McGill

Department of Mechanical Engineering

MECH 573: Mechanics of Robotic Systems
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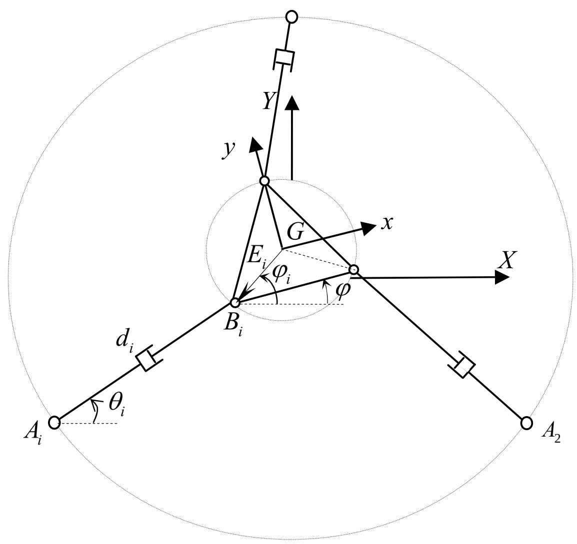
Consider the planar 3RPR manipulator shown in the following figure for continuing analysis. In this manipulator, the base points A_i's are located on a circle with radius R_A, with an equivalent angle of 120^o. Similarly, the moving platform points B_i's are located on a circle with radius R_B. The fixed coordinate frame is located at the center of circle R_A, whose x axis is parallel to A_1A_2 and its y axis along A_3. The moving platform position and orientation is represented by the vector $\boldsymbol{x} = [x_G, y_G, \varphi]^T$, and the input joint variables are the extendible limb lengths d_i's, $i=1,2$ and 3. The joint variables vector is represented by $\boldsymbol{q} = [d_1, d_2, d_3]^T$ and the angle of each limb is denoted with θ_i's. The vector E_i denotes the vector representation of GB_i.



As a result of the singularity analysis accomplished in Part I, it is observed that the manipulator is in singularity configuration if $\varphi=0,\pm180^o$. Therefore, to avoid singularity consider the maximum workspace of this manipulator as $\varphi= \varphi_o \pm 60^o$ in which $\varphi_o=90^o$ for all the remaining simulations in Part II and III of the project.

In this part of the project dynamic analysis of the manipulator and its general simulation is developed as following.

1) Perform acceleration analysis of the manipulator. Using the kinematics and Jacobian analysis performed in Part I, derive equations relating $\ddot{d}_i$'s and $\ddot{\alpha}_i$'s as a function of $\ddot{\mathbf{x}} = [\ddot{x}_G, \ddot{y}_G, \ddot{\varphi}]^T$ and the state variables $\boldsymbol{x}$ and $\boldsymbol{q}$ and their first derivatives.

2) Using the Newton-Euler method, find the inverse dynamic formulation of the manipulator In order to simplify the formulation consider the prismatic limbs as extendible but slender

homogeneous rods, whose inertias are changing in time due to elongation. Also consider that the gravity vector is perpendicular to the plane of motion. Perform the dynamic analysis following these steps:

2-1) Derive the velocities and accelerations of the center of mass of each limb.

2-2) Write the Newton-Euler equation for each limb. Notice the changing mass and inertia of each limb due to its elongation. As a result of this analysis derive the tangential and normal forces that the limbs are exerting to the moving platform, as a function of kinematics and differential kinematics variables and the actuator force f_i's.

2-3) Consider the mass of the moving platform denoted as M, and its inertia about the center of mass denoted as I. Moreover, consider a disturbance force-moment acting on the center of mass of the moving platform as $\boldsymbol{F_d} = [f_{dx},\ f_{dy},\ \tau_d]^T$. This force can also be considered as the resulting force that the manipulator is exerting to the environment. Write the Newton-Euler equation for the moving platform, and use the forces derived in above to generate the equation of motion of the moving platform.

3) The resulting equation of motion has an implicit form of $F(\ddot{x}, \dot{x}, x, \ddot{q}, \dot{q}, q, f_i) = F_d$ and cannot be directly integrated using usual Runge-Kutta numerical method developed in Matlab, or Simulink. As explained in detail in the class you may use implicit ODE solver to numerically integrate the above equations. Use **`ode15i`** to simulate the dynamic equation of the system with the following numerical values and use **`decic`** to generate a set of consistent initial values for the integration state.

$R_B=1,\ R_A=10,\ M=10,\ I = 0.1,$ and the length density of the limbs $\rho=0.1\ (kg/m)$.
Initial condition and simulation time: $\mathbf{x}(0)=[0,\ 0,\ 90^o]^T,\ 0 < t < 1$.
Open-loop simulation: control inputs f_i's $= 0$.

1) No Disturbance input: $\boldsymbol{F_d} = 0$.
2) Disturbance inputs as following: viscous friction plus pulses:

$$f_{dx} = \begin{cases} -c_x\dot{x} + 1000 & \text{for } 0.4 \le t < 0.5 \\ -c_x\dot{x} & \text{otherwise} \end{cases};\quad f_{dy} = \begin{cases} -c_y\dot{y} + 1000 & \text{for } 0.1 \le t < 0.3 \\ -c_y\dot{y} - 1000 & \text{for } 0.3 \le t < 0.5; \\ -c_y\dot{y} & \text{otherwise} \end{cases}\quad \tau_d = -c_\phi\dot{\varphi}\ \ \forall t.$$

In which the viscous friction coefficients are: $c_x = c_y = c_\varphi = 1000$. Assume appropriate values for any other parameter which is not given explicitly.

4) Examine the resulting motion of the moving platform and the limbs and verify it by physical insight. Provide sufficient plots, a concise report, and your developed program with your analysis report.

"Good Luck"