MECH 573: Mechanics of Robotic Systems
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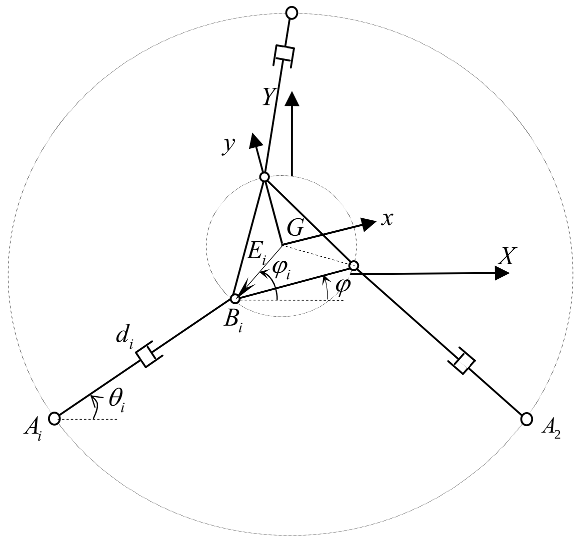
Consider the planar 3RPR manipulator shown in the following figure.



In this manipulator, the base points A_i's are located on a circle with radius R_A, with an equivalent angle of 120^o. Similarly, the moving platform points B_i's are located on a circle with radius R_B. The fixed coordinate frame is located at the center of circle R_A, whose x axis is parallel to A_1A_2 and its y axis along A_3. The moving platform position and orientation is represented by the vector $\boldsymbol{x} = [x_G, y_G, \varphi]^T$, and the input joint variables are the extendible limb lengths d_i's, $i=1,2$ and 3. The joint variables vector is represented by $\boldsymbol{q} = [d_1, d_2, d_3]^T$ and the angle of each limb is denoted with θ_i's. The vector E_i denotes the vector representation of GB_i.

1) Derive expressions for inverse kinematics, i.e. suppose $[x_G, y_G, \varphi]^T$ is given find d_i's.

2) Solve the forward kinematics meaning d_i's are given find expressions for $[x_G, y_G, \varphi]^T$.

3) Derive the Jacobian matrices J_x and J_q, and discuss on the inverse and direct kinematics singularities, the Jacobian matrices are defined as $J_x \dot{x} = J_q \dot{q}$.

4) Derive the stiffness matrix of the manipulator, assuming that the moving platform is at a central location. Under what conditions will the stiffness matrix become decoupled? Assume unit stiffness at each limb.

5) Develop a program in Matlab with the following numerical values for parameters, in which for a given moving platform trajectory ***x(t)***, the limb lengths $d_i(t)$'s are found, notice that due to multiple solution in inverse kinematics, the solution must be found in such a way that no jumps are observed in the manipulator trajectories. Then give the time trajectories found in the first part, and solve the forward kinematics of the manipulator. Compare the calculated ***x(t)*** with the original one to verify your programs. Submit your developed programs via WebCT, and provide the illustrative plots of $d_i(t)$'s, and possibly ***$x_d(t)$ - x(t)*** in your project report, for verification purpose.

$R_B=1$, $R_A=10$, $\mathbf{x}(0)=[0, 0, 0]^T$, $\delta x=-0.5$, $\delta y=0.5$, $\omega=0.25$, $0 < t < 1$.
$x_d(t)= \delta x\ (3-2t)\ t^2$, $y_d(t)= \delta y\ (3-2t)\ t^2$, $\varphi_d(t)= \omega\ t$.

6) For the above trajectories, evaluate the Jacobian matrix $J = J_q^{-1} J_x$ and its singular values (`svd` function in Matlab). Does the lowest singular value vanish in any configuration and the system experience any singularities? To further analyze the singularities of the mechanism, extend the trajectories, into a larger 3D workspace for the moving platform and give 3D plot of the locations where the system experience singularities, if any.

7) Try similar analysis as in (6) but examine the eigenvalues of the stiffness matrix instead of the singular values of the Jacobian matrix, what do you observe?

"Good Luck"