6.2 Force Control

In previous sections we considered the problem of motion tracking using elementary and advanced control methods. These position control schemes are adequate for tasks, that the manipulator is not interacting with environment, such as flight simulator application of SGP. However, tasks such as grinding or milling like in variant structure in machine centers, involves in interacting forces between the E-E tool and the environment, and usually the amount of forces being transmitted must be controlled. Force control schemes is one that modifies position trajectories based on sensed or estimated forces exerted to the environment by the manipulator.

To measure the Force it is possible to use 6DOF F/T sensors which are mounted on the endeffector, which results into measuring the E-E wrench in 6 coordinates, 3 Forces along 3 axis & 3 torques about 3 axis. However, usually it is easier to measure the actuator forces, by single degree of freedom force sensors or load cells which are mounted on each actuator axis. Jacobian is always the best tool to change the joint space coordinates to task space if necessary

$$F = J^T C$$

In this section we present control algorithms, that not only provide position control, but also controls the forces that the end effector exerts on the environment.

6.2.1. Stiffness Control

Industrial robots are designed as rigid as possible to allow the robot control designer, to obtain reasonable positional accuracy by utilizing simple and high gain control laws. For a rigid robot, though interaction with a stiff environment is very difficult and sometimes dangerous. assume a robot which is solely position controlled with high gain controllers and rigid structure; If in the course of his pre-planned trajectory an obstacle appears, the robot will hit the obstacle, and since the purpose of his control routine, is to preserve his pre-planned trajectory, it may damage the obstacale totally (or kill the human!), to reach to its original trajectory by his rigid structure; Hence it is always good to have some elastisity in robot structure, and/or control to avoid such problems.

Elasticity in the robot design is a new approach, which is dominated robots with light-weight and flexible link & joints

Although their control schemes are much more difficult.
But given elasticity to the control law, was accomplished
from beginning in the designs. This is called "Stiffness Control"
of the robot by means of the control law. It is essential to
include forces exerted to the environment (by those laws into)
the Equation of motion of the robot manipulator. to under
stand the idea, let us first give a 1DOF Simple robotic
application, where Stiffness Control is implemented. After discussion
on the characteristics and specification of stiffness Control
we may generalize it to a NDOF general Case.

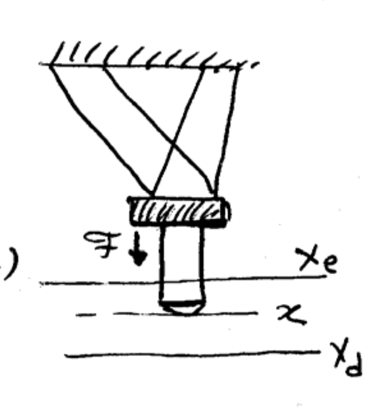
A) Stiffness Control of a Single DOF Manipulator

Consider a 1DOF parallel (or Simple) Structure interacting
with the environment. The Contact model is Considered as a
linear spring with elastic stiffness K_e hence the interacting
force is

$$f_e = K_e(x - x_e) = K_e \Delta x \qquad (2.1)$$

The Robot Dynamics in general is

$$\mathcal{F} = M(x)\ddot{x} + C(x,\dot{x})\dot{x} + G(x) + \mathcal{F}_e \qquad (2.2)$$



We will show as follows that the simple PD + IDC proposed earlier for the position control can be used here as a stiffness control routine. Hence for 1DOF system the Dynamic Eq. simplifies to

$$\tau = m(x)\ddot{x} + C(x,\dot{x})\dot{x} + g(x) + k_e(x - x_e) \qquad (2.3)$$

use PD+IDC as control (as in 6.1.6)

$$\left\{ \begin{array}{l} \tau = \underbrace{C(x,\dot{x})\dot{x} + g(x)}_{\text{F. Linearizing}} - k_p\tilde{x} - k_v\dot{x} \qquad (2.4) \\ \tilde{x} = x - x_d \quad \text{as befor} \end{array} \right.$$

Hence No force measurement is needed here only special attention should be given to $x_d(t)$ which will be explained later. The structure of controller is similar to position control.

Now derive closed-loop Eq. of motion by substituting (2.4) into (2.3) considering perfect feedback linearization

$$m(x)\ddot{x} + k_e(x - x_e) + k_p(x - x_d) + k_v\dot{x} = 0 \qquad (2.5)$$

this simplifies to

$$m\ddot{x} + k_v\dot{x} + (k_p + k_e)x = k_p x_d + k_e x_e \qquad (2.6)$$

using Laplace transform, the linearized System can be written in transfer function form; in which $k_p x_d$ & $k_e x_e$ have the dimension of force:

$$\frac{x(s)}{f(s)} = \frac{1}{ms^2 + k_v s + (k_p + k_e)} \qquad (2.7)$$

or in terms of the output position of the System

$$x(s) = \frac{k_p x_d(s) + k_e x_e(s)}{ms^2 + k_v s + (k_p + k_e)} \qquad (2.8)$$

Let us examine the steady state behavior of such manipulator in which $x_d(t)$ & $x_e(t)$ are constant @ steady state (for a constant step input $X_d(s) = \frac{\bar{X}_d}{s}$ And/or $X_e(s) = \frac{\bar{X}_e}{s}$)

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} s\,X(s) = \lim s \frac{k_p(\bar{X}_d/s) + k_e(\bar{X}_e/s)}{ms^2 + k_v s + (k_p + k_e)}$$

$$x_{ss} = \lim_{s\to 0} \frac{k_p \bar{x}_d + k_e \bar{x}_e}{ms^2 + k_v s + (k_p + k_e)}$$

$$\bar{X} = \frac{K_p \bar{X}_d + K_e \bar{X}_e}{K_p + K_e} \qquad (2.9)$$

The steady state force will be

$$\bar{f} = k_e(\bar{x} - \bar{x}_e) = k_e \bar{x} - k_e \bar{x}_e$$

$$= \frac{k_e(k_p \bar{x}_d + \cancel{k_e \bar{x}_e})}{K_p + K_e} - \frac{k_e \bar{x}_e(\cancel{k_p} + k_e)}{k_p + k_e} = \frac{k_p k_e(\bar{x}_d - \bar{x}_e)}{k_p + k_e}$$

hence
$$\bar{f} = \frac{k_p k_e (\bar{x}_d - \bar{x}_e)}{k_p + k_e} \qquad (2.10)$$

But note that the environment stiffnes k_e is order of magnitudes more than k_p, meaning $k_e \gg k_p$, hence $\bar{f}$ can be approximately found (in steady-state) as:

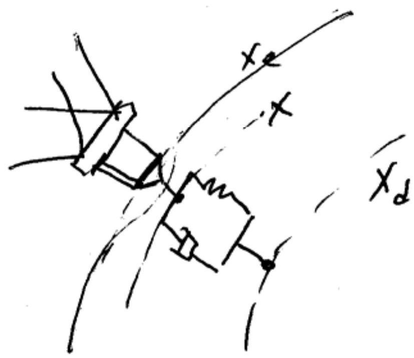
$$\bar{f} \cong \frac{k_p k_e (\bar{x}_d - \bar{x}_e)}{k_e} = k_p(\bar{x}_d - \bar{x}_e) \qquad (2.11)$$

It is interesting that although the interacting stiffnes k_e was very large, by using PD+IDC control the relation between the interacting force $\bar{f}$ (@steadystate) and environment deflection $\bar{\Delta x} = \bar{x}_d - \bar{x}_e$ is controlled with the controller gain K_p and hence can be manipulated as we wish in diffrent applicati

$$\bar{f} = k_p(\bar{x}_d - \bar{x}_e) \qquad (2.11)$$

Note the importance of defining x_d in the above equati. It is sufficient to use a PD+IDC position control + defining x_d a bit inside the environment to result in the manipulator stiffnes being k_p, which can be controlled by the designer wish.

To visulize this concept graphically a virtual spring damper is considred to connect the End Effector to the inside of the contact point in the stiff environment, producing the required force in the manipulator. Note that x never reaches x_d and that's why $x_d(t)$ is considred as the virtual position to control the interacting force component



x_d is considred inside of the Body

B) General Stiffness Control

As before model the environment Stiffness as a linear spring but in 6DOF dimension

$$\mathcal{F}_e = K_e(\underset{\sim}{x} - x_e) \quad (2.12)$$

in which K_e is an $n \times n$ ~~for~~ (6x6 for 6DOF case) diagonal matrix positive definite to represent the translation and angular Stiffness of the environment.

The general Eq. of Motion of a parallel manipulator in Contact with the environment is

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} + G(x) + \mathcal{F}_e = J^T\tau = \mathcal{F} \quad (2.13)$$

use the PD + IDC structure in control

$$\mathcal{F} = J^T\tau = C(x,\dot{x})\dot{x} + G(x) - K_v\dot{x} - K_p\tilde{x} \quad (2.14)$$

as stated before calculation of $C(x,\dot{x})$ terms are very difficult Hence this part can be also excluded from the control law

$$\mathcal{F} = J^T\tau = G(x) - K_v\dot{x} - K_p\tilde{x}$$

$$\text{OR} \quad \tau = J^{-T}\{G(x) - K_v\dot{x} - K_p\tilde{x}\} \quad (2.12)$$

Note again No force measurement is done here.

The closed loop Eq. of motion is

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} = -F_e - K_v\dot{x} - K_p\tilde{x}$$

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} = -K_v\dot{\tilde{x}} - K_p\tilde{x} - K_e(x - x_e) \qquad (2.13)$$

to analyse asymptotic stability, choose the following Lyapunov function candidate

$$V(x) = \frac{1}{2}\dot{x}^T M(x)\dot{x} + \frac{1}{2}\tilde{x}^T K_p \tilde{x} + \frac{1}{2}(x - x_e)^T K_e (x - x_e) \qquad (2.14)$$

determine its time derivative

$$\dot{V}(x) = \frac{1}{2}\dot{x}^T \dot{M}\dot{x} + \dot{x}^T M(x)\ddot{x} + \dot{\tilde{x}}^T K_p \tilde{x} + \dot{x}^T K_e (x - x_e) \qquad (2.15)$$

use $\tilde{x} = x - x_d$ and assume x_d & x_e are constant,

$$\dot{V}(x) = \frac{1}{2}\dot{x}^T \dot{M}\dot{x} + \dot{x}^T M(x)\ddot{x} + \dot{x}^T K_p (x - x_d) + \dot{x}^T K_e (x - x_e)$$

$$= \frac{1}{2}\dot{x}^T \dot{M}\dot{x} + \dot{x}^T\{-C(x,\dot{x})\dot{x} - K_v\dot{x} - \cancel{K_p\tilde{x}} - \cancel{K_e(x - x_e)}\}$$
$$+ \cancel{\dot{x}^T K_p (x - x_d)} + \cancel{\dot{x}^T K_e (x - x_e)}$$

$$= \frac{1}{2}\dot{x}^T\{\dot{M} - C(x,\dot{x})\}\dot{x} - \dot{x}^T K_v \dot{x}$$

= 0 skew-symmetric property

$$\dot{V}(x) = -\dot{x}^T K_v \dot{x} \leq 0 \qquad (2.16)$$

Using Lasalle's Theorem yields to asymptotic convergence of $x \xrightarrow[t\to\infty]{} 0$

this yields to the following Eq, considering Eq (2.13) @ steady-state where $\dot{x} = \ddot{x} = 0$

$$\lim_{t\to\infty} [-K_p \tilde{x} - K_e(x - x_e)] = 0 \rightarrow \lim_{t\to\infty} [-K_p(x-x_d) - K_e(x-x_e)]$$

or Equivalently

$$\lim_{t\to\infty}\{[-K_p - K_e]x + K_p x_d + K_e x_e\} = 0$$

$$\bar{x} = \lim_{t\to\infty} \tilde{x} = [K_p+K_e]^{-1}[K_p\bar{x}_d + K_e\bar{x}_e] \qquad (2.17)$$

again the NDOF force is found from 2.12 $F_e = K_e(x-x_e)$

@ steady-state

$$\bar{F}_e = K_e(\tilde{x} - \bar{x}_e)$$

$$= K_e[K_p+K_e]^{-1}\{K_p\bar{x}_d + K_e\bar{x}_e\} - K_e\bar{x}_e$$

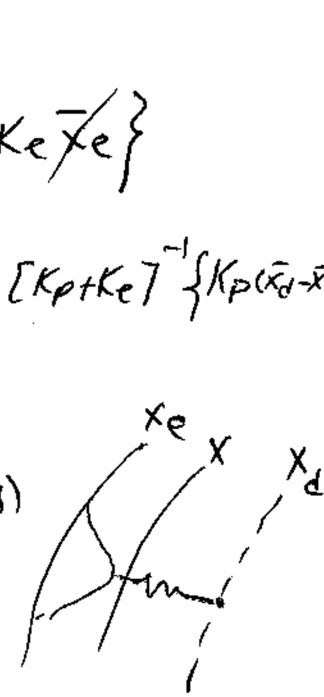
Since $K_e \gg K_p$ in general then

$$\bar{F}_e = [K_p+K_e]^{-1}\{K_eK_p\bar{x}_d + \cancel{K_e^2\bar{x}_e} - [K_p+\cancel{K_e}]K_e\cancel{\bar{x}_e}\}$$

$$= \underbrace{[K_p+K_e]^{-1}}_{\simeq K_e^{-1}}\{\cancel{K_e}K_p\bar{x}_d - K_pK_e\bar{x}_e\} - K_e[K_p+K_e]^{-1}\{K_p(\bar{x}_d-\bar{x}_e)\}$$

$$\cong K_p(\bar{x}_d - \bar{x}_e) \qquad (2.18)$$

$\bar{F}_e \cong K_p(\bar{x}_d - \bar{x}_e)$: Virtual Spring



The same stiffnen property for the control scheme is observed as for 1DOF, but here in a general N-space.

Note that if the manipulator is not experiencing any environment constraint, $K_e = 0$ during the trajectory and substituting this in steady-state representation yields to

$$\lim_{t \to \infty} \underset{\sim}{x} = \underset{\sim}{x}_d \qquad (2.191)$$

As we expected the position control produces tracking performance as required. But as the system comes in contact with the environment, the stiffnen of the manipulator - environment is set to K_p and the forces are calculated from $F_e \simeq K_p(\bar{x}_d - x_e)$

In this case $\underset{\sim}{x}$ never reaches x_d, to preserve the force/position requirement set by stiffness control.

Example:

Dynamic Analysis and Control of SPM : lebret, liu and Lewis
Journal of Robotic Systems 10(5) 629-655 _1993.
First a detail Dynamic analysis is given. Then,
As it is proposed the 6-component forces between the cutter and the environment is considered as F_c, and in eq(57)

$$M(x)\ddot{x} + V_m(x,\dot{x})\dot{x} + G(x) = F - J_n^{-T} F_c \qquad (2.20)$$

$J_n^{-T} F_c$ is used here as F_{ext}, to map the forces at cutting tool into the Center of mass of the moving platform, hence J_n^{-T} is the Jacobian derived from cutting point to C.of.mass of the m.p.

Then only 3 linear Spring is considered for Contact environment (58

$$F_c = K_t \begin{bmatrix} x - x_e \\ \underset{\sim}{0} \end{bmatrix} \qquad (58) \rightarrow (2.21)$$

neglecting the rotational spring that could be considered for the environment, F_{cd} is also generated using a virtual displacement x_d inside the body as depicted in Figure 6.

The Lyapunov function is given and Stability analysis is done for the controller in form of

$$F = J^{-T}\tau = G(x) + J_n^{-T}\tilde{F}_c - K\dot{x} \qquad (66) \rightarrow (2.22)$$

describing this function as what we have before in (2.12) Compare:

$$\mathcal{F} = G(x) + \underbrace{J_n^{-T} K_t (x - x_d)}_{\text{P control}} - \underbrace{K\dot{x}}_{\text{D control}} \tag{2.23}$$

hence the Controller is PD + $G(x)$ compensation as in (2.12), the only difference is the use of J_n^{-T} to interpret the cutting tool coordinate into the M.p. coordinate. the Lyap Candidate is

$$V(x) = \frac{1}{2}\dot{x}^T M \dot{x} + \frac{1}{2}\tilde{F}_c^T K_t^{-1} \tilde{F}_c \qquad 60 \rightarrow (2.24)$$

$$= \frac{1}{2}\dot{x} M \dot{x} + \frac{1}{2}(\tilde{x})^T K_t^T K_t^{-1} K_t (\tilde{x})$$

$$= \frac{1}{2}\dot{x}^T M \dot{x} + \frac{1}{2}\tilde{x}^T K_t^T \tilde{x}$$

$\tilde{F}_c - F_c =$

$K_t(x - x_e) - K_t(x_d - x_e)$

$- K_t(x - x_e)$

$= - K_t(x_d - x_e)$

$$\dot{V}(x) = \frac{1}{2}\dot{x}^T \dot{M}\dot{x} + \dot{x}^T M \ddot{x} + \dot{x}^T K_t^T \tilde{x}$$

$$= \frac{1}{2}\dot{x}^T \dot{M}\dot{x} + \dot{x}^T\left\{-\cancel{V_m}\dot{x} - \cancel{G(x)} + \cancel{G(x)} + J_n^{-T}\tilde{F}_c - K\dot{x} - J_n^{-T} F_c\right\} + \dot{x}^T K_t^T \tilde{x}$$

$$= \underbrace{\frac{1}{2}\dot{x}^T\{\dot{M} - 2V_m\}\dot{x}}_{0 \text{ Skew-Symmetricity}} - \dot{x}^T K \dot{x} \mp \dot{x}^T J_n^{-T} K_t (x_d - x_e) + \dot{x}^T K_t^T (x - x_d)$$

$$= -\dot{x}^T K \dot{x} + \cdots$$ Not as stated in the paper!