MIDTERM EXAM

Introduction to Computer Systems (COMP 273)

Tues. Feb. 7, 2012

Professor Michael Langer

STUDENT NAME: ________________________________ ID: __________________________

The exam consists of eight questions, for a total of 20 points.

No calculators or notes are allowed.

You may use this sheet to sketch out solutions.

1. **(2 points)**

   (a) Convert `0xde` to binary, then treat it as an *unsigned* integer and convert to decimal.

   SOLUTION:
   0xde $= (1101\ 1110)_2 = (222)_{10}$.

   (b) Binary is base 2, *octal* is base 8, decimal is base 10, and hexadecimal is base 16. Convert the decimal number 211 to octal.

   SOLUTION:
   Converting to binary is $(11\ 010\ 011)_2$. Converting to octal is $(323)_8$.

   GRADING: 1 point for (a). 1 point for (b).

2. **(3 points)**

Write the number -0.3 in IEEE single precision floating point format.
Give your answer in hexadecimal.

Hints:

- The three fields for IEEE single precision format are of size (1,8,23).
- The bias for the exponent is 127.
- For this example, the significand has a simple form when written in hexadecimal.
- You will have to round off the number. Round towards zero, i.e. round down the absolute value.

SOLUTION:

$$
\begin{aligned}
.3 &= 0.3 \times 2^{0} \\
&= (00).6 \times 2^{-1} \\
&= (001)_2.2 \times 2^{-2} \\
&= (0010)_2.4 \times 2^{-3} \\
&= (00100)_2.8 \times 2^{-4} \\
&= (001001)_2.6 \times 2^{-5}
\end{aligned}
$$

The right side has now cycled back to where it was earlier (at the 2^{-1} term). So we know that the bit sequence 1001 will repeat infinitely many times. Thus,

$$0.3 = (001001).1001\ 1001\ 1001\ 1001\ \cdots \times 2^{-5}$$

and we then normalize by multiplying and dividing by 2^{-3}, giving

$$0.3 = 1.001\ 1001\ 1001\ 1001\ 1001\ \cdots \times 2^{-2}$$

The significand is 001 1001 1001 1001 1001 .... The exponent code is 127-2, or 125, which in binary is 01111101. The sign bit is 1. So, the 32 bits (sign, exponent, significand) are

$$1011\ 1110\ 1001\ 1001\ 1001\ 1001\ 1001\ 1001 = \text{0xbe999999}.$$

GRADING: 1 point for writing 0.3 in binary. 0.5 point for normalized it. 0.5 for computing the exponent 125. 1 point for packing the bits and converting to hex.

3. **(2 points)**

Recall that $\oplus$ is "exclusive or". Define $Y = (\overline{A \oplus B}) \cdot C$.

(a) Rewrite Y as a sum-of-products where each product depends on all three variables A, B, C. Use a truth table to compute your solution.

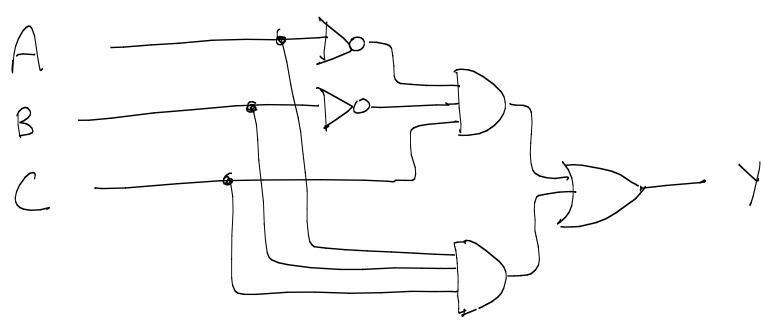
SOLUTION:

A	B	C	$A \oplus B$	$\overline{A \oplus B}$	Y
0	0	0	0	1	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	0	0
1	1	0	0	1	0
1	1	1	0	1	1

$$Y = \overline{A} \cdot \overline{B} \cdot C + A \cdot B \cdot C$$

(b) Draw a circuit for the sum-of-products in (a).

SOLUTION:



GRADING: 1 point for correct expresion for Y in (a). 1 point for (b). **If (a) is incorrect, then (b) is graded based on the answer for Y given in (a).**

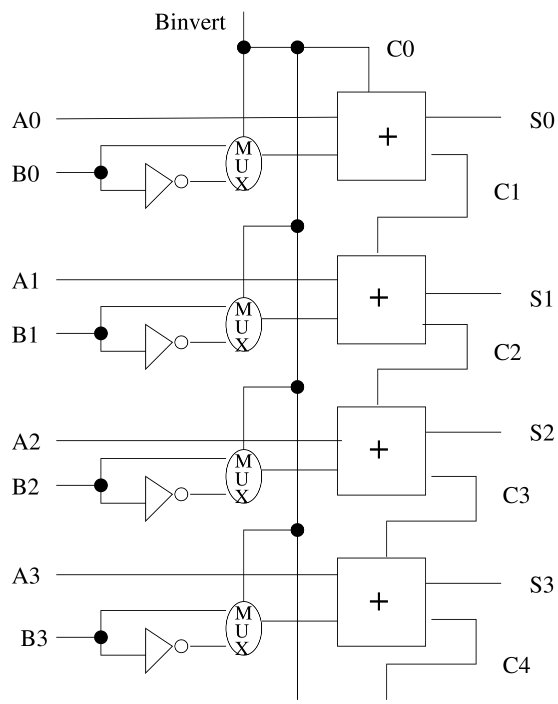
4. **(3 points)**

(a) Let $A = -27$ and $B = 13$. Write these numbers in binary using the twos complement representation, and using *one byte each*.

SOLUTION:
A = $(11100101)_2$. B = $(00001101)_2$.

(b) This part of the question depends on part (a). We will give you 0 points on this part, if your answer to (a) was incorrect. So doublecheck your answer to (a) now.

Consider the low order four bits of an "adder" circuit, shown below. Give the binary values on the input and output lines, assuming the adder is computing $A - B$ for the values given above.



SOLUTION:

$$(A_3, A_2, A_1, A_0 = Binvert) = (0, 1, 0, 1), \qquad (B_3, B_2, B_1, B_0) = (1, 1, 0, 1).$$

The four low order carry and sum bits are:

$$(C_4, C_3, C_2, C_1, C_0 = Binvert) = (0, 1, 1, 1, 1), \qquad (S_3, S_2, S_1, S_0) = (1, 0, 0, 0).$$

GRADING:

1 point for (a). 1 point for carry bits including C_0 or Binvert. 1 point for S bits.

5. **(3 points)**

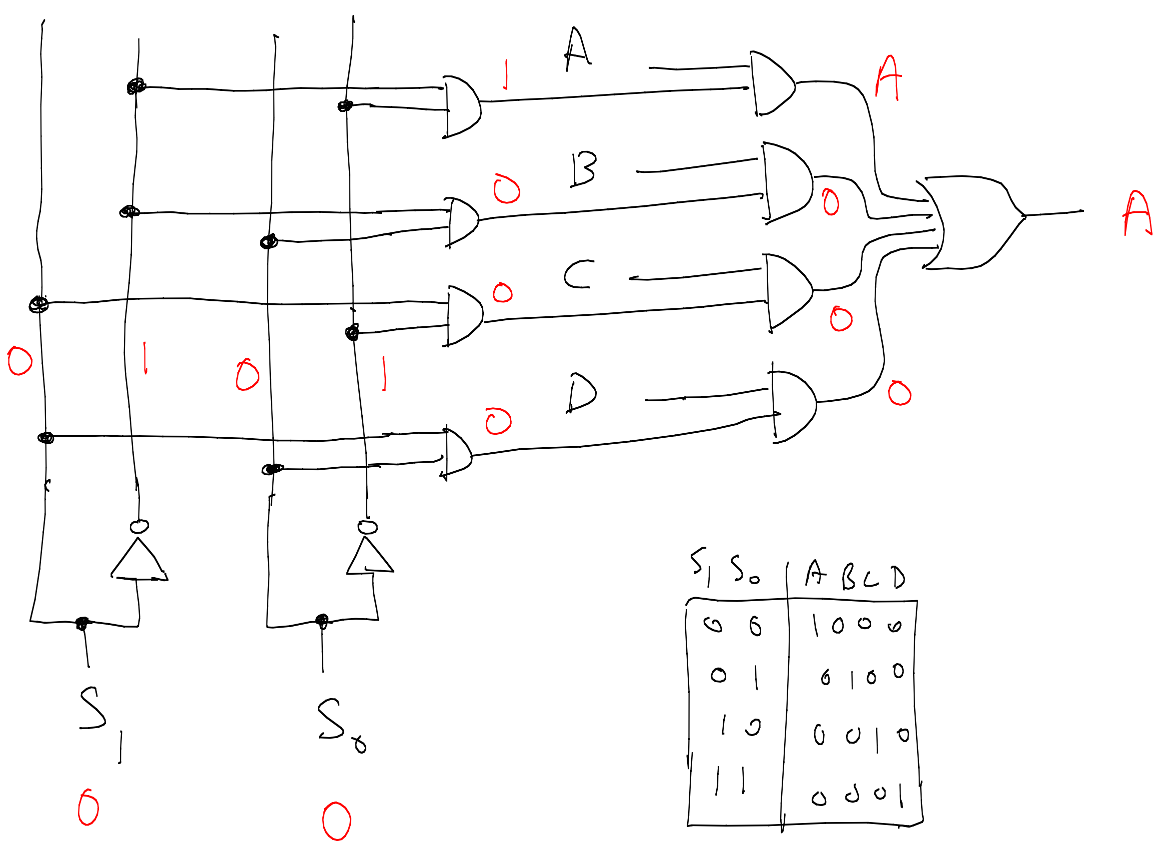
(a) Draw the *complete* combinational circuit of a multiplexor that selects from 4 possible input bits (A, B, C, D).

You need to draw the complete circuit for a 2-to-4 decoder and connect this to the part of the multiplexor circuit that was given in class (and which you must reproduce).

Sketch out the solution on the front page of this exam, before writing your final answer here.

(b) Show the values on all the lines, for the case that A is selected.

You don't know the value of A, so just label the selected value as A.



S_1	S_0	A	B	C	D
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

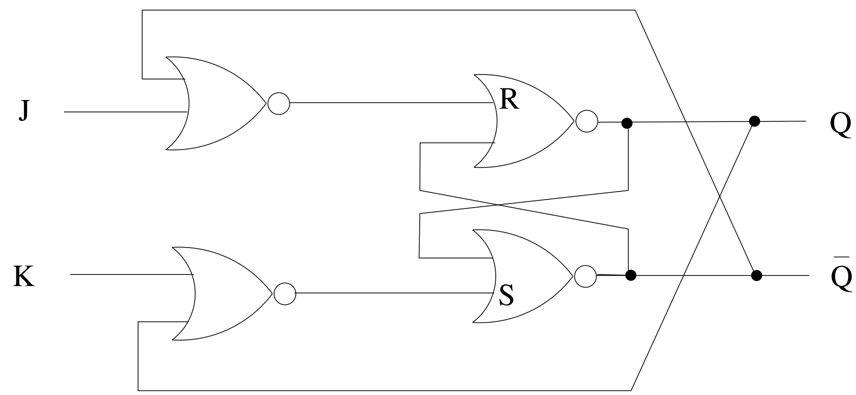
GRADING: 1 point for the decoder part. 1 point for the selector part. 1 point for showing the values on all the lines.

6. **(3 points)**

Recall from lecture 5 that an RS latch has this behavior:

R	S	Q	$\overline{Q}$	
0	0	hold		← remember
0	1	1	0	←"set"
1	0	0	1	←"reset"
1	1	0	0	← not allowed

Note that there is an RS latch embedded in the circuit below.



Fill in the right two columns of the following table. For example, in the first row, assume that $(Q, \overline{Q})$ *currently* have values $(0, 1)$ and fill in what would be the next values of $(Q, \overline{Q})$, if (J, K) were given values $(0, 0)$. Note: the next values might be the same as the current values.

Hint: Take advantage of the up-down symmetry of the figure, when possible.

SOLUTION:
The way to approach this question is to ignore the details within the RS latch and instead to use the table at the top of the page.

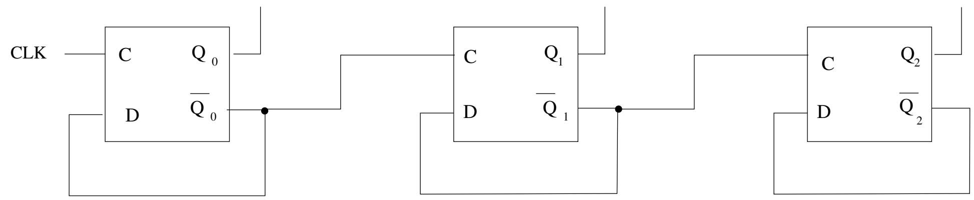
You can use the up-down symmetry by filling in rows (without tracing bits) or by verifying the consistency of two rows where you did get the solutions by tracing bits.

			current		next		
	J	K	Q	$\overline{Q}$	Q	$\overline{Q}$	
(0)	0	0	0	1	1	0	
(1)	0	0	1	0	0	1	toggle
(2)	0	1	0	1	0	1	
(3)	0	1	1	0	0	1	reset
(4)	1	0	0	1	1	0	
(5)	1	0	1	0	1	0	set
(6)	1	1	0	1	0	1	
(7)	1	1	1	0	1	0	hold

GRADING: 1 point for (0)-(1). 1 point for (2)-(5). 1 point for (6)-(7).

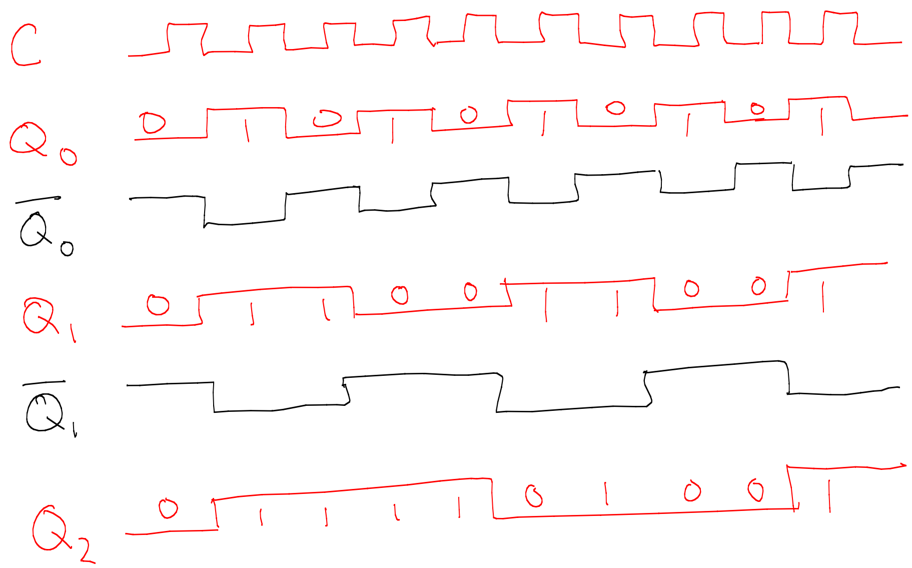
7. **(2 points)**

(a) Draw a timing diagram of the real clock input CLK, and data values Q_0, Q_1, Q_2. Assume the D flip-flops are *falling edge* triggered, and that the starting (Q_2, Q_1, Q_0) values are (0,0,0).



(b) What does this circuit do?

SOLUTION:
This circuit is slightly different from the one given in class. Here, the C input for the Q_{i+1} come from $\overline{Q_i}$. This makes a difference.



Its a timer (count down)

GRADING: 1 point for timing diagram. It was sufficient to give the C and Q_2, Q_1, Q_0 values. 1 point for saying its a timer.

If you treated the circuit as if it were the one given in class, then you would get a counter. I gave 1 point for this (assuming you gave the circuit and said it was a counter).

8. **(2 points)**

What is the product $x \times y$ of the following two binary numbers?

$$x = (1.0101)_2 * 2^{-14} \qquad y = (1.101)_2 * 2^{19}$$

Write your answer as a normalized number.

Note: we are *not* asking for the IEEE floating point representation.

SOLUTION:

```
      10101 x 2^{-18}
       1101 x 2^{16}
       ----
      10101
    10101
   10101
   --------
  100010001  x 2^{-2}  =  1.00010001 x 2^6
```

GRADING : 1 point for any non-normalized version. (Only 0.5 if you got the significand correct, but exponent was wrong.) 1 point for normalized version.