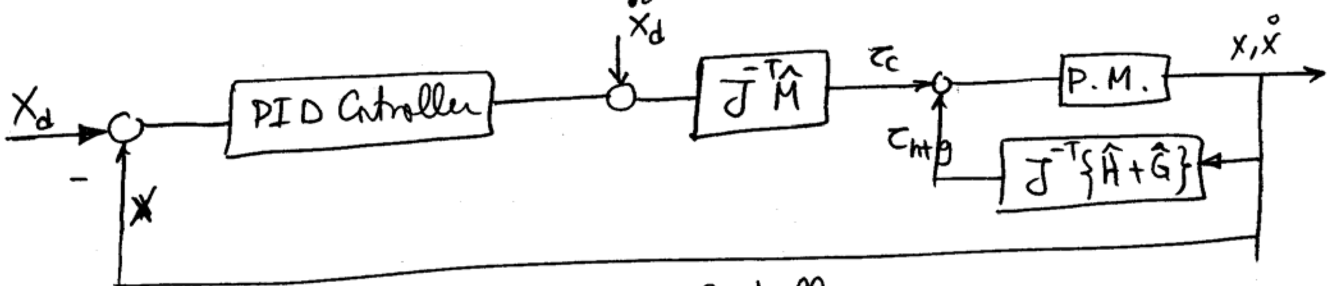
before implementing IDC it is examined that the variation of $H(x,\dot{x})$ on the trajectories is less than 10%, and its contribution to the dynamics (compared to $M\ddot{x}+G$) is less than 1%. hence two topologies of Control is implemented:

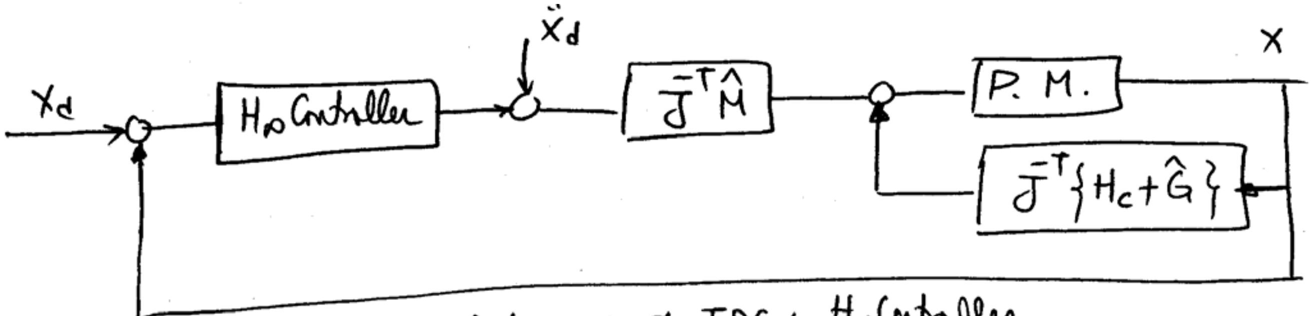
1) Full IDC + a PID Controller
2) Approximate IDC + a Robust H_∞ Controller

in which the topologies are shown in the following figure:



1) Full IDC + PID Controller

In this topology all the varying terms of $\hat{M}, \hat{H}, \hat{G}$ are computed online and implemented into Control



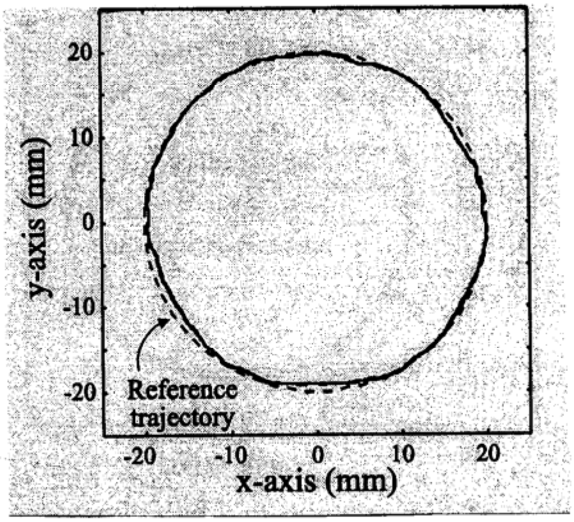
2) Approximate IDC + H_∞ Controller

in which the full information on $H(x,\dot{x})$ is not used and only a fixed H_c is used instead of $H(x,\dot{x})$. This makes the computational effort much less, hence increasing the sampling time in digital implementation.

The experimental results are shown below for two topologies

1) Slow tracking

Minimal Diffrence



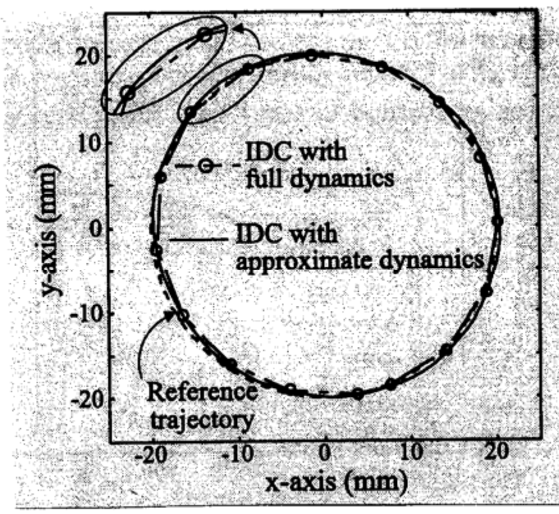
2) Fast tracking

Approximate IDC is better

because of shorter Sampling time

Full IDC $T_S = 400ms$

App. IDC $T_S = 30ms$



3) Rectangular trajectory

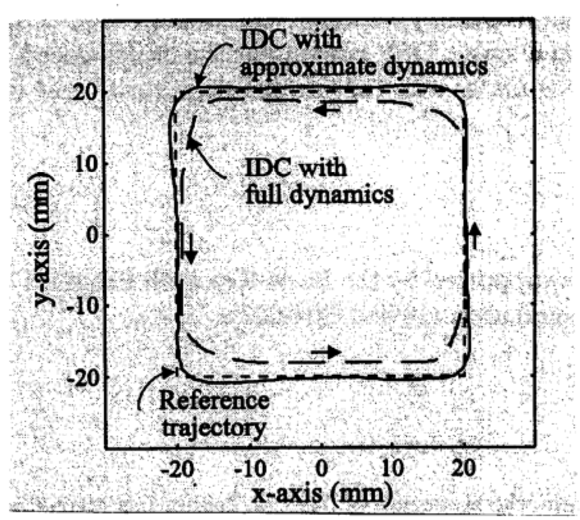
Approximate IDC is better

Again because of slow

Sampling time in Full IDC

fast transients cannot

be expected



6.1.4. Robust IDC

The inverse dynamics approach relies on exact cancellation of nonlinearities in the manipulator equation of motion. The practical implementation of inverse dynamics control requires consideration of varrious sources of uncertainties such as modelling error, unknown loads, and computation errors. Since the Dynamic eq. of Motion of a parallel manipulator is nonlinear, nonlinear robust Controller designs can be advised to preserve the important characteristics of closed-loop System, such as stability and performance in presence of those uncertainties; A general approach in N.R.C is to consider the uncertainties norm bounded and to use a Lyapunov based control law, to preserve robust stability, in what follows such a algorithm is explained in detail:

Start with the Dynamic Eq. of a parallel Manipulator in task space

$$\tau = J^{-T}\{D(x)\ddot{x} + H(x,\dot{x}) + G(x)\}$$

in which $\underset{\sim}{x}$ is considered as the M.P. position/orientation coordinate. multiply J^{-T} and Simplify

$$M(x)\ddot{x} + C(x,\dot{x}) + g(x) = u \qquad (1)$$

in which :

$$M(x) = J^{-T} D(x)$$
$$C(x,\dot{x}) = J^{-T} H(x,\dot{x})$$
$$g(x) = J^{-T} G(x)$$

and $u = C$ as the control input. we use a general IDC routine to design u

$$u = \hat{M}(x) a_q + \hat{C}(x,\dot{x}) + \hat{g}(x) \qquad (2)$$

in which the $\hat{(\cdot)}$ represents the Nominal values of $(\cdot)$ and indicates that exact IDC cannot be achieved in practice due to the uncertainties in the System.

a_q is the linear + robust control part of the controller that must be designed to overcome the uncertainties. Let us denote the error or mismatch as $\tilde{(\cdot)} = \hat{(\cdot)} - (\cdot)$

put (2) into (1) and do some manipulations we reach to

$$\ddot{x} = a_q + \eta(x,\dot{x},a_q) \qquad (3)$$

where $\eta = (M^{-1}\hat{M} - I)a_q + \tilde{C}(x,\dot{x}) + \tilde{g}(x) \qquad (4)$

is called the uncertainty

Let us define $\tilde{M}$ & E as

$$E := M^{-1}\tilde{M} = M^{-1}\hat{M} - I \quad \text{then} \tag{5}$$

$$\eta = Ea + M^{-1}(\tilde{C} + \tilde{g}) \tag{6}$$

Note the general Idea to linearize the System via Feedback and put it into a double integrator is considered in eq. (3) but due to the uncertainties this eq. is still Nonlinear and coupled. Therefore, ∄ guarantee that the outer loop given by $a = \ddot{x}^*$ in usual IDC will satisfy the desired tracking performance and even robust stability. Several approaches are advised by researcher to design a robust controller "a" for the System here we follow a Lyapunov design method: Consider $a = \ddot{x}^* + \delta_a$ to preserve robustness:

$$a = \ddot{x}_d(t) - K_0\tilde{x} - K_1\dot{\tilde{x}} + \delta_a \tag{7}$$

where the additional robust term δa must be designed in terms of tracking error

$$e = \begin{bmatrix} \tilde{x} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} x - x_d \\ \dot{x} - \dot{x}_d \end{bmatrix} \tag{8}$$

we may write Equations (3) & (7) in form of a state space representation

$$\mathring{e} = Ae + B\{\eta + \delta_a\} \qquad (9)$$

$$\dot{\underset{\sim}{e}} = \begin{bmatrix} 0 & I \\ -K_0 & -K_1 \end{bmatrix} \underset{\sim}{e} + \begin{bmatrix} 0 \\ I \end{bmatrix} \{\eta + \delta_a\}$$

Thus the double integrator is first stabilized by linear PD control term $-K_0\tilde{x} - K_1\mathring{\tilde{x}}$, and the additional control term δ_a should be designed to overcome the potentially destabilizing effect of the uncertainty η by δ_a

As mentioned before, assume the uncertainty is norm-bounded

$$\|\eta\| < \rho(e,t) \qquad (10)$$

we design δ_a to gurantee ultimate boundedness of the error trajectory $e(t)$ in equation (9). Note that in general ρ is a function of the tracking error e and time.

Substitute a from eq. (7) into definition of η in eq(6)

$$\begin{aligned} \eta &= Ea + \bar{M}^{-1}(\tilde{C} + \tilde{g}) \\ &= E\delta_a + E[\ddot{x}_d - K_0\tilde{x} - K_1\mathring{\tilde{x}}] + \bar{M}^{-1}(\tilde{C} + \tilde{g}) \end{aligned} \qquad (11)$$

Let us assume that we can find $\alpha < 1$ & γ_1, γ_2, possibly time varying γ_3 $\ni$

$$\|\eta\| \leq \alpha\|\delta_a\| + \gamma_1\|e\| + \gamma_2\|e\|^2 + \gamma_3 \qquad (12)$$

Note: the condition $\alpha < 1$ determines how close our estimate $\hat{M}$ must be to the true Matrix M, since

$$\alpha := \| E \| = \| M^{-1} \hat{M} - I \| \text{ must be } < 1$$

This is always possible to satisfy by choosing $\hat{M}$ as following:

Suppose M^{-1} satisfies $\quad \underline{M} \leqslant \| M^{-1} \| \leqslant \overline{M} \qquad (13)$

If we choose

$$\boxed{\hat{M} = \frac{2}{\overline{M} + \underline{M}} \cdot I} \qquad (14)$$

Then it can be shown that

$$\| M^{-1} \hat{M} - I \| \leqslant \frac{\overline{M} - \underline{M}}{\overline{M} + \underline{M}} < 1$$

Next assume that $\| \delta_a \| \leqslant \rho(e,t)$.

this must be checked a posteriori, after the design of the robust Controller

It follows that (from (12))

$$\| \eta \| \leqslant \alpha \rho(e,t) + \gamma_1 \| e \| + \gamma_2 \| e \|^2 + \gamma_3 =: \rho(e,t) \qquad (15)$$

which since $\alpha < 1$ defines ρ as

$$\rho(e,t) = \frac{1}{1-\alpha} \left(\gamma_1 \| e \| + \gamma_2 \| e \|^2 + \gamma_3 \right) \qquad (16)$$

Now that $\rho(e,t)$ has been determined from the norm bounds of errors and the above parameters, δ_a can be designed based on $\rho(e,t)$.

But K_0 & K_1 must be chosen first, that the Matrix A in Eq. (9) is Hurwitz (strictly stable). If this is the case then from Lyapunov Equation for Stable Systems we may choose $Q > 0$ and find $P > 0$ be the unique symmetric pos. def. matrix satisfying Lyapunov Eq. for System (9)

$$A^T P + PA = -Q \qquad (17)$$

Defining Control δ_a according to

$$\delta_a = \begin{cases} -\rho(e,t)\dfrac{B^T P e}{\|B^T P e\|} & \text{if } \|B^T P e\| \neq 0 \\ 0 & \text{if } \|B^T P e\| = 0 \end{cases} \qquad (18)$$

By this choice it follows that the Lyapunov function

$$V = e^T P e \qquad (19)$$

Satisfies $\dot{V} < 0$ along solution trajectory of Eq. (9)

proof: $\dot{V} = -e^T Q e + 2e^T P B \{\delta_a + \eta\}$

Set $w = B^T P e$ and consider $w^T\{\delta_a + \eta\}$ in the above expression

If $w = 0$ this terms vanishes and $\dot{V} < 0$

for $w \neq 0$ we have $\delta_a = -\rho \dfrac{w}{\|w\|}$ and hence the Cauchy-Schwartz inequality

$$w^T\left(-\rho\frac{w}{\|w\|} + \eta\right) \leq -\rho\|w\| + \|w\|\cdot\|\eta\|$$
$$= \|w\|\left(-\rho + \|\eta\|\right) \leq 0$$

Since $\|\eta\| \leq \rho$ then

$$\dot{V} \leq -e^T Q e < 0$$

→ Note that $\|\delta\| < \rho$ as required.

Note that although the controller defined in (18) is ultimately bounding the error but since it is discontinuous in physical implementation it produces unwanted oscillations in the output which is called chattering; to avoid chattering the Controller is continuously approximated By

$$\delta = \begin{cases} -\rho(e,t)\dfrac{B^T P e}{\|B^T P e\|} & \text{if } \|B^T P e\| > \epsilon \\ -\rho(e,t)\, B^T P e & \text{if } \|B^T P e\| \leq \epsilon \end{cases} \qquad (20)$$

By this change the control effort remains continuous, and the following result can be proven

Theorem: All trajectories of the System (19) are U.U.B (uniformly ultimately bounded) using the continuous control law (20).

proof in Spong 2005 page 304

6.1.5 Adaptive Inverse Dynamics

The first adaptive algorithm used for robot manipulators, were based on inverse dynamics control. Consider again the Robot manipulator dynamics given as

$$M(x)\ddot{x} + C(x,\dot{x}) + g(x) = u \qquad (1)$$

and the Control is given by equation (2)

$$u = \hat{M}(x)\,a + \hat{C}(x,\dot{x}) + g(x) \qquad (2)$$

But Suppose that the parameters appearing in Eq (2) is not fixed, and are time-varying estimates of the true parameters. Substitute (2) in (1) and Set

$$a = \ddot{x}_d - K_1(\dot{x} - \dot{x}_d) - K_0(x - x_d) \qquad (3)$$

it can be shown that using the linear parametrization property

$$\ddot{\tilde{x}} + K_1\dot{\tilde{x}} + K_0\tilde{x} = \hat{M}^{-1}Y(x,\dot{x},\ddot{x})\tilde{\theta} \qquad (4)$$

where Y is the regressor function and $\tilde{\theta} = \hat{\theta} - \theta$, where $\hat{\theta}$ is the estimate of the parameter vector θ. In State space we can write the System (4) as following

$$\mathring{e} = Ae + B\phi\tilde{\theta} \qquad (5)$$

where

$$A = \begin{bmatrix} 0 & I \\ -K_0 & -K_1 \end{bmatrix}; \; B = \begin{bmatrix} 0 \\ I \end{bmatrix}, \; \phi = \hat{M}^{-1} Y(x, \dot{x}, \ddot{x}) \qquad (6)$$

with K_0 & K_1 choses as before as diagonal matrices of positive gains $\Rightarrow$ matrix A becomes Hurwitz. Let P be the unique Solution to the System Lyapunov Equation:

$$A^TP + PA = -Q \qquad (7)$$

and choose the parameter update law as

$$\mathring{\hat{\theta}} = -\Gamma^{-1}\phi^T B^T Pe \qquad (8)$$

where Γ is a constant, symmetric, pos. def matrix. Then, global Convergence to zero of the tracking error with all internal signals remaining bounded can be shown using the Lyapunov function:

$$V = e^TPe + \frac{1}{2}\tilde{\theta}^T\Gamma\tilde{\theta} \qquad (9)$$

it can be shown that

$$\mathring{V} = -e^TQe + \tilde{\theta}^T\{\phi^T B^T Pe + \Gamma\mathring{\hat{\theta}}\} \qquad (10)$$

the latter term following since θ is constant $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$. Using the parameter update law we have

$$\mathring{V} = -e^TQe \qquad (11)$$

From this it follows that the position tracking errors converge to zero asymptotically and the parameter estimation errors remain bounded.

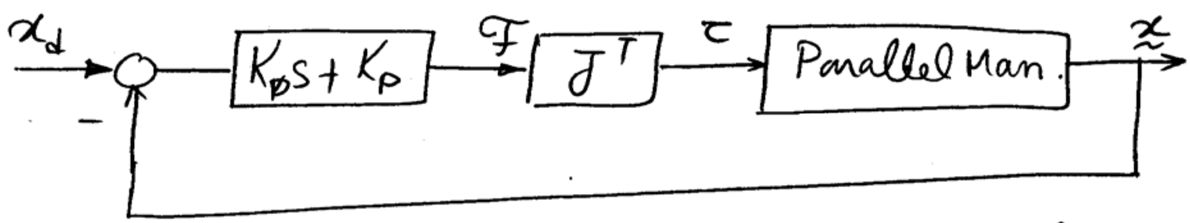
In order to implement such adaptive law, one notes that the acceleration $\ddot{x}$ is needed in the dynamics scheme, however, this is a serious challenge for implementation. Also $\hat{M}$ must be invertible, where in time varying parameters, $\hat{M}$ may approach to singularity. this can be avoided by resetting the parameter estimates whenever $\hat{\theta}$ leads into a singular $\hat{M}$.

These two drawbacks, minimizes the extensive use of adaptive mechanisms in the present form in practice. On a newer Algorithms, passivity of Nonlinear System is used as the basis of the Controller design, by this means the above mentioned drawbacks are removed. a short review of these methods are given below in next section.

The passivity approaches needs skew-symmetricity of $\dot{M}-2C$ in general which is shown to be satisfied for parallel manipulators.

6.1.6 PD Control in task space

Before elaborating passivity control algorithms, revisit the simples control advised for parallel manipulators. Consider a PD Control but in task space:



Note that two differences are here from the previous form, one the feedback is x in task space & the controller output is $\mathcal{F}$ again in task space. in industrial applications even simpler version is used in which

$$u = -K_P \tilde{x} - K_D \dot{x}$$

there $\qquad \tilde{x} = x - x_d$

and the derivative of the desired trajectory is not used in the feedback, for the purpose of ease of tuning. K_P & K_D are diagonal matrices of positive gains.

It is shown that by feedback linearizing the Gravity term $G(x)$ and using so simple Controller, asymptotic tracking for the manipulator is achieved

The manipulator Dynamics

$$\begin{cases} M(x)\ddot{x} + C(x,\dot{x})\dot{x} + G(x) = F \\ F = G(x) - K_P\tilde{x} - K_D\dot{x} \end{cases}$$

$$\Longrightarrow \quad M(x)\ddot{x} + C(x,\dot{x})\dot{x} + K_P\tilde{x} + K_D\dot{x} = 0$$

At this point Consider a Lyapunov function Candidate

$$V(x) = \frac{1}{2}\dot{x}^T M(x)\dot{x} + \frac{1}{2}\tilde{x}^T K_P \tilde{x}$$

the kinetic energy + the proportional term, which is a positive definite fuction. take the derivative

$$\dot{V}(x) = \underbrace{\dot{x}^T M(x)\ddot{x}} + \frac{1}{2}\dot{x}^T\dot{M}(x)\dot{x} + \dot{x}^T K_P\tilde{x}$$

put the System dynamics in $M(x)\ddot{x}$, we have

$$\dot{V}(x) = \dot{x}^T\left(-C(x,\dot{x})\dot{x} - K_P\tilde{x} - K_D\dot{x}\right) + \frac{1}{2}\dot{x}^T\left(\dot{M}(x)\right)\dot{x} + \dot{x}^T K_P\tilde{x}$$

$$= \dot{x}^T\left(-\cancel{K_P\tilde{x}} - K_D\dot{x} + \cancel{K_P\tilde{x}}\right) + \frac{1}{2}\dot{x}^T\left(\dot{M}(x) - 2C(x,\dot{x})\right)\dot{x}$$

Skew symmetry

because of the skew-symmetry property of the System

$$\dot{V}(x) = -\dot{x}^T K_D\dot{x} \leq 0$$

Using Lasalle's theorem, this leads to an asymptotic stability leading to tracking performance without steady state error.

6.1.7 Passivity-based Motion Control

The methods based on passivity do not rely on cancelling the N.L. terms, and hence do not lead to a linear closed-loop system even in the exact case of no uncertainty, However, they have other advantages such as G.A. stability, with less Model-dependent Control law.

Consider the general Dynamics Equation as:

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} + G(x) = u \qquad (I)$$

and choose u according to

$$\text{in which} \begin{cases} u = M(x)a + C(x,\dot{x})v + G(x) - Kr \\ v = \dot{x}_d - \Lambda\tilde{x} \\ a = \dot{v} = \ddot{x}_d - \Lambda\dot{\tilde{x}} \\ r = \dot{x} - v = \dot{\tilde{x}} - \Lambda\tilde{x} \end{cases} \qquad (2)$$

where K & Λ are diagonal matrices of constant, positive gains. Substitute the control law (2) into (I) and form the closed loop equation of motion leads to

$$M(x)\dot{r} + C(x,\dot{x})r + Kr = 0 \qquad (3)$$

Note that in contrast to IDC, the closed loop system is still a coupled N.L. System. Stability and asymptotic convergence of the tracking error to zero, therefore, are detailed by an Lyapunov based analysis.

Consider the Lyapunov function candidate

$$V = \frac{1}{2} r^T M(x) r + \tilde{x}^T \Lambda K \tilde{x} \qquad (4)$$

Calculating $\dot{V}$ yields

$$\dot{V} = r^T M \dot{r} + \frac{1}{2} r^T \dot{M} r + 2 \tilde{x}^T \Lambda K \dot{\tilde{x}} \qquad (5)$$

$$= - r^T K r + 2 \tilde{x}^T \Lambda K \dot{\tilde{x}} + \underbrace{\frac{1}{2} r^T (\dot{M} - 2C) r}_{0 \text{ Skew-symmetricity}}$$

$$= -(\dot{\tilde{x}} - \Lambda \tilde{x})^T K (\dot{\tilde{x}} - \Lambda \tilde{x}) + 2 \tilde{x}^T \Lambda K \dot{\tilde{x}}$$

$$= -\dot{\tilde{x}}^T K \dot{\tilde{x}} - \tilde{x}^T \Lambda^T K \Lambda \tilde{x} - \cancel{2 \tilde{x}^T \Lambda K \dot{\tilde{x}}} + \cancel{2 \tilde{x}^T \Lambda K \dot{\tilde{x}}}$$

$$= -\dot{\tilde{x}} K \dot{\tilde{x}} - \tilde{x}^T \Lambda^T K \Lambda \tilde{x}$$

$$= -e^T Q e < 0 \qquad (6)$$

where $e = \begin{bmatrix} \tilde{x} \\ \dot{\tilde{x}} \end{bmatrix}$ & $Q = \begin{bmatrix} \Lambda^T K \Lambda & 0 \\ 0 & K \end{bmatrix} \qquad (7)$

hence the equilibrium $e=0$ in error space is globally asymptotically stable. up to here the passivity approach does not give any tangible advantage to IDC, and is harder to analyze. but in Robust passivity-based approach the assumption $\|E\| = \|M^{-1}\hat{M} - I\| < 1$ can be eliminated, and in adaptive version the need of acceleration measurement and boundedness of $\hat{M}$ can be eliminated, hence making it very attractive in practice.

6.1.8 Passivity based Robust Control

In this section we use the passivity-based controller designed in previous section and combine it to robust IDC, and achieve to a robust controller synthesis, which is much easier in design & implementation.

First Modify the control (2) to

$$u = \hat{M}(x)a + \hat{C}(x,\dot{x})v + \hat{G}(x) - Kr \qquad (8)$$

where K, Λ, v, a, r are given as before. In terms of linear parametrization of the robot dynamics, (8) becomes:

$$u = Y(x,\dot{x},a,v)\,\hat{\theta} - Kr \qquad (9)$$

put it into System dynamics equation (I) leads to

$$M(x)\,\mathring{r} + C(x,\dot{x})r + Kr = Y(\hat{\theta}-\theta) \qquad (10)$$

We now choose $\hat{\theta}$ in equation (9) as

$$\hat{\theta} = \theta_0 + \delta\theta \qquad (11)$$

where θ_0 is a fixed nominal parameter vector and $\delta\theta$ is an additional Control term. The System (10) then becomes:

$$M(x)\,\mathring{r} + C(x,\dot{x})r + Kr = Y(x,\dot{x},a,v)(\tilde{\theta}+\delta\theta) \qquad (12)$$

where $\tilde{\theta} = \theta_0 - \theta$ is a constant vector and represent the parametric uncertainty in the system. If the uncertainty can be bounded by finding a nonnegative constant $\rho \geq 0 \ni$

$$\|\tilde{\theta}\| = \|\theta - \theta_0\| \leq \rho \qquad (13)$$

then the additional term $\delta\theta$ can be designed according to

$$\delta\theta = \begin{cases} -\rho \dfrac{Y^T r}{\|Y^T r\|} & \text{if } \|Y^T r\| > \epsilon \\ -\dfrac{\rho}{\epsilon} Y^T r & \text{if } \|Y^T r\| \leq \epsilon \end{cases} \qquad (14)$$

using the same Lyapunov Candidate as before

$$V(x) = \frac{1}{2} r^T M(x) r + \tilde{x}^T \Lambda K \tilde{x} \qquad (15)$$

leads to a negative definite $\dot{V}(x)$ in form of

$$\dot{V}(x) = -e^T Q e + r^T Y(\tilde{\theta} + \delta\theta) \qquad (16)$$

Comparing this approach to robust IDC, it is clear that finding a constant bound ρ for the constant vector $\tilde{\theta}$ is much simpler than finding a time-varying bound η. The bound ρ in this case depends only on the inertia parametric uncertainty, while $\rho(x,t)$ in previous approach depends on the manipulator state vector and the reference trajectory.

Finally the assumptions required on the estimate of $\hat{M}(q)$ is released here.

6.1.9 Passivity-based adaptive Control

In adaptive approach we use the same control structure as before:

$$u = M(x)a + C(x,\dot{x})v + G(x) - Kr$$

in which
$$u = Y(x,\dot{x},a,v)\hat{\theta} - Kr \qquad (17)$$

the vector $\hat{\theta}$ in this equation is taken to be the time-varying estimate of the true parameter vector θ. Combing the control law into Eq. of Motion (I) yields to

$$M(x)\dot{r} + C(x,\dot{x})r + Kr = Y\tilde{\theta} \qquad (18)$$

The parameter estimate $\hat{\theta}$ may be computed using standard method of adaptive control such as gradiant or least squares. For example use gradiant update law

$$\dot{\hat{\theta}} = -T^{-1}Y^T(x,\dot{x},a,v)r \qquad (19)$$

together with the Lyapunov function

$$\vartheta = \frac{1}{2}r^T M(x) r + \tilde{x}^T \Lambda K \tilde{x} + \frac{1}{2}\tilde{\theta}^T T \theta \quad (20)$$

this results into global convergence of the tracking error to zero and boundedness of the parameter estimates.

The difference between adaptive approach & robust approach is that the state vector in here includes $\tilde{\theta}$ by state-space representation given in (19), hence $\frac{1}{2}\tilde{\theta}^T T\theta$ is also included in Lyap. funct.

Now Compute $\mathring{V}$ along the System trajectory

$$\mathring{V} = -\tilde{x}^T \Lambda^T K \Lambda \tilde{x} - \mathring{\tilde{x}}^T K \mathring{\tilde{x}} + \tilde{\theta}^T \{ \Gamma \mathring{\hat{\theta}} + Y^T r \} \quad (21)$$

Substituting the expression for $\mathring{\hat{\theta}}$ from the gradient update law (19) yields to

$$\mathring{V} = -\tilde{x}^T \Lambda^T K \Lambda \tilde{x} - \mathring{\tilde{x}}^T K \mathring{\tilde{x}} = -e^T Q e \leqslant 0$$

showing that the closed loop system is stable in Lyap. Sence.
Notice that $\mathring{V}$ is neg. semidefinite, and this doesn't guarantee asymptotic convergence of estimated parameters or tracking error to zero; But it can be shown that $e(t)$ is square integrable and under some mild additional assumptions must tend to zero as $t \to \infty$.

Hence in practice although the estimated parameters may not yield to their exact values but the tracking error approaches to zero without any problem.