MECH 577 Optimum Design

Project # 3: Inequality-Constrained Optimization–The Optimum Design of a Mechanism-Housing for a Pitch-Roll Wrist

Assigned: October 28, 2004 Due: November 30, 2004

A pitch-roll wrist (PRW) is a robotic device intended to produce two-degree-of-freedom rotations of a robot gripper about a fixed point, the wrist centre. Conventional means of producing such motions rely on a bevel-gear differential train, similar to those found in automotive driving-wheel axles. Moreover, these trains in robotic wrists invariably bear straight-tooth bevel gears, which are the source of noise and significant power losses. In an attempt to overcome these drawbacks, a PRW is being designed at the Robotic Mechanical Systems Laboratory, as depicted in Fig. 1, with gears replaced by cam-roller pairs.

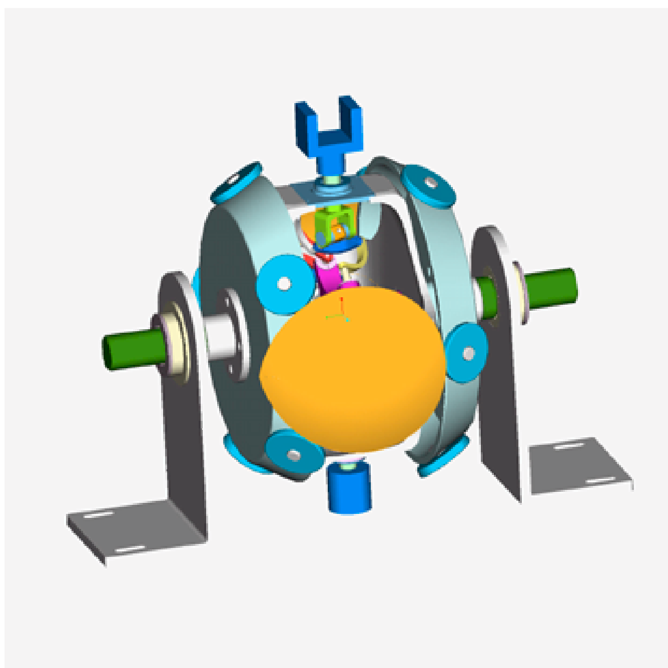

Figure 1: An innovative PRW with bevel gears replaced by cam-roller pairs

The key component of the mechanism is an array of spherical Stephenson linkages used to transmit the power from the two independent cam rotations to the gripper, as illustrated in Fig. 2. The two cams are driven by the motion of the two roller-carrying disks of Fig. 1, which are rigidly coupled to their respective motors. When the two face-to-face motors turn at opposite angular velocities of identical absolute values, the whole array turns about the common axis of the two cams, as a single rigid body (pitch); when the two motors turn at identical angular velocities, the plane Π containing the four spherical-linkage centres remains stationary, but the gripper turns about its axis (roll.) The array must be supported by a housing that doubles as a protection means to isolate the spherical linkages from the environment dust and dirt.

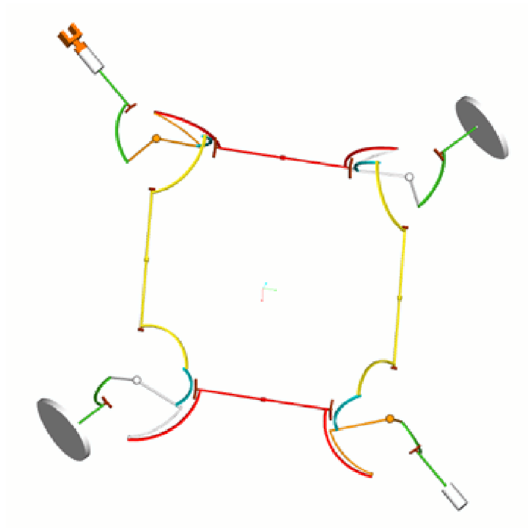

Figure 2: The four Stephenson linkages of the PWR

A housing has been proposed, as shown in Fig. 3, that is based on Lamé curves (Gardiner, 1965). These curves are given by the implicit function

$$\left|\frac{x}{a}\right|^p + \left|\frac{y}{b}\right|^p = 1$$

where p is an integer and a and b are real numbers that determine the dimensions, $2a \times 2b$, of the box circumscribing the curve. Notice that, for $a = b$ and $p = 1$, the corresponding curve is a square centred at the origin of the x-y plane and rotated 45° with respect to the coordinate axes. For $p = 2$, the curve is an ellipse with semiaxes of lengths a and b. For $p \to \infty$ and $a = b$, the curve is a square of side $2a$ centred at the origin of the x-y plane.

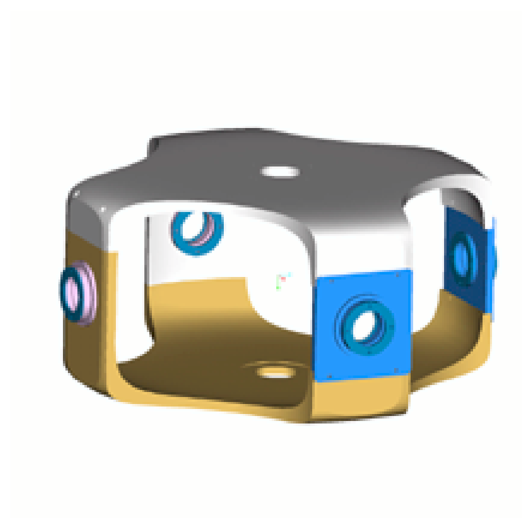

Figure 3: The Stephenson-linkage housing made up of two identical covers

A cross section of the proposed housing, illustrating the geometry of the Lamé curves, is displayed in Fig. 4, the assembly of the two identical halves of the housing being displayed in Fig. 3. Notice that, for assemblability, two bearing housings, not integral parts of the two halves, are mounted between the halves. Furthermore, the dimensions of the Stephenson-linkage array dictate that the housing be contained in a cylinder of a 200 mm radius and a height of 200 mm.

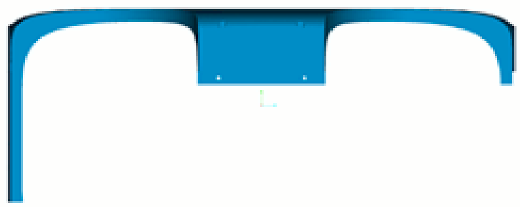

Figure 4: A cross-section of the proposed housing of the Stephenson-linkage array based on Lamé curves

While the Lamé curves provide a thicker cross section at the points of maximum curvature of the curves, a plus for the curves, the additional material, placed at a distant location from the cam axis, adds significantly to the moment of inertia of the whole device. Moreover, the blending of the Lamé curves with the straight parts housing the bearings occurs at points on the curves of low, but non-zero curvature values. The outcome is that a jump curvature discontinuity at the blending points arises, which brings about high stress concentrations, and hence, a potential for failure.

The purpose of the project is thus to replace the current housing shape by an alternative shape, to be determined so as to eliminate the drawbacks of the Lamé curves. The project objective is, therefore,

> *To produce a cross section of the housing with both uniform thickness and zero curvature at the blending points with the straight bearing housings.*

While the new shape can be determined in multiple ways, it is highly advisable that you use non-parametric cubic splines (Angeles and López-Cajún, 1991). To help you with the project, a C-code package, `CURSYN`, has been developed and posted on the course website. This code resorts to another C package, `ODA`, for the search of the optimum shape.

Notice that your optimum solution must include a provision to avoid non-convex shapes, which would a) add to the fabrication costs and b) lead to high stresses due to the curvature sign changes. A convex shape can be implemented by means of an inequality constraint preventing negative curvature values of the mid-curve of the housing.

Time permitting, you are encouraged to produce von Mises-stress distributions of both the current design, made of an inner Lamé curve with parameter $p = 4$ and an outer

Lamé curve with parameter $p = 6$, and then compare this stress distribution with that of your proposed design. Comment on the issue of stress concentrations. The loading to be assumed is one of a couple produced by two 1000 N forces acting at the centres of the bearing housings of Fig. 3 and normal to the common cam axis. Assume that the material of the housing is aluminium AL6061.

The procedure to adopt here for the finding of the optimum shape should parallel that reported in (Bidaud, Teng and Angeles, 2001).

Reference:

Angeles, J. and López-Cajún, C.S., 1991, *Optimization of Cam Mechanisms*, Kluwer Academic Publishers B. V., Dordrecht.

Bidault, F., Teng, C. P. and Angeles, J., 2001, "Structural optimization of a spherical parallel manipulator using a two-level approach," *Proc. ASME 2001 Design Engineering Technical Conferences*, Pittsburgh, PA, Sept. 9-12, CD-ROM DETC 2001/DAC-21030.

Gardiner, M., 1965, "The superellipse: a curve that lies between the ellipse and the rectangle", *Sci. Am.*, Vol. 21, pp. 222–234.

Hernandez, S., 2004, *The Optimum Design of Epicyclic Trains of Spherical Cam-Roller Pairs*, M.Eng. Thesis, McGill University, Montreal.