McGill
Department of Mechanical Engineering
MECH 573: Mechanics of Robotic Systems
Winter 2006: TR 11:35-12:55 ENGTR 1090
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Assignment No. 3 Due date: 2006/02/09

1) Figure (1) shows the schematic diagram of a planar 2 DOF five bar manipulator, which is constructed with one prismatic and four revolute joints. Find the end effector position $\boldsymbol{q}$ as a function of the two input joint variables, d_1 and θ_2.

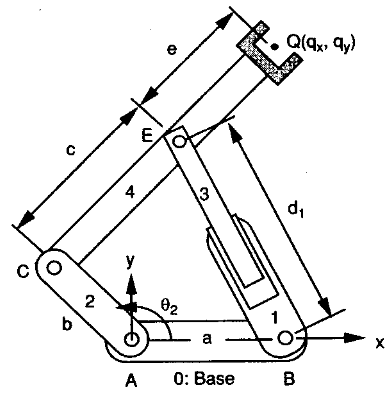


Figure 1. Five-bar, 4R1P manipulator.

2) A planar 2 DOF pantograph mechanism is shown in figure (2). Let links (1) and (2) be the input links, and link (6) be the output one. Also assume $BC \parallel DE$, $BE \parallel CD$, and points A, B, and Q lie on a straight line. Find the end effector position $\boldsymbol{q}$ as a function of the two linear displacements x_1, and y_2 of the two input links.

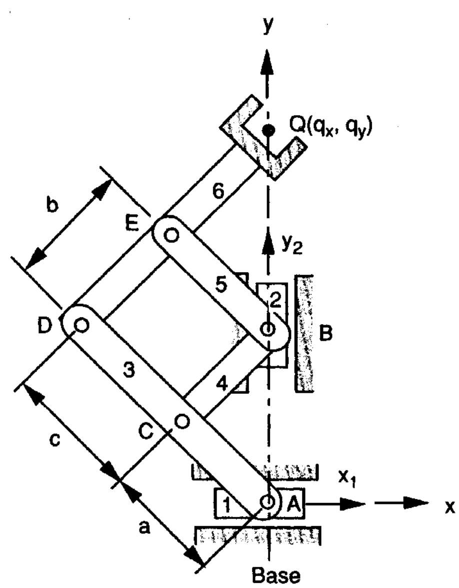


Figure 2. 2 DOF pantograph manipulator.

3) Figure (3) illustrates the schematics of a spatial 3UPU platform manipulator, in which three identical limbs connect the moving platform to the fixed base by universal joints at points B_i and A_i, $i=1, 2,$ and 3, respectively. Each limb consists of an upper member and a lower member that are connected by a prismatic joint. The three base-connected axes of the universal joints are coplanar. Similarly, the three moving platform connected axes of the universal joints are coplanar. The second axes of the universal joints that are directly attached to the upper and lower members of each limb are parallel to each other, and are both perpendicular to the axis of the prismatic joint.

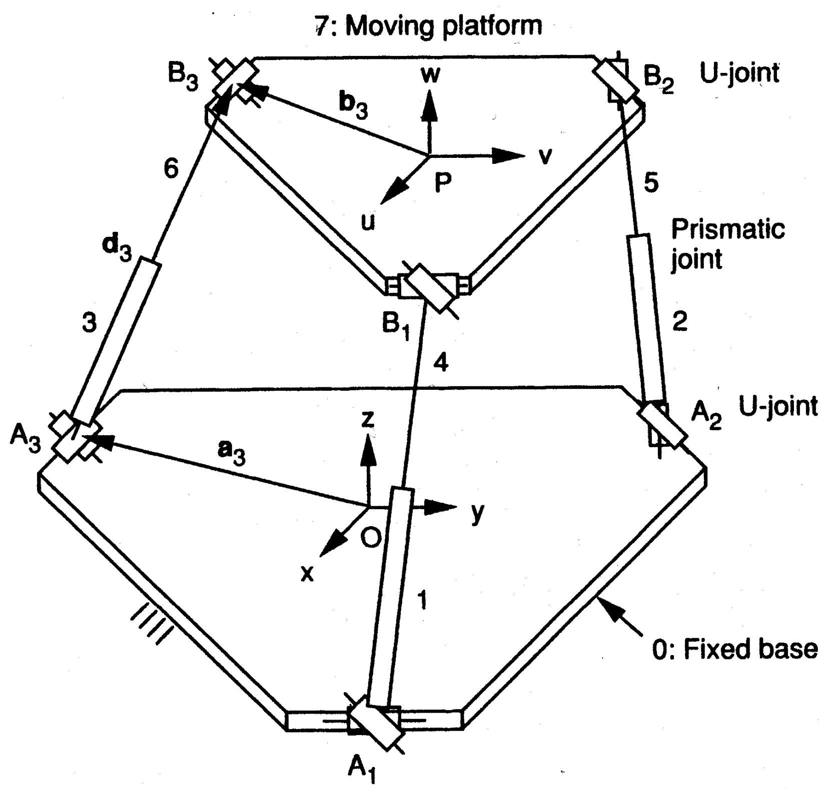
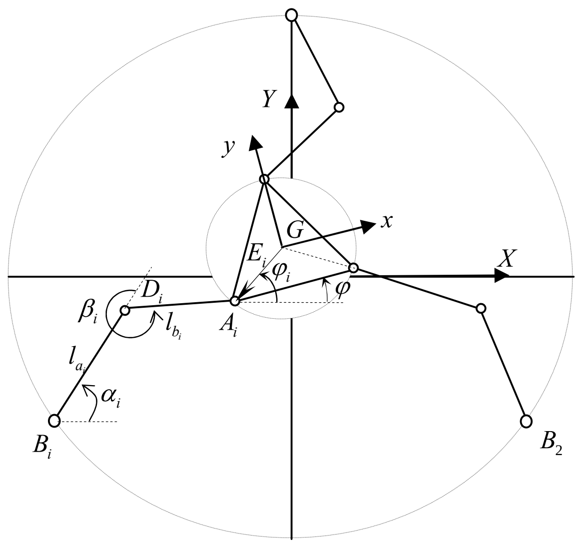


Figure 3.The schematics of a 3 DOF 3U3P parallel manipulator.

As you have shown in assignment No.1 this manipulator has three translational degrees of freedom. Considering the three prismatic joints as the input joints, solve the inverse kinematics problem for this manipulator.

4) In this problem it is intended to develop Matlab programs to solve inverse and forward kinematics of the 3RRR manipulator, whose analysis is given in the class. For simplicity of the programming and possibility to extend it to the course project, consider the following notations and frame assignments, which has small differences from what is given in the book:



In this manipulator, the base points B_i's are located on a circle with radius R_B, with an equivalent angle of 120^o. Similarly, the moving platform points A_i's are located on a circle with radius R_A. The fixed coordinate frame is located at the center of circle R_B, whose x axis is parallel to B_1B_2 and its y axis along B_3. The moving platform position and orientation is represented by the vector $\boldsymbol{x} = [x_G, y_G, \varphi]^T$, and the input joint angles are α_i's, $i=1,2$ and 3. The second link angle of each limb is denoted with β_i's. The limb link lengths are given as l_{ai}'s and l_{bi}'s as illustrated in the figure. The vector E_i denotes also the vector representation of GA_i.

First derive expressions for inverse kinematics, meaning that $[x_G, y_G, \varphi]^T$ is given find α_i's. Then solve the forward kinematics meaning α_i's are given find expressions for $[x_G, y_G, \varphi]^T$. Develop a program in Matlab with the following numerical values for parameters, in which for a given moving platform trajectory $\boldsymbol{x(t)}$, the joint angles $\alpha_i(t)$'s are found, notice that due to multiple solution in inverse kinematics, the angles must be found in such a way that no jumps are observed in the manipulator trajectories.

Then, give the time trajectories found in above, and solve the forward kinematics of the manipulator. Compare the calculated $\boldsymbol{x(t)}$ with the original one to verify your programs. Submit your developed programs in a diskette or via WebCT, and provide the illustrative plots of $\alpha_i(t)$'s, and possibly $\boldsymbol{x_d(t)} - \boldsymbol{x(t)}$ in your assignments, for verification purpose.

$R_B=10,\ R_A=1,\ l_{ai}=l_{bi}=5,\ \mathbf{x}(0)=[0\ 0\ 0]^T,\ \delta x=-0.5,\ \delta y=0.5,\ \omega=0.25,\ 0< t <1.$
$x_d(t)= \delta x\ (3-2t)\ t^2,\ y_d(t)= \delta y\ (3-2t)\ t^2,\ \varphi_d(t)= \omega\ t.$

"Good Luck"