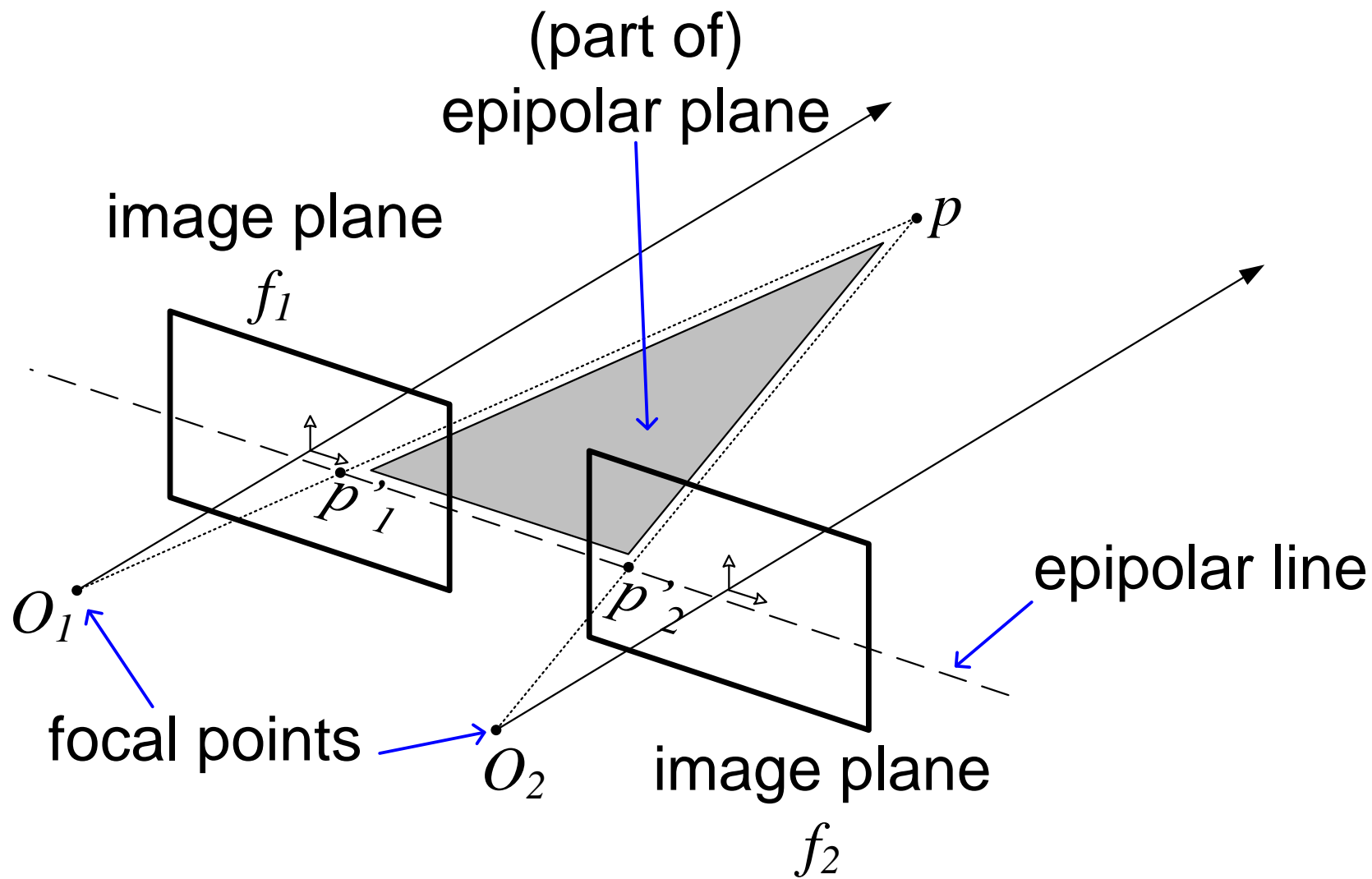
Stereo Vision: Pinhole Camera



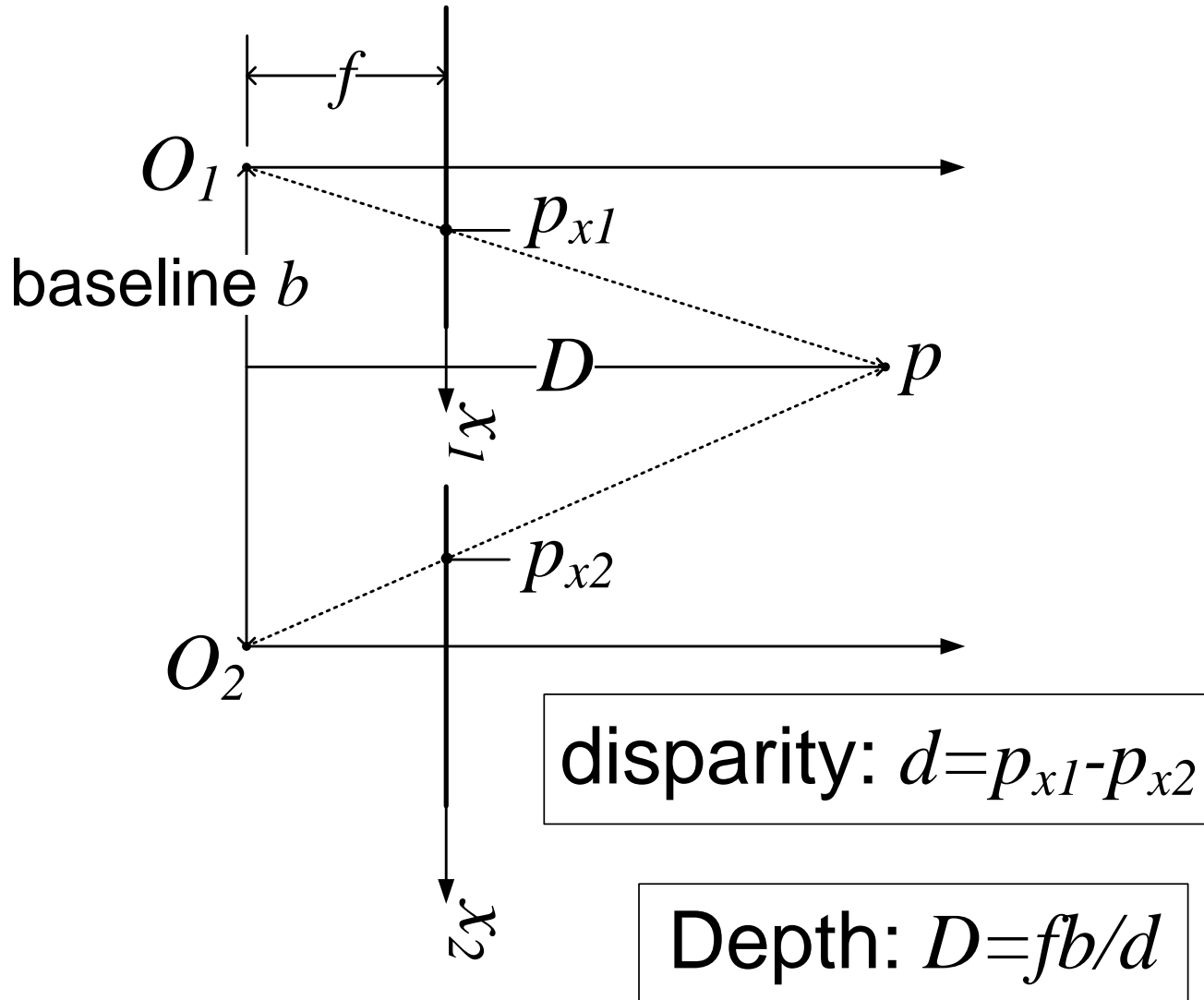
Stereo Vision: Pinhole Camera



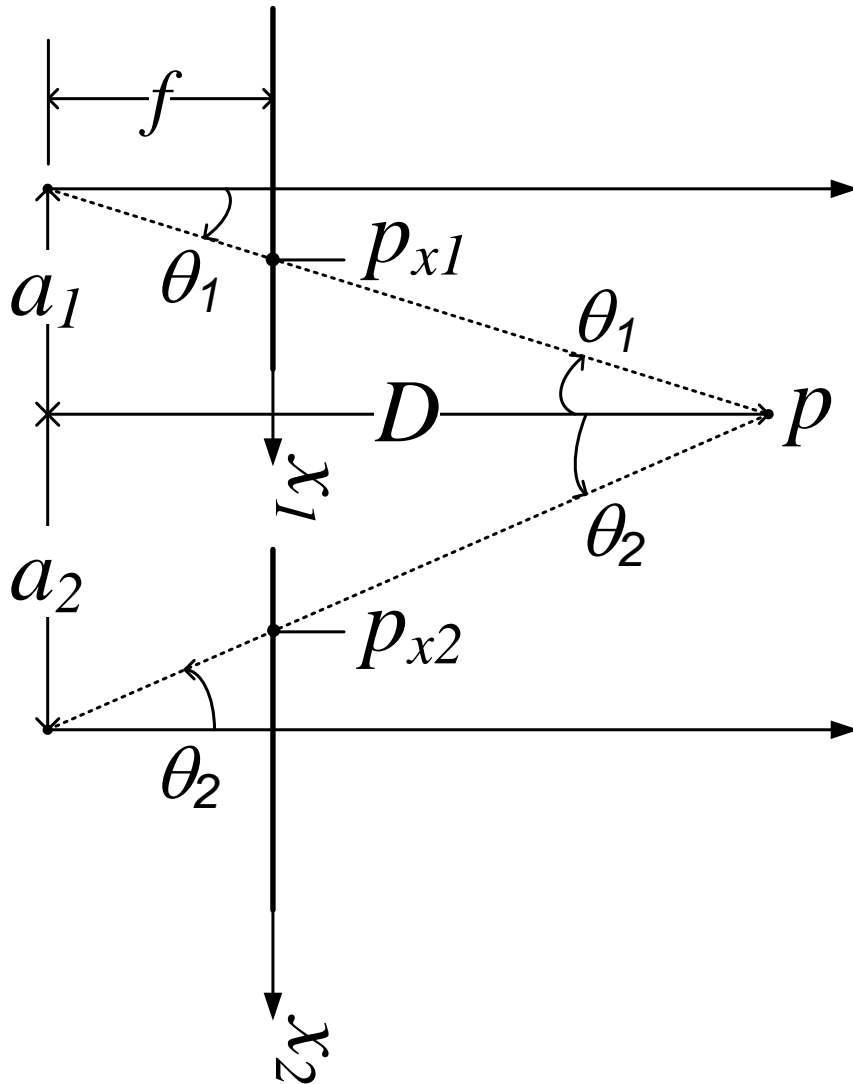
Stereo Vision: Pinhole Camera



Stereo Vision: Pinhole



Stereo Vision: Pinhole



$$\frac{p_{x1}}{f} = \frac{a_1}{D}$$

$$\frac{p_{x2}}{f} = \frac{a_2}{D}$$

$$a_1 + a_2 = b$$

Large Baseline



Stereo: Disparity Map



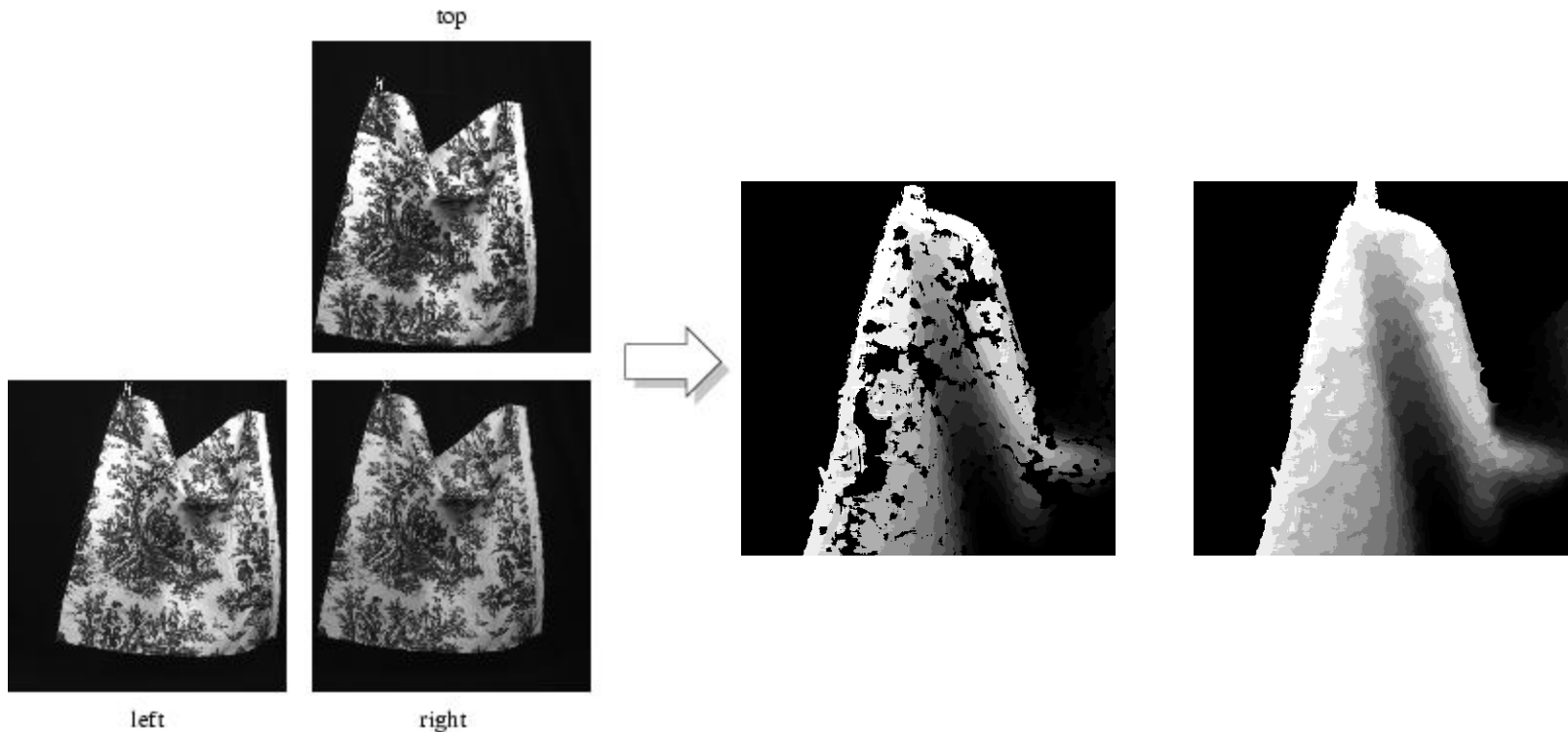
Using real-time stereo vision for mobile robot navigation

Don Murray

Jim Little

Computer Science Dept.
University of British Columbia
Vancouver, BC, Canada V6T 1Z4

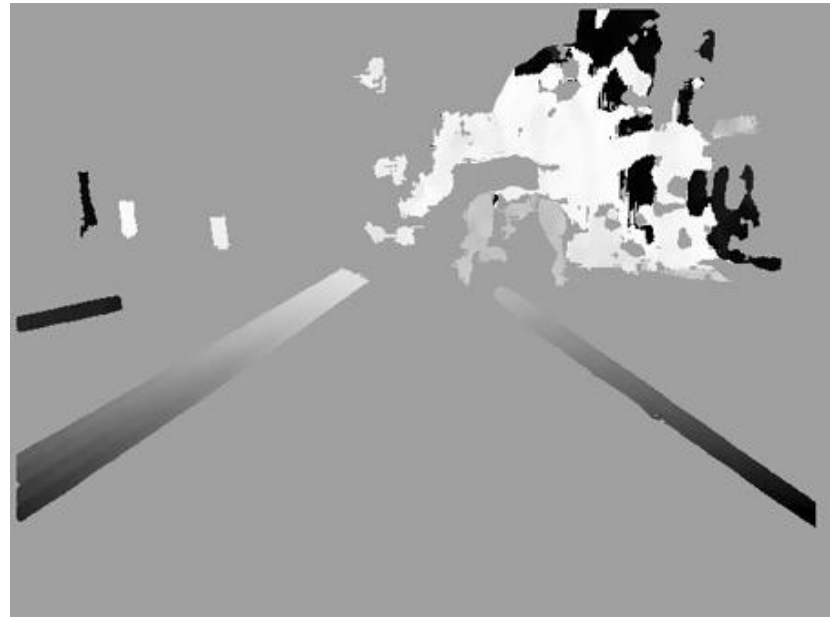
Another Example (Hole Filling)



Cloth Parameters and Motion Capture by David Pritchard

B.A.Sc., University of Waterloo, 2001

Depth Map in a City



Stereo Vision

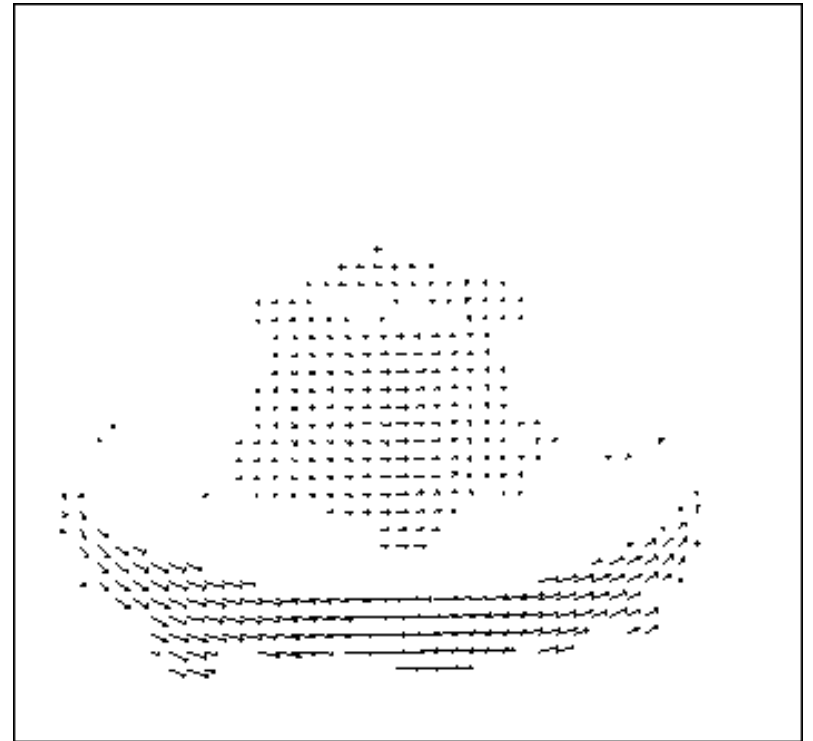
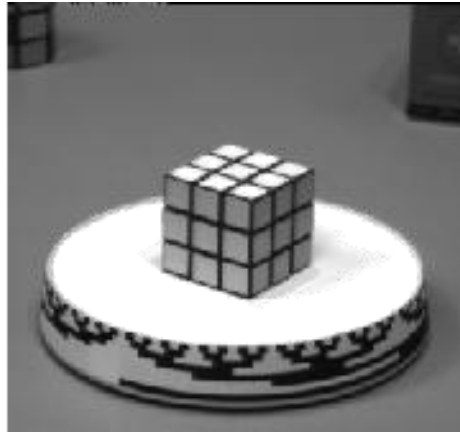
- Large number of algorithms out there:
<http://vision.middlebury.edu/stereo/>

rank 43 different algorithms.

Optical Flow

- Definition:
 - *the pattern of apparent motion of objects, surfaces, and edges in a visual scene caused by the relative motion between an observer (an eye or a camera) and the scene.*

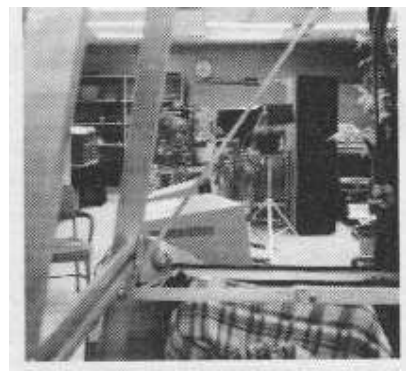
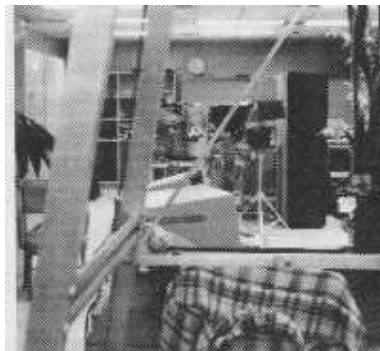
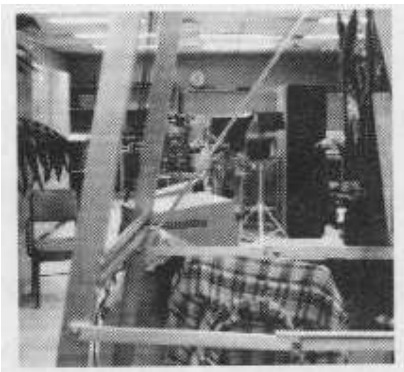
Optical Flow Field



Optical flow

Information about *image motion* rather than the *scene*. This is a classic **reconstruction** problem.

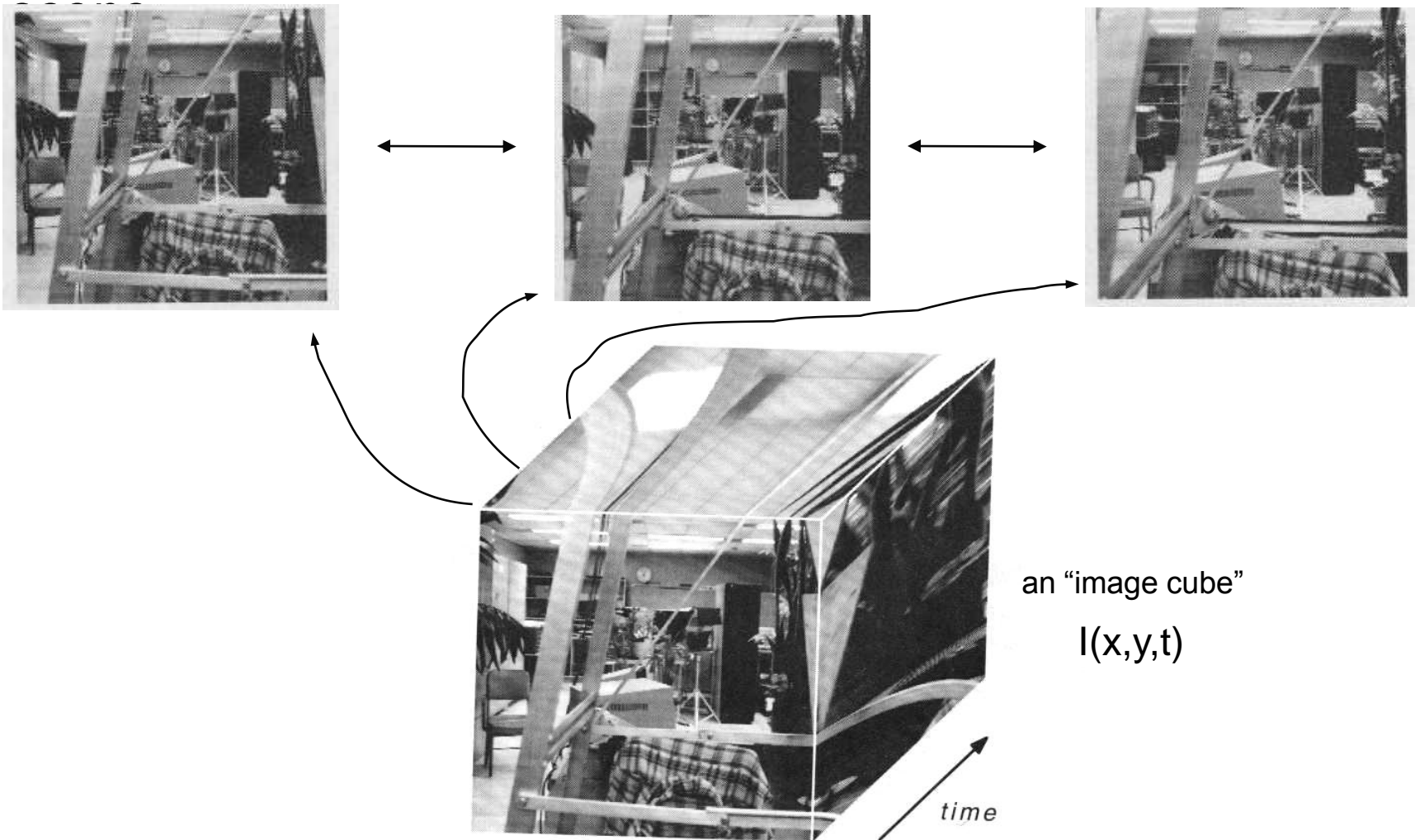
This next step might be to use the image motion to infer scene motion, robot motion or 3D layout.



time sequence of images

Optical flow

Information about *scene motion* rather than the

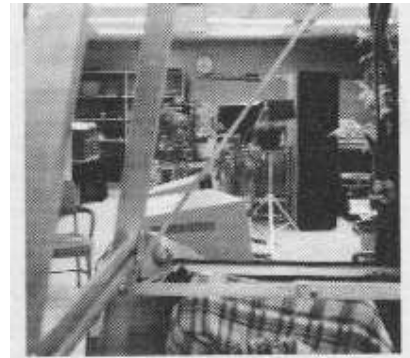
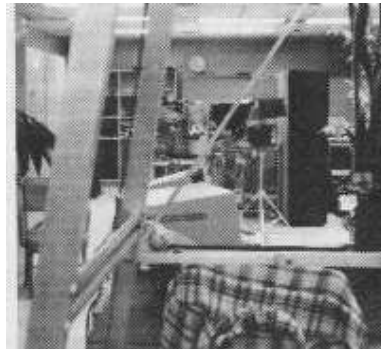
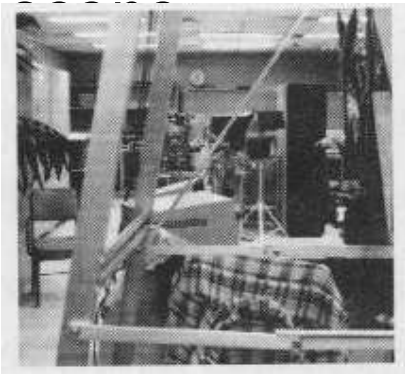


an "image cube"

$I(x,y,t)$

Optical flow

Information about *scene motion* rather than the

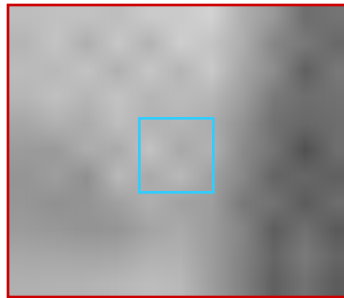
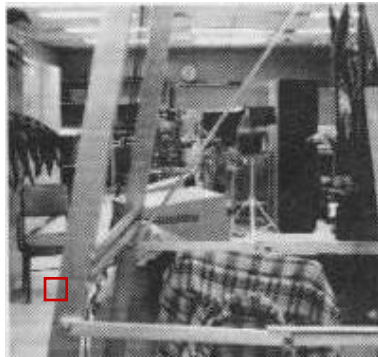


optical flow

How ?

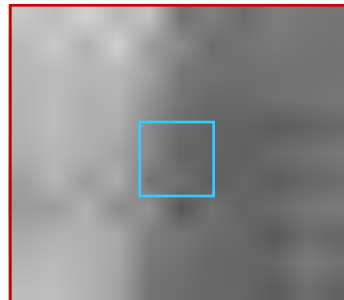
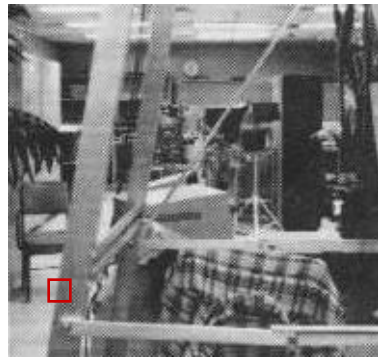
Optical Flow

By measuring the direction that intensities are moving...



$I(x,y,t)$

99	90	90	70	40
95	90	70	40	40
90	90	70	40	40
90	90	70	40	40
90	70	50	40	30



90	90	70	40	25
90	70	40	40	25
90	70	40	40	25
90	70	40	40	20
70	50	40	30	15

We can estimate things...

Observations & Warnings

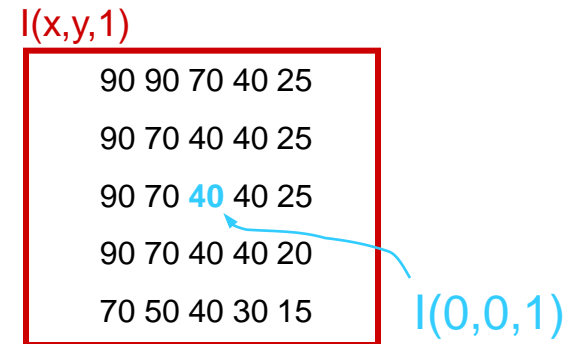
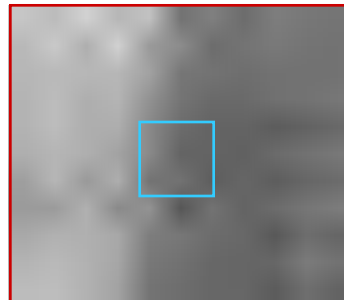
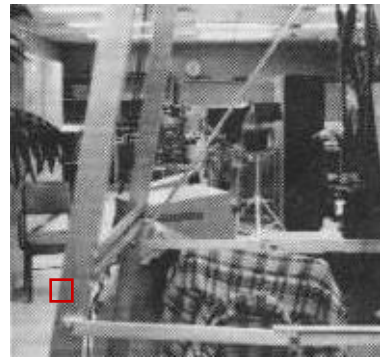
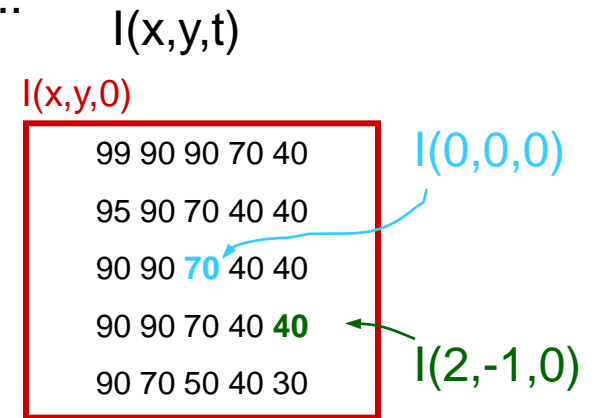
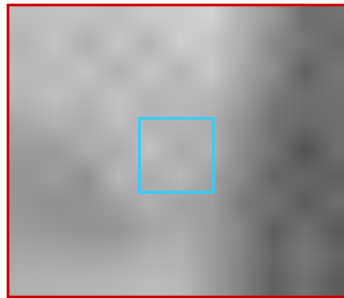
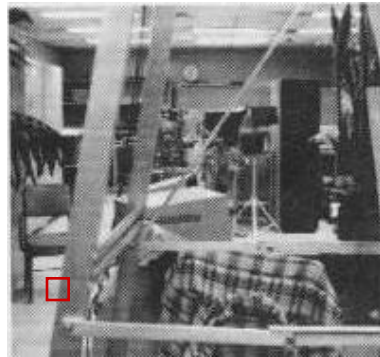
- How can we do this?
- Assume the scene itself is static.
- Find matching chunks in the images.
- An instance of *correspondence*.

BUT

- World really isn't static.
- Lightning might change even in a static scene.

Optical Flow

By measuring the direction that intensities are moving...

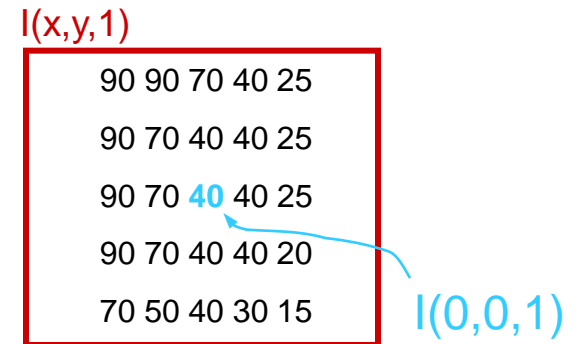
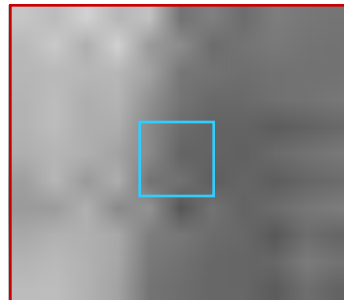
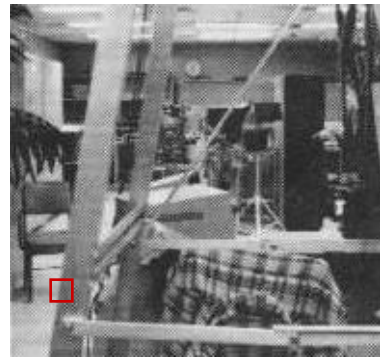
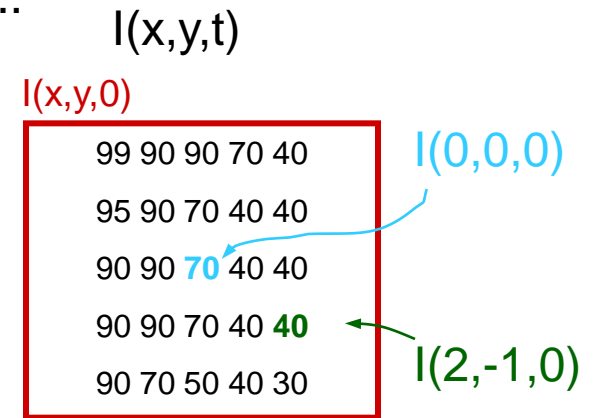
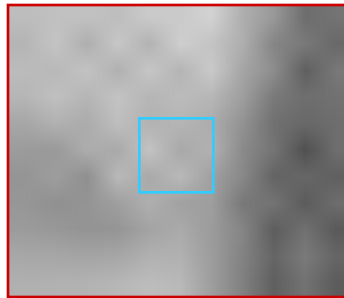
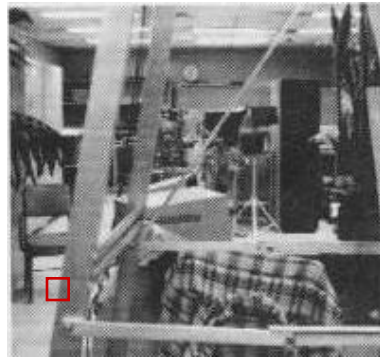


We can estimate things ...

$$\frac{dl}{dx} = I_x \quad \text{at } (0,0,0)$$

Optical Flow

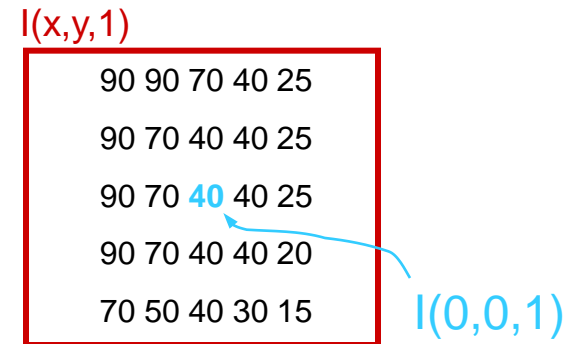
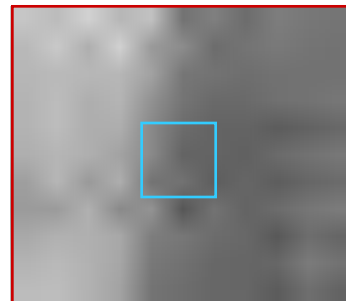
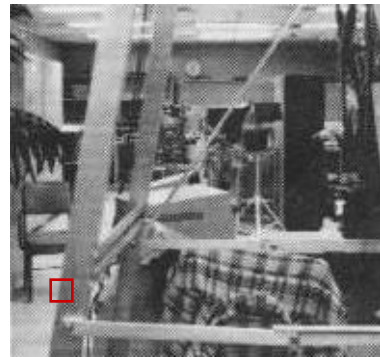
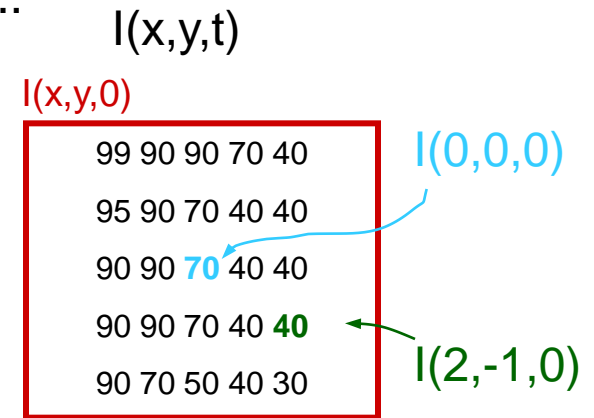
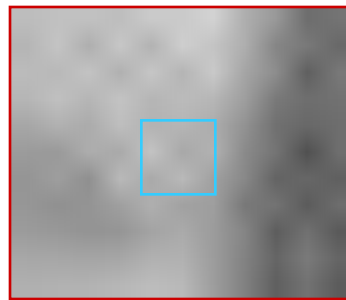
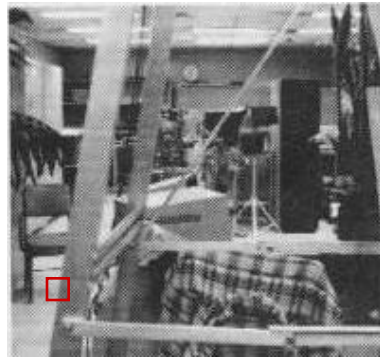
By measuring the direction that intensities are moving...



We can estimate things like $\frac{dl}{dx} = I_x$ at $(0,0,0) = \frac{\Delta I}{\Delta x} = \frac{I(1,0,0) - I(0,0,0)}{1 - 0} = -30$

Optical Flow

By measuring the direction that intensities are moving...



We can estimate things like

$$\frac{dl}{dx} = I_x$$

$$\frac{dl}{dy} = I_y$$

$$\frac{dl}{dt} = I_t$$

so...

Measuring Optical Flow

Let $I(x,y,t)$ be the sequence of images.

Simplest assumption (constant brightness constraint):

$$I(x,y,t) = I(x + dx, y + dy, t + dt)$$

(x,y,t)

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95	90	70	40	40
90	90	70	40	40
90	90	70	40	40
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Reminder: $f(x + dx) = f(x) + f'(x) dx + f''(x) dx^2 / 2 + \dots$

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$$I(x,y,t) = I(x,y,t) + I_x dx + I_y dy + I_t dt + \text{2nd deriv.} + \text{higher}$$

ignore these terms

$$0 = I_x dx + I_y dy + I_t dt$$

Measuring Optical Flow

Let $I(x,y,t)$ be the sequence of images.

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$$I(x,y,t) = I(x,y,t) + I_x dx + I_y dy + I_t dt + \underbrace{2\text{nd deriv.} + \text{higher}}_{\text{ignore these terms}}$$

ignore these terms

$$0 = I_x dx + I_y dy + I_t dt$$

$$-I_t = I_x \frac{dx}{dt} + I_y \frac{dy}{dt}$$

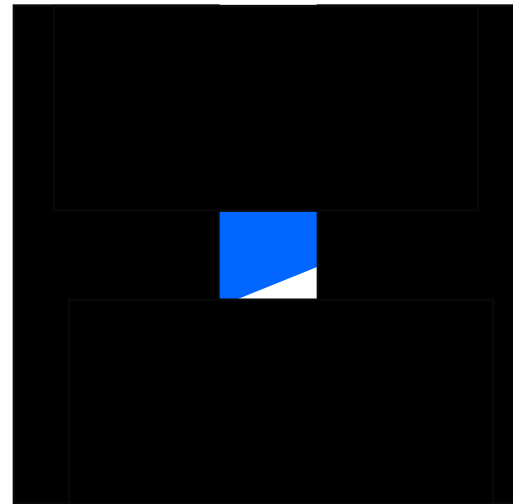
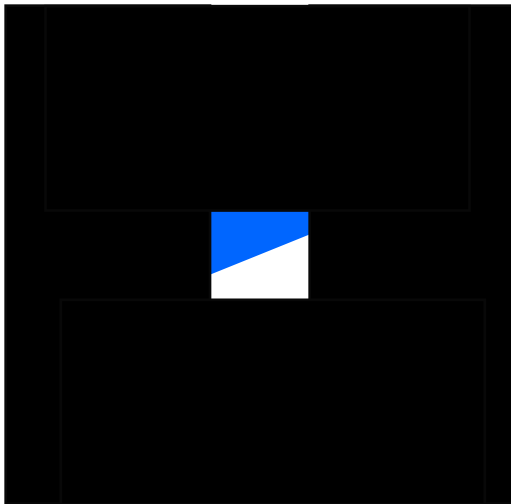
intensity-flow equation

The “aperture” problem

$$-I_t = I_x \frac{dx}{dt} + I_y \frac{dy}{dt}$$

- The intensity-flow equation provides only one constraint on *two* variables (x-motion and y-motion)

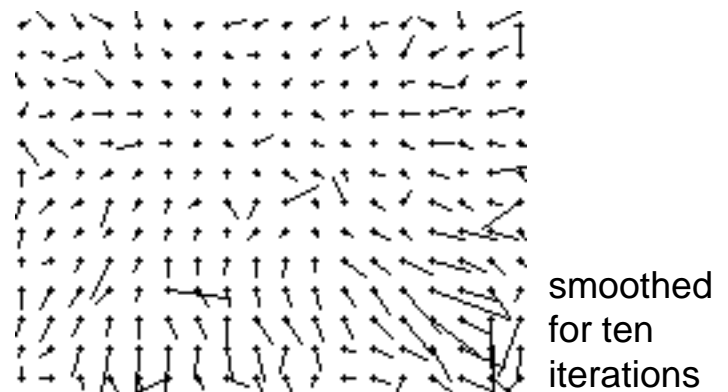
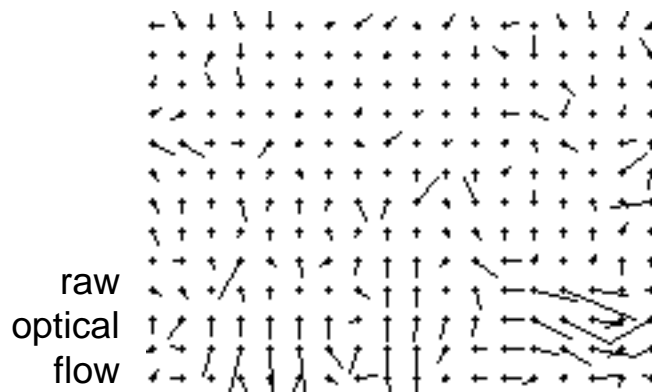
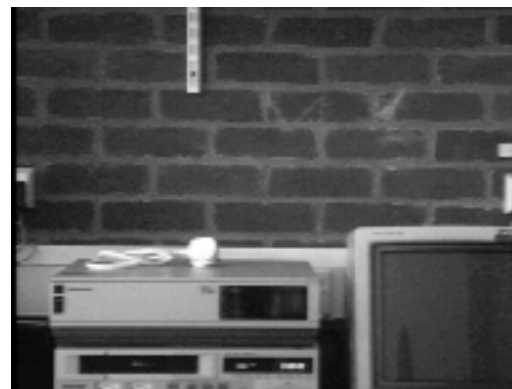
→ It is only possible to find optical flow in one direction...



The aperture problem

→ It is only possible to find optical flow in one direction...

*at any **single** point in the image !*



Smoothing can be done by incorporating neighboring points' information.

SIFT



Optical Flow Application

- Visual Odometry
 - Wheel slip detection on future Mars Rovers

Image Downsampling

