Thesis Proposal

Design of Stabilizing Feedback Controls for Strongly Nonlinear Systems

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Abstract

This work addresses the problem of stabilizing feedback design for strongly nonlinear systems, i.e. systems whose linearization about their equilibria is uncontrollable and for which there does not exist a smooth or even continuous stabilizer. The construction of stabilizing controls for these systems is often further complicated by the presence of a drift term in the differential equation describing their dynamics. The development of stabilizing laws for bilinear systems is also considered in this research. The relevance of bilinear systems stems from the fact that they result from the linearization of certain nonlinear control systems with respect to the state only. The proposed methodologies yield time-varying control laws whose construction is based on Lie algebraic techniques.

Two systematic approaches to the construction of time-varying feedback laws for nonlinear systems with drift are proposed: (1) a continuous time-varying control strategy, partially drawing on the ideas by Coron and Pomet for driftless systems, and (2) a discontinuous time-varying feedback based on computationally feasible Lie algebraic techniques.

The continuous time-varying control law is a combination of a periodic time-varying control providing for critical stabilization with an asymptotically stabilizing feedback "correction" term. The periodic control is obtained through the solution of an open loop, finite horizon, control problem on the associated Lie group which is posed as a trajectory interception problem in the Philip Hall coordinates of flows for the system and its Lie bracket extension. The correction term is calculated to be a control which decreases a Lyapunov function whose level sets contain the periodic orbits of the system stabilized by the time-periodic feedback.

The second method constitutes the first attempt to provide a systematic methodology for the synthesis of discontinuous time-varying feedback, and thus overcomes some of the practical implementation difficulties presented by the first method. The control law comprises two modes. In one mode the control is a smooth state feedback that guarantees an instantaneous decrease of a chosen control Lyapunov function. This mode is applied whenever there exists a smooth control. The other mode considers an open loop piece-wise constant control which decreases the control Lyapunov function on average, after a finite period of time. The synthesis of the smooth state feedback is based on the standard Lyapunov approach, thus the emphasis is put on the construction of the time-varying discontinuous control. The Lie algebraic control is composed of a sequence of constant controls whose values are calculated as the solution to a non-linear programming problem. Two approaches to the formulation of the non-linear programming problem are presented. In the first approach the formulation of the non-linear programming problem results from the direct application of the Campbell-Baker-Hausdorff formula for composition of flows, while in the second approach, the non-linear programming problem is formulated by posing the original control problem in terms of a relaxed control problem in the associated logarithmic coordinates.

These approaches are general and applicable to a large class of nilpotent systems which do not lend themselves to successful linearization (be it through state-feedback transformations, or else simply around some operating points).

Two strategies for the stabilization of homogeneous bilinear systems with unstable drift are proposed. The first method considers the construction of a time-invariant feedback for the Lie bracket extension of the original system. The original system controls are then obtained as a solution to an open loop, finite horizon, control problem posed in terms of a finite horizon interception problem of the logarithmic coordinates for flows. Under such controls the trajectories of the open loop system and the extended system intersect after a finite time, independently of their common initial condition. Thus, the "average motion" of the original system corresponds to the motion of the controlled extended system. The speed of convergence of the system trajectory to the desired terminal point is dictated by the static feedback for the extended system.

The second approach to the stabilization of bilinear systems comprises two phases: the *reaching phase* and the *sliding phase*. In the reaching phase the state of the system is steered to a selected stable manifold by employing a suitably designed control Lyapunov function in conjunction with the discontinuous time-varying Lie algebraic control proposed for general nonlinear systems with drift. The latter is necessary when there do not exist controls which generate instantaneous velocities decreasing the Lyapunov function. Once the set of stable manifolds is reached the control is switched to its sliding phase whose task is to confine the motion of the closed loop system to the latter set, making it invariant under limited external disturbances.

The computationally feasible approaches proposed in this research necessitated the development of a set of software tools for symbolic manipulation of expressions with Lie brackets. The novel software package constitutes a contribution towards the automated construction of Philip Hall bases, simplification of any Lie bracket expression, composition of flows via the Campbell-Baker-Hausdorff formula and other Lie algebraic manipulations.

Conditions under which the constructed feedback laws render the corresponding systems asymptotically stable are analyzed. The applicability and effectiveness of the proposed approaches is demonstrated through computer simulations of several nonlinear systems, including well known nonholonomic systems without drift, such as the kinematic models of a unicycle and a front-wheel drive car, and systems with drift like the challenging angular velocity stabilization of a satellite in actuator failure condition.

1 Research Area and Motivation

This work addresses the problem of stabilizing feedback design for systems whose dynamics on $\mathbb{R}^n$ is modeled by nonlinear ordinary differential equations of the form:

$$\Sigma : \qquad \dot{x} = f_0(x) + \sum_{i=1}^{m} f_i(x) u_i \tag{1.1}$$

where the $f_i : \mathbb{R}^n \to \mathbb{R}^n$, $i = 0, \ldots, m$, are smooth vector fields and u_i, $i = 1, 2, \ldots, m$, with $m < n$, are the control inputs.

This research is also relevant to the development of stabilizing control laws for bilinear systems for which $f_i(x) = A_i x$, with $A_i \in \mathbb{R}^{n \times n}$, are linear vector fields. Bilinear systems are of interest since they correspond to the linearization of (1.1) with respect to the state only.

Systems described by (1.1) often occur in practice, typical examples being the nonholonomic systems and systems that become underactuated due to the failure of some of its physical components [1].

The motivation to the proposed research is provided by observing the following:

- The existence of relatively few general methods for stabilization of nonlinear systems with drift.
- The fact that strongly nonlinear systems often result in uncontrollable linearizations about their equilibria and do not satisfy the necessary conditions for smooth [3, 4] or even continuous [5] stabilization.
- The lack of computationally feasible methods for the construction of discontinuous feedback and the computational complexity of the feedback laws based on Lie algebraic approaches developed so far.
- The lack of constructive approaches to stabilization of general (higher order $n > 2$) multi-input bilinear systems for which the matrix $A_0 + \sum_{i=1}^{m} u_i A_i$ is unstable for all choices of constants u_i.

2 Research Objective

The main objective of this research is the development of:

1. Algorithms for the construction of time-varying stabilizing controls for a wide class of systems with drift based on Coron's [6, 7] approach of critical orbits for driftless systems.
2. Computationally feasible algorithms for the construction of time-varying discontinuous feedback controls for systems with drift.
3. Algorithms for the stabilization of bilinear systems with unstable drift.

3 Previous Approaches Pertaining to the Research Objective

The stabilization problem for systems described by (1.1) has found considerable interest. Most methods begin by finding a Lyapunov function for some type of linearization of (1.1) [2, 9], or else assume the existence of a suitable control Lyapunov function which can be decreased to zero by an adequate choice of controls in (1.1), cf. [10] and references therein.

Lie algebraic methods have already been applied with great success to general systems without drift [8]. However, relatively few attempts exist which utilize Lie algebraic approaches for the stabilization of systems with drift [11, 12, 13]. These methods are computationally complex, thus not very attractive for practical implementation purposes.

Some methods are based on physical considerations of the problem and particular characteristics of the system's vector fields [14, 15, 16, 17], hence these results are of limited scope and cannot be generalized.

Regarding the stabilization of bilinear systems, numerous approaches have been proposed [18, 19, 20, 21, 23, 24, 25, 26], see also the survey by Elliott [27]. However there is no universal solution proposed yet. Several methods assume that the drift term is stable, [18, 26], i.e. A_0 has no eigenvalues in the open right-half of the complex plane.

Stabilization of homogeneous bilinear systems in the plane has been fully analyzed. Bacciotti and Boieri [22], have used constant, linear, and quadratic feedbacks, and Chabour et al. [23], have used feedbacks differentiable except at zero, to give complete stabilizability conditions for single input bilinear systems on $\mathbb{R}^2 - \{0\}$. The methods of analysis in these papers again involve Lyapunov functions, center manifolds, and properties of curves in the plane.

For higher dimensional systems relatively few methods for feedback stabilization are available. In [24], Wang gives a sufficient condition for stabilizability of systems in $\mathbb{R}^n$ by piece-wise constant controls, however, no general procedure for their construction is provided.

4 Research Progress

In relation to the above objective, the following progress is reported:

1. Time-varying stabilization of systems with drift [P2]:
   The proposed feedback law is a composition of a periodic time-varying control which provides for critical stabilization, and an asymptotically stabilizing feedback "correction" term. The method partially draws on the ideas of Coron and Pomet, see [6, 7], who constructed time-periodic stabilizing controls for systems without drift.

   The periodic control is obtained through the solution of an open loop, finite horizon, control problem on the associated Lie group which is posed as a trajectory interception problem in the Philip Hall coordinates of flows for (1.1) and its Lie bracket extension [8]. The correction term is calculated to be a control which decreases a Lyapunov function whose level sets contain the periodic orbits of the system stabilized by the time-periodic feedback. This approach an its successful application to a strongly nonlinear nilpotent system has been reported in [P2].

2. Computationally feasible algorithms for discontinuous time-varying feedback stabilization of systems with drift [P4, P5]:
   The proposed control law comprises two modes. In one mode the control is a smooth state feedback $u(x)$ that guarantees an instantaneous decrease of a chosen control Lyapunov function $V(x)$. This mode is applied whenever there exists a control $u(x)$ such that $\dot{V}(x, u(x)) < 0$. In the other mode, an open loop piece-wise constant control $\bar{u}(x, t)$ which decreases the control Lyapunov function on average, after a finite period of time T, is applied. The synthesis of $u(x)$ is based on the standard Lyapunov approach, thus the emphasis is put on the construction of $\bar{u}(x, t)$ by means of Lie algebraic techniques.

Two approaches to the synthesis of the Lie algebraic control $\bar{u}$ have been proposed. In both approaches the control is composed of a sequence of constant controls whose values are calculated as the solution to a non-linear programming problem. In the first approach the formulation of the non-linear programming problem results from the direct application of the Campbell-Baker-Hausdorff formula for composition of flows, while in the second approach, the non-linear programming problem is formulated by posing the original control problem in terms of a relaxed control problem in the associated logarithmic coordinates.

The theoretical background for this approach is discussed in [P5]. The method has been successfully applied to the control of nonholonomic systems: the unicycle and the front-wheel drive car [P5], as well systems with drift: angular velocity stabilization of a satellite in actuator failure mode [P4, P5].

3. Stabilization of bilinear systems with unstable drift [P1, P3, P6]:
   Two approaches to the stabilization problem have been investigated. Both methods make use of the Lie bracket extension of the system.

   The first method, [P1], considers the construction of a time-invariant feedback for the extended system, which is a relatively simple task under reasonable assumptions. The original system controls are then obtained as a solution to an open loop, finite horizon, control problem posed in terms of a finite horizon interception problem of the logarithmic coordinates for flows [8]. The open loop controls so generated are such that the trajectories of the open loop system intersect those of the controlled extended system after a finite time T, independent of their common initial condition. Thus, the "average motion" of the original system corresponds to the motion of the controlled extended system. The speed of convergence of the system trajectory to the desired terminal point is dictated by the static feedback for the extended system.

   The second approach comprises two phases: the *reaching phase* and the *sliding phase*. In the reaching phase the state of the system is steered to a selected stable manifold by employing a suitably designed control Lyapunov function in conjunction with a Lie algebraic control. The latter is necessary when there do not exist controls which generate instantaneous velocities decreasing the Lyapunov function. The Lie algebraic control is constructed using the first method proposed in [P5]. Conditions are given under which the constructed feedback control renders the stable manifold globally attractive and attainable in finite time. Once the set of stable manifolds is reached the control is switched to its sliding phase whose task is to confine the motion of the closed loop system to the latter set, making it invariant under limited external disturbances. Two examples corresponding to different dimension of the stable manifolds are presented in [P3, P6] to demonstrate the effectiveness of the approach.

4. A software package for symbolic manipulation of elements in Lie algebraic theory [28]:
   The computationally feasible approaches proposed in this research necessitated the development of a set of software tools for symbolic manipulation of expressions with Lie brackets.

   To this end, a software package has been implemented in Maple. The module is called Lie Tools Package (LTP) [28], and among other functions it enables the following automated Lie algebraic manipulations:

   - Construction of Philip Hall bases.
   - Simplification of any Lie bracket expression.
   - Composition of flows via the Campbell-Baker-Hausdorff formula.
   - Set up of the logarithmic-coordinates equation.

5 Originality of the Proposed Research

The proposed approaches constitute an original contribution to the stabilization of (1.1) in that:

- The novel synthesis method for time-varying stabilizing controls [P2] is general and applicable to a large class of nilpotent systems which do not lend themselves to successful linearization (be it through state-feedback transformations, or else simply around some operating points).
- The approach in [P6] constitutes the first attempt to provide a systematic methodology for the synthesis of discontinuous time-varying feedback based on computationally feasible Lie algebraic techniques.
- The Lie algebraic approaches to the synthesis of stabilizing feedback control for homogeneous bilinear systems are completely new. Unlike existing methods, the proposed approaches consider systems with unstable drift which cannot be stabilized by any constant control. Sufficient conditions for the existence of the proposed control law are given.
- The symbolic manipulation procedures developed in LTP also constitute a novel tool. The existing software for Lie algebraic manipulations are very specialized, e.g. [29], and Maple's *liesymm package* [30], and do not provide any of the functionality listed above (Waterloo Maple Inc. was contacted without any positive response, and other major computer algebra systems: Reduce, Macsyma, Mathematica were evaluated less favorably).

6 Publications

[P1] H. Michalska, M. Torres-Torriti, Time varying stabilizing feedback design for bilinear systems, *Proceedings of the IASTED International Conference on Modelling, Identification and Control - MIC'2000*, Innsbruck, Austria, February 2000.

[P2] H. Michalska, M. Torres-Torriti, Time-varying stabilizing feedback control for nonlinear systems with drift, *Proceedings of the* 40^{th} *Conference on Decision and Control*, Orlando, Florida, December 2001.

[P3] M. Torres-Torriti, H. Michalska, Stabilization of bilinear systems with unstable drift, accepted in *ACC-2002*.

[P4] M. Torres-Torriti, H. Michalska, Computationally feasible Lie algebraic approaches for stabilization of systems with drift: the satellite example, to be submitted to the *CDC-2002*.

[P5] H. Michalska, M. Torres-Torriti, Computationally feasible Lie algebraic approaches for stabilization of nonlinear affine systems with drift, *journal paper in preparation*.

[P6] H. Michalska, M. Torres-Torriti, Stabilization of bilinear systems with unstable drift, *journal paper in preparation*.

References

[1] W.S. Levine, *Control System Applications*. CRC Press LLC, USA, 2000.

Books in Nonlinear Control Theory

[2] A. Isidori, *Nonlinear Control Systems: An Introduction*, Springer-Verlag, 1989.

Stabilizability

[3] R.W. Brockett, Asymptotic stability and feedback stabilization, in *Differential Geometric Control Theory*, R.W. Brockett, R.S. Millman and H.J. Sussmann eds., Birkhäuser, Boston, pp. 181-191, Basel, 1983.

[4] E.P. Ryan, On Brockett's condition for smooth stabilizability and its necessity in a context of nonsmooth feedback, *SIAM J. Control and Optimization*, v. 32, n. 6, pp. 1597-1604, 1994.

[5] J. Zabczyk, Some comments on stabilizability, Applied Mathematics and Optimization, v. 19, pp. 1-9, 1989.

Stabilization of Systems without Drift

[6] J.-M. Coron and J.-B. Pomet, A remark on the design of time-varying stabilizing feedback laws for controllable systems without drift. *IFAC Nonlinear Control Systems Design Symposium '92 (NOLCOS92)*, Bourdeaux, France, 24-26 June 1992, pp. 413-417.

[7] J.-M. Coron, Global asymptotic stabilization for controllable systems without drift. *Mathematics of Controls Signals and Systems*, v. 5, 1992, pp. 295-312.

[8] G. Lafferriere and H. J. Sussmann, A differential geometric approach to motion planning, in *Nonholonomic Motion Planning*, Z. Li and J. F. Canny eds., Kluwer Academic Publishers, 1993, pp. 235-270.

Stabilization of Systems with Drift

[9] D. Aeyels, Stabilization of class of nonlinear systems by a smooth feedback control. *Systems & Control Letters*, v.5, pp. 289-294, 1985.

[10] M. Malisoff and E.D. Sontag, Universal formulas for feedback stabilization with respect to Minkowski balls, *Systems & Control Letters*, v.40, pp. 247-260, 2000.

[11] H. Hermes, On the synthesis of a stabilizing feedback control via Lie algebraic methods. *SIAM J. Control and Optimization*, v. 18, n. 4, July 1980, pp. 352-361.

[12] S. Čelikovský and E. Aranda-Bricaire, Constructive nonsmooth stabilization for a class of nonlinear single-input systems, *Proceeding of the* 36^{th} *Conference of Decision & Control.* San Diego, California, U.S.A, December 1997, pp. 1734-1735.

[13] H. Michalska and F. U. Rehman, Time varying feedback synthesis for a class of nonhomogeneous systems, *Proceeding of the* 36^{th} *Conference of Decision & Control.* San Diego, California, U.S.A, December 1997, pp. 4018-4021.

[14] H.J. Sussmann, Local controllability and motion planning for some classes of systems with drift. *Systems and Control Letters*, v. 12, 1989, pp. 213-217.

[15] J.-M. Godhavn, A. Balluchi, L.S. Crawford and S.S. Sastry, Control of nonholonomic systems with drift terms. *1997 IEEE American Control Conference*, pp. 0-42.

[16] A. Bacciotti, Local Stabilizability of Nonlinear Control Systems, World Scientific Publishing Co., Singapore, 1992.

[17] D.O. Popa and J.T. Wen, Feedback stabilization of nonlinear affine systems. *Proceeding of the* 38$^{\text{th}}$ *Conference of Decision & Control.* Phoenix, Arizona, U.S.A, December 1999, pp. 1290-1295.

Bilinear Systems

[18] M. Slemrod, Stabilization of bilinear control systems with applications to nonconservative problems in elasticity. *SIAM J. Control and Optimization*, Vol. 16, No. 1, 1978, pp. 131-141.

[19] R. Longchamp, Stable feedback control of bilinear systems, *IEEE Transactions on Automatic Control*, Vol. AC-25, No. 2, pp. 302-306, April 1980.

[20] Per-Olof Gutman, Stabilizing controllers for bilinear systems, *IEEE Transactions on Automatic Control*, Vol. AC-26, No. 4, pp. 917-922, August 1981.

[21] S. Čelikovský, On the stabilization of the homogeneous bilinear systems, *Systems & Control Letters*, No. 21, pp. 503-510, North-Holland, 1993.

[22] A. Bacciotti, and P. Boieri, A characterization of single-input planar bilinear systems which admit a smooth stabilizer, *Systems & Control Letters*, No. 16, pp. 139-144, North-Holland, 1991.

[23] R. Chabour, G. Sallet, J. C. Vivalda, Stabilization of nonlinear systems: A bilinear approach, *Mathematics of Control, Signals and Systems*, No. 6, pp. 224-246, 1993.

[24] H. Wang, Feedback stabilization of bilinear control systems, *SIAM Journal on Control and Optimization*, Vol. 36, No. 5, pp. 1669-1684, September 1998.

[25] M.-S. Chen, Exponential stabilization of a constrained bilinear system, *Automatica*, Vol. 34, No. 8, pp. 989-992, 1998.

[26] G. Lu, Z. Yufan, Z. Cishen, Y. Xinghuo, Global asymptotic stabilization of MIMO bilinear systems with undamped natural response, *Proceedings of the 37th Conference on Decision and Control*, pp.1989-1994, Tampa, Florida, December 1998.

[27] D. L. Elliott, Bilinear Systems, in the *Encyclopedia of Electrical Engineering*, edited by John Webster, John Wiley and Sons, 1999.

Computer Algebra

[28] M. Torres-Torriti, *Lie Tools Package - User's Guide.* Technical Report, McGill University, Canada, 2001.

[29] M.A.A. van Leeuwen, LiE, A software package for Lie group computations, Euromath Bulletin 1, n. 2, 1994.

[30] B.W. Char, et al. *Maple V Library Reference Manual.* Springer-Verlag, New York, 1991.