MECH 593 Design Theory and Methodology

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A Method to Specify Tolerances Using Taguchi's Robust Engineering

Taguchi (1993) proposes a model for *productivity loss* L_p:

$$L_p = L_q + C \tag{1}$$

where

L_q: *Quality loss*

C: *Production cost*

L_p: *Productivity loss*

Moreover, the *productivity index* I_p is defined as

$$I_p = \frac{1}{L_p} \tag{2}$$

As a means to increase productivity, Taguchi recommends to minimize the productivity loss by means of properly specifying tolerances. According to Taguchi,

> *Variations in the objective functions of products (or technologies) are primarily due to three sources: environmental effects, deteriorative effects, and manufacturing imperfections. The purpose of robust design is to make the products and the processes less* [rather, least] *sensitive to these effects.*

The above quoatation means that sources of *noise* are of two kinds, *internal* and *external*. Internal are (*i*) congenital—manufacturing imperfections—and (*ii*) aging—deteriorative effects. External are due to the *environment*.

Taguchi's methodology is based on a *black box*—as opposed to a model—to which the designer can provide inputs and obtain (measurable) outputs. The black box is usually a *physical prototype*—methodology was proposed in the fifties!—either under redesign or built from scratch with the purpose of conducting experiments. In fact, Taguchi's methodology is based on what is known as *design of experiments*, within the framework of the *scientific method*. In this vein, the designer has at her or his disposal a "black box" to conduct experiments, but the designer can actually open this box and change its parameters at will, within certain ranges.

The results of the experiments are then recorded in tables termed "orthogonal arrays" in Taguchi's terminology. Each entry of the array records values of inputs and outputs.

"The L50 point" is frequently used. This "point" is defined as: "the point" at which the probability of failure—or falling outside tolerance specifications—is 50%.

One more concept used by this methodology: The economical safety factor ϕ, defined as

$$\phi = \sqrt{\frac{A_0}{A}} \tag{3}$$

where

A_0: Average financial loss when objective characteristic of products exceeds functional limits

A: Average financial loss when objective characteristic just exceeds tolerance specifications of production plans

Notice that ϕ is a kind of economic parameter, with financial implications determined by both engineering and economic means, rather than simply an engineering parameter.

Further, let Δ_0 be the functional limit, positive or negative, of an objective characteristic of a product or component. The tolerance specification Δ for this characteristic is then

$$\Delta = \frac{\Delta_0}{\phi} \tag{4}$$

Example

Let the L50 point of an ingredient of a prescription drug that might cause side effects be 8000 ppm (parts per million), which is Δ_0. The economical safety factors and the tolerance specifications of production facilities for this ingredient are calculated below:

First, the average cost of the death of one Japanese is calculated as

$$A_0 = J_{\mathrm{GNP}} \times J_{\mathrm{ave}} \tag{5}$$

where (data from the eighties)

J_{GNP}: annual Japan's GNP, 2×10^6 Yen/year (per individual)

J_{ave}: Japan's average life, 77.5 years

Hence,

$$A_0 = 2 \times 10^6 \times 77.5 = 1.55 \times 10^8 \text{ Yen}$$

If we assume that the cost of the drug is 300 Yen ($A = 300$), then the economical safety factor is

$$\phi = \sqrt{1.55 \times \frac{10^8}{300}} = 719$$

Hence, tolerance specification for drug manufacturers for this ingredient is

$$\Delta = \frac{8000}{719} = 11 \text{ ppm}$$

Reference

Taguchi, G., 1993, *Taguchi on Robust Technology Development. Bringing Quality Engineering Upstream*, ASME Press, New York.