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The Gear Selection Problem for Electric Vehicles: an Optimal Control Formulation

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***Abstract*—Optimal gear selection problem for energy minimization for two-gear electric vehicles is formulated within the framework of optimal control theory. Methods for restricting the number of switchings are analyzed and a computational example is presented.**

I. Introduction

Energy efficiency is an important factor in performance evaluation of vehicles. For Electric Vehicles (EV's), in particular, more efficient operation is equivalent to longer running times on a single charge of the battery. Efficiency improvement can be provided through various kinds of methods including improvement of the motor performance, decreasing transmission losses, etc., but once the vehicle is designed, these performance factors are fixed. However, energy efficiency can still be improved through control strategies. An electric motor (EM) equipped with a certain number of gears is able to provide the power requested by the driver in more than one of its operating points and hence there exists the possibility of optimization of energy consumption through gear shifting.

The energy optimization problem for Hybrid Electric Vehicles (HEV's) has been the subject of numerous studies. In the case of HEV's the importance of power distribution between the internal combustion engine (ICE) and the EM dominates the optimal control problem while the problem of optimal gear selection appears as an implicit or a secondary problem. The control strategy of Barsali, Miulli and Possenti [1] to minimize fuel consumption results in an algorithm for turning the generator on and off. The minimization of energy consumption analyzed by Gong, Li and Peng [2] and the optimization problem considered by Jeon, Jo, Park and Lee [3] result in strategies based on road conditions. The dynamic programming approach by Pérez, Bossio, Moitre and Garcia [4] also focuses on power distribution. In contrast, the work of Sciarretta, Back and Guzzella [5] considers an a priori gear sequence in the optimization problem and the work of Stockar, Marano, Canova and Guzzella [6] and Lee, Sul, Cho and Lee [7] describe rule-based gear selection strategies. The criterion used by Pisu and Rizzoni [8] results in similar strategies because gear optimization is performed to the operation of ICE.

When power distribution is not a part of control problem, e.g. for EV's or fuel vehicles, the literature provides limited information concerning energy optimization. Kock, Welfers and Passenberg [9], Passenberg, Kock and Stursberg [10] and Fu and Bortolinb [11] study the gear shifting problem for fuel operated vehicles together with optimal throttle control and speed scheduling. This type of problem formulation, which replaces the driver's command by the controller's optimal decision, has applications for the operation of off-road vehicles. For passenger vehicles, however, the only optimality decision is determining the gear number. The individual importance of gear selection in the optimal performance of vehicles has been the subject of limited study. For ICE, for example, the study by Blagojević, Vorotović, Ivanović, Janković and Popović[12] results in a rule-based shifting algorithm and for EM's a similar rule-based schedule is derived by Xiong, Xi, Zhang, Jin and Chen [13] for an electric bus.

In this paper, the problem of optimal gear selection is formulated for electric vehicles and the individual importance of gear selection on the energy consumption is demonstrated. The minimum possible energy consumption is subject to several consecutive switchings in short periods of time which is undesirable due to physical limitations and performance efficiency. Hence, restrictions on switching counts are introduced which in general result in the dependance of the gear selection decision to the whole time interval of optimization. This dependance makes the control strategy applicable only to offline control problems. In order to make the optimality condition depend only on short sub-intervals, a gear selection strategy is developed which restricts the switching commands to at most one gear change in every time interval of length Δt. The method is implemented for an EV undergoing UDDS driving cycle and it is demonstrated that the proposed method eliminates numerous unnecessary switchings while the overall energy consumption remains close to the minimum possible value.

II. System Dynamics

Based on Newton's second law of motion, with m being the effective mass of the vehicle and z the coordinate along the road, the car's acceleration $a = dv/dt$ depends on the resultant of the traction force F_{tr}, the aerodynamic force $\frac{1}{2}\rho_a C_d A_f v^2$, the gravitational force along the road $mg\sin\gamma(z)$ and the rolling resistance force $mgC_r\cos\gamma(z)$. Thus the system dynamics is described by

$$
\begin{aligned}
\frac{dz}{dt} &= v \\
\frac{dv}{dt} &= \frac{1}{m}F_{tr} - \frac{1}{2m}\rho_a C_d A_f v^2 - g\sin\gamma(z) - gC_r\cos\gamma(z)
\end{aligned}
\tag{1}
$$

At each time, the driver observes (senses) the current velocity v and decides whether to accelerate, decelerate or maintain the speed based on either environmental (i.e. traffic, road condition, slope, etc.) or personal reasons. The controller has no influence on the driver's decision and it is supposed to provide the car with the traction force (or equivalently the acceleration) demanded by the driver. The only freedom the controller has is the choice of the gear which is to be used in order to minimize the total energy consumption.

In each gear, the motor speed is proportional to the vehicle speed according to

$$\omega = \frac{v}{r_w g(q)} \tag{2}$$

where r_w is the wheel radius and $g(q)$ is the overall (motor-to-wheel) gear ratio at gear number q. The requested acceleration is interpreted for the motor as a torque request given by

$$T = \frac{r_w g(q) F_{tr}}{\eta_{trans}} \tag{3}$$

with η_{trans} being the overall transmission efficiency. The electric power required for providing each speed-torque set is determined by the mechanical power and the efficiency of the motor, computed as

$$
\begin{aligned}
P^e(q, F_{tr}, v) &= T\omega\eta^{-1}(T,\omega) \\
&= \tfrac{1}{\eta_{trans}} F_{tr} v\, \eta^{-1}\left(\tfrac{r_w g(q) F_{tr}}{\eta_{trans}}, \tfrac{v}{r_w g(q)}\right)
\end{aligned}
\tag{4}
$$

where $\eta(T,\omega)$ is the motor efficiency as a known function of T,ω which is a characteristic of the motor.

In order to represent the system in the formal hybrid system definition [14], [15], the hybrid system $\mathbb{H}$ representing the electric vehicle dynamics is defined as

$$\mathbb{H} = \{H := Q\times\mathbb{R}^n, I := \Sigma\times U, \Gamma, A, F, \mathscr{M}\} \tag{5}$$

where $Q := \{1,2\} \equiv \{q_1, q_2\}, |Q| = 2$ is the finite set of *discrete states* representing the gears.

$H := Q\times\mathbb{R}^2$ with $q\in Q$ and $x := (z,v)\in\mathbb{R}^2$ composes the *(hybrid) state space* of the hybrid system $\mathbb{H}$.

$I := \Sigma\times U$ is the set of system input values, where $|\Sigma| < \infty$.

$U\subset\mathbb{R}$ is the set of *admissible values* for F_{tr} where U is an open bounded set in $\mathbb{R}$.

$\mathscr{U}(U) := PC([t_0, T_*), U)$ which is the set of all piece-wise continuous functions that are (i) bounded, (ii) continuous on $[t_0, T_*), T_* < \infty$, except possibly at a countable number of points and (iii) are continuous from the right.

$\Gamma : H\times\Sigma\to H$ is the time independent (partially defined) *discrete state transition map* which is the identity on the second ($\mathbb{R}^2$) component.

$A : Q\times\Sigma\to Q$ denotes both a finite automaton and the automaton's associated transition function, on the state space Q and event set Σ, such that for a discrete state $q\in Q$ only the discrete controlled and uncontrolled transitions into the q-dependant subset $\{A(q,\sigma), \sigma\in\Sigma\}\subset Q$ occur under the projection of Γ on its Q components: $\Gamma : Q\times\mathbb{R}^n\times\Sigma\to H|_Q$. In other words, Γ can only make a discrete state transition in a hybrid state (q,x) if the automaton A can make the corresponding transition in q.

F is the indexed collection of *vector fields* $\{f_q\}_{q\in Q}$. Since the discrete state q does not appear explicitly or implicitly in Eq. (1), $f := f_1 = f_2$ is the only member of F. It can easily be examined that $f\in C^k\left(\mathbb{R}^2\times U\to\mathbb{R}^2\right), k\geq 1$ if $\gamma\in C^k(\mathbb{R}\to\mathbb{R})$. In addition f satisfies a uniform Lipschitz (in $x = (z,v)^T$) condition, i.e. there exists $L_f < \infty$ such that $\|f(x_1,F_{tr}) - f(x_2,F_{tr})\| \leq L_f\|x_1 - x_2\|,\ x_1,x_2\in\mathbb{R}^2,\ F_{tr}\in U$. It can also be shown that there exists $K_f < \infty$ such that $\sup\limits_{F_{tr}\in U}\|f(0,F_{tr})\| \leq K_f$.

$\mathscr{M}$ is the collection of (autonomous) *switching manifolds* which is empty here.

III. The Optimal Control Problem

We are interested in minimizing the total energy consumption only through the selection of gears. The controller does not influence the car maneuver and the input F_{tr} is directly associated to the driver's control input through the acceleration pedal. As stated earlier, $F_{tr}(.)\in\mathscr{U}(U)$ is a piece-wise continuous function which in this paper is assumed to be known a priori independent of the gear switching decision.

The total electrical energy consumed for each maneuver is determined by

$$J\left(S\left(t_0,t_f\right)\right) = \int_{t_0}^{t_f} P^e(q(\tau), F_{tr}(\tau), v(\tau))\,d\tau \tag{6}$$

where $P^e(q, F_{tr}, v)$ calculated from (4) is continuous in its arguments F_{tr} and v for each q and is bounded above and below.

Consider the initial time t_0, final time $t_f < \infty$ and initial state $(q(0), x(0), v(0))$. For simplicity of notation, we assume that $\gamma(z) = 0$, i.e. assuming zero grading road that gives the dynamics for v decoupled from that of z. Let the (discrete-state) control input be defined as

$$
\begin{aligned}
S\left(t_0,t_f\right) &= ((t_0,\sigma_0), (t_1,\sigma_1), \cdots, (t_i,\sigma_i), \cdots) \\
&\equiv ((t_0,q_0), (t_1,q_1), \cdots, (t_i,q_i), \cdots)
\end{aligned}
\tag{7}
$$

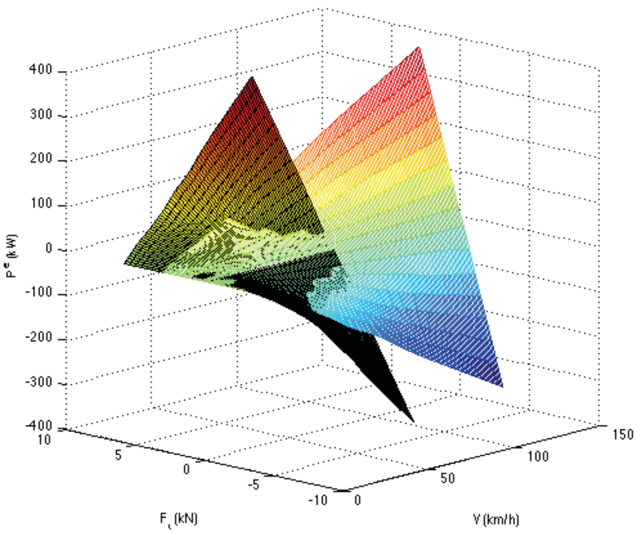

Fig. 1. Direct comparison of electric power P^e for an EV with two gears of relative ratio of 2

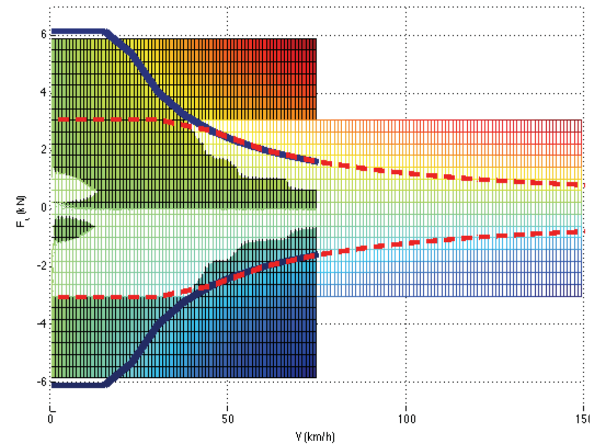

Fig. 2. The bottom view of Fig. 1 determines the optimal operation regions. The dark grid represents the first gear and the light grid represents the second. The grid observed from the bottom view (this figure) corresponds to the optimal gear for its corresponding set of F_{tr} and v. The dark-blue solid envelope is the maximum steady torque vs. speed for the first gear and the red dotted line is the one for the second gear

where the possibly countable sequence $\{t_i\}_0^\infty$ satisfies $t_0 < t_1 < \cdots < t_i < t_{i+1} < \cdots < t_f$. Denoting the set of all inputs in the form of (7) by $\mathbb{S}$, the optimal control problem is to find

$$J^o = \inf_{S(t_0,t_f)\in\mathbb{S}} J\left(S\left(t_0,t_f\right)\right) \tag{8}$$

A. Unrestricted Switching Problem

By assumption $F_{tr}(\tau)$ is piece-wise continuous in $\tau \in \left[t_0,t_f\right]$, $v(\tau)$ is continuous in $\tau \in \left[t_0,t_f\right]$ and for each $q \equiv \sigma$, $P^e(q,F_{tr},v)$ is continuous in its arguments F_{tr} and v. Hence, $P^e(q^o(\tau),F_{tr}(\tau),v(\tau))$ is piece-wise continuous in $\tau \in \left[t_0,t_f\right]$ where

$$q^o(\tau) := \arg\min P^e\left(q(\tau),F_{tr}(\tau),v(\tau)\right), \quad \tau \in \left[t_0,t_f\right] \tag{9}$$

If the cardinality of the control input S is unbounded then the optimal control problem (8) can be reformulated as

$$\begin{aligned} J^o = \inf_{S(t_0,t_f)} \int_{t_0}^{t_f} P^e\left(q(\tau),F_{tr}(\tau),v(\tau)\right) d\tau \\ = \int_{t_0}^{t_f} \inf_q P^e\left(q(\tau),F_{tr}(\tau),v(\tau)\right) d\tau \\ = \int_{t_0}^{t_f} \min_q P^e\left(q(\tau),F_{tr}(\tau),v(\tau)\right) d\tau \\ = \int_{t_0}^{t_f} P^e\left(q^o(\tau),F_{tr}(\tau),v(\tau)\right) d\tau \end{aligned} \tag{10}$$

i.e. the optimal gear selection over the time interval $\left[t_0,t_f\right]$ is equivalent to determining the gear that provides the requested mechanical power-demand by the lowest electric power at each instnat $\tau \in \left[t_0,t_f\right]$.

Thus the control problem can be solved a priori and implemented online with no information required from the past performance and no prediction of the future conditions. For a two-gear electric vehicle this can be done by direct comparison of the values of $P^e(q,F_{tr},v)$ as illustrated in Fig. 1.

At each pair of F_{tr} and v for $q \in \{1,2\}$ the lowest surface corresponds to the optimal gear q^o. Taking into account the permissible steady-state operation regions and their corresponding envelopes, the optimality regions are illustrated in Fig. 2.

To demonstrate the performance of such a control strategy, consider an EV with two gears with a relative gear ratio of 2. As an illustrative driving scenario, assume that F_{tr} is such that the vehicle undergoes the UDDS driving cycle as shown in Fig. 3. The corresponding gear selection strategy is demonstrated in Fig. 4. As can be seen, the gear changes 182 times in a time interval of 23 minutes long and several consecutive gear changes occur. Because of the integral form of the energy consumption, constant gear periods with short durations have negligible effect on the total energy consumption (8) but their corresponding switchings can cause wearing of the gearbox components as well as increased energy loss, contrary to the initial purpose of saving energy.

B. Cardinality Restriction of the Switching Command

There are several ways to redefine the optimal control problem such that the unconstrained number of switchings is prevented. One powerful method is to associate a cost to each time the gear is switched. Theoretical results on the influence of switching cost on the optimal control solutions are presented in [15], [16]. To write down the resulting control problem one can redefine it as

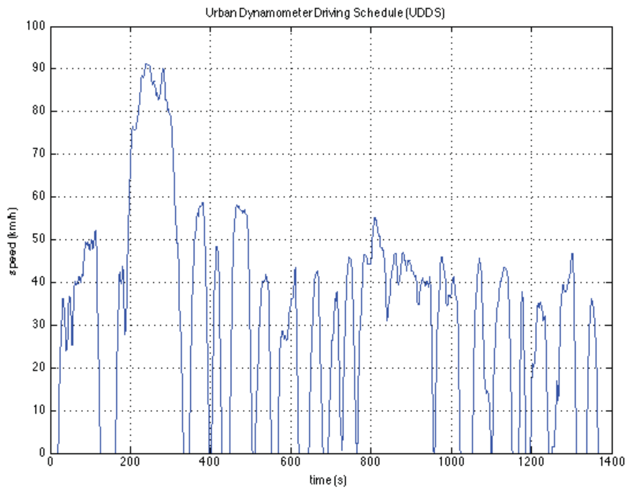

Fig. 3. Urban Dynamometer Driving Schedule (UDDS)

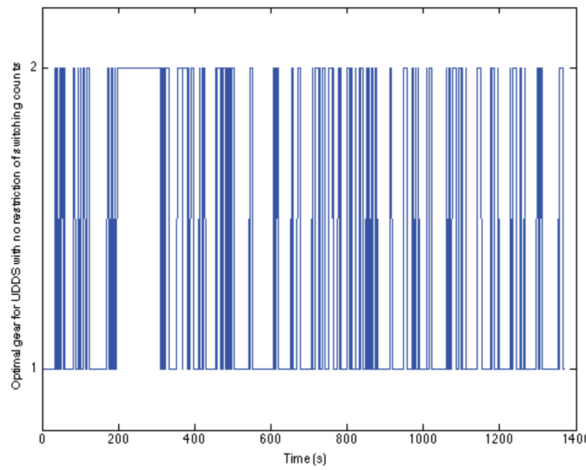

Fig. 4. The optimal gear corresponding to the minimum energy consumption for UDDS when no restriction is imposed on the number of switchings

$$\min_{S} \int_{t_0}^{t_f} P^e\left(q, F_{tr}(\tau), v(\tau)\right) d\tau + \sum_{j=1}^{N} c_\sigma\left(F_{tr}(t_j), v(t_j)\right) \quad (11)$$

If c_σ is taken to be the required electrical energy to perform the gear shifting and compensate for the losses, the above problem has a physical interpretation as the total energy required for the whole maneuver by the car, but in general c_σ can be defined to be any positive valued switching cost function. The utilization of switching costs to limit the number of switchings is beyond the scope of this paper.

Another method is to restrict the class of inputs $\mathbb{S}$ to the class $\mathbb{S}_L$ with cardinality of at most L, i.e.

$$S_k\left(t_0, t_f\right) = \left(\left(t_0, \sigma_0\right), \left(t_1, \sigma_1\right), \cdots, \left(t_k, \sigma_k\right)\right) \in \mathbb{S}_L \quad (12)$$

only if $k \leq L$ and $t_k < t_f$. Note that since $\mathbb{S}_L$ is a proper subset (strict subset) of $\mathbb{S}$ the following inequality holds

$$\begin{aligned} &\min_{S\in\mathbb{S}} \int_{t_0}^{t_f} P^e\left(q, F_{tr}(\tau), v(\tau)\right) d\tau \\ &\leq \min_{S\in\mathbb{S}_L} \int_{t_0}^{t_f} P^e\left(q, F_{tr}(\tau), v(\tau)\right) d\tau \end{aligned} \quad (13)$$

Replacing the minimization in (8) over $S \in \mathbb{S}_L$ instead of $S \in \mathbb{S}$ as well as utilization of switching costs as in (11) results in the loss of property (10). Instead, these optimal control problems require the information of the maneuver in the whole time interval $\left[t_0, t_f\right]$. Since in practice, the gear sequence decision cannot be made and fixed a priori for a long period of time, methods that require the information of shorter periods are of special interest. To develop such methods, consider the value function V for the optimal control problem (8) which is defined to be the optimal cost to go from time $t \in \left[t_0, t_f\right]$ to t_f, i.e.

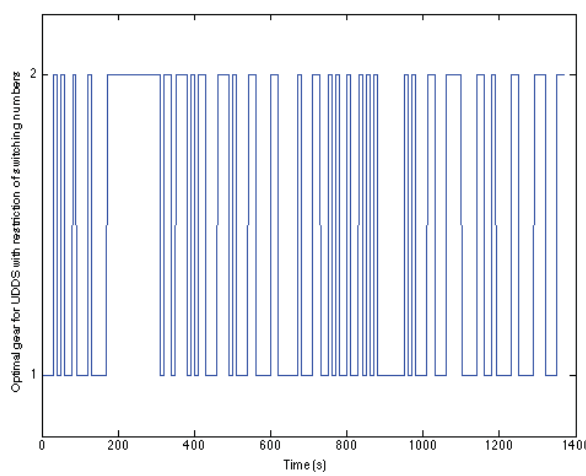

Fig. 5. The optimal gear corresponding to the minimum energy consumption for UDDS with the restriction of at most one switching every 10 seconds

$$V\left(t, q, v(t)\right) = \min_{S\left(t, t_f\right)} \int_{t}^{t_f} P^e\left(q, F_{tr}(\tau), v(\tau)\right) d\tau \quad (14)$$

The control input S is defined as in Eq. (7) but in the interval $\left[t, t_f\right]$ where $t_0 \leq t \leq t_f$. For consistency of the definition, the following compatibility condition must hold

$$\left[\left(t, \sigma_i\right) \in S\left(t_i, t\right)\right] \wedge \left[\left(t, \sigma_j\right) \in S\left(t, t_j\right)\right] \Rightarrow \sigma_i = \sigma_j \quad (15)$$

with $t_i < t < t_j$. In the case where t is not a switching time the element $\left(t, \sigma(t)\right) \equiv \left(t, q(t)\right)$ is defined to be an identity element giving

$$(t,\sigma(t)) = (t,id) \equiv (t,q(t)) = \left(t, \lim_{\tau\to t} q(\tau)\right) \tag{16}$$

Consider an arbitrary time interval $\Delta t > 0$ to write

$$\begin{array}{c} V(t,q,v(t)) \\ = \min_{S(t,t+\Delta t)} \int_t^{t+\Delta t} P^e(q,F_{tr}(\tau),v(\tau))\,d\tau \\ +V(t+\Delta t,q,v(t+\Delta t)) \end{array} \tag{17}$$

Because of the system's dynamics (1) the value of $v(t+\Delta t)$ is the resultant of the maneuver the car is taking and is not influenced by the control input $S(t,t+\Delta t)$. Thus $V(t+\Delta t,q,v(t+\Delta t))$ is independent of $S(t,t+\Delta t)$ if the input $S(t+\Delta t,t_f)$ is independent of $S(t,t+\Delta t)$. Hence, any switching restriction method with this property would only require the information of the maneuver in $[t,t+\Delta t]$ for gear selection in this period.

Although Δt is not required to be uniformly constant in the interval $[t_0,t_f]$, this assumption simplifies the notation and thus from now on Δt is considered as a fixed constant value. Defining $t_n = t_0 + n\Delta t$ the relation (17) becomes

$$\begin{array}{c} V(t_n,q,v(t_n)) \\ = \min_{S(t_n,t_{n+1})} \int_{t_n}^{t_{n+1}} P^e(q,F_{tr}(\tau),v(\tau))\,d\tau \\ +V(t_{n+1},q,v(t_{n+1})) \end{array} \tag{18}$$

and the corresponding optimal gear selection problem becomes

$$\begin{array}{c} \min_S \int_{t_0}^{t_f} P^e(q,F_{tr}(\tau),v(\tau))\,d\tau \\ = \min_S \sum_{n=0}^{N} \int_{t_n}^{t_{n+1}} P^e(q,F_{tr}(\tau),v(\tau))\,d\tau \end{array} \tag{19}$$

with $t_{N+1} = t_f$, $S_n := S(t_n,t_{n+1})$ and $S = \bigcup_{n=0}^{N} S_n$.

As mentioned earlier, if S_n's are independent, then

$$\begin{array}{c} \min_S \sum_{n=0}^{N} \int_{t_n}^{t_{n+1}} P^e(q,F_{tr}(\tau),v(\tau))\,d\tau \\ = \sum_{n=0}^{N} \min_{S_n} \int_{t_n}^{t_{n+1}} P^e(q,F_{tr}(\tau),v(\tau))\,d\tau \end{array} \tag{20}$$

If for example S_n is taken to be $S_n = (t_n,\sigma_n) \equiv (t_n,q_n)$ with σ_n extended by the identity member id, then the property (20) holds and the optimal gear selection problem can be solved independently for periods of the length Δt.

As mentioned earlier, the driver's input $F_{tr}(\tau)$ and hence the trajectory $v(\tau)$ for $\tau \in [t_n,t_{n+1}]$ are assumed to be known a priori. Methods for calculation of this information based on feedback informations and observations are discussed in [17]. It is just mentioned here that for short values of Δt, even rough estimates based on the knowledge of road conditions (urban vs. highway, road slope, etc.) are adequate for the purpose of optimal gear selection. This is because v is a slowly changing process for vehicles having large masses (see Eq. (1)) and hence in short time intervals the variations of F_{tr} are the main factor determining the region in which the points of operation lie in Fig. 2.

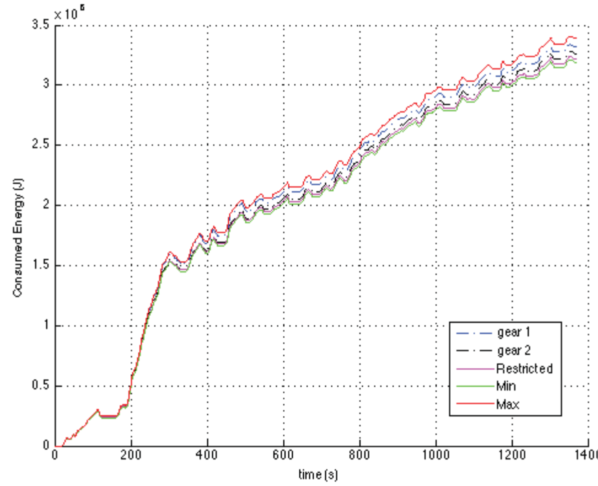

Fig. 6. The comparison of the restricted switching controller for UDDS with other gear shifting strategies. The gear selection strategy with at most one switching per 10 seconds is denoted by "Restricted; "gear 1" and "gear 2" correspond to single gear runs on the first and second gears respectively; "Min" and "Max" correspond to unrestricted switching commands to determine, respectively, the lower bound and the upper-bound for the energy consumption.

For the same UDDS driving cycle example as in section A, the switching problem is solved for $\Delta t = 10$ and $S_n = (t_n,\sigma_n) \equiv (t_n,q_n)$. This restricts the number of gear changes to at most one switching every 10 seconds. The results are compared in Fig. 6 with the gear shifting strategies below.

The case where the EV runs in its first gear only is denoted by "gear 1" and the case it operates in its second gear only is denoted by "gear 2". The case with unrestricted switching numbers is denoted by "Min" as it provides the ideal lower bound for the energy consumption. The worst case scenario where the vehicle always runs on the more energy consuming gear is denoted by "Max". As can be observed in Fig. 6, the restricted switching input results in an energy consumption very close to the lower bound for the overall energy consumption. At the end of the run for UDDS driving cycle, for example, the difference between the restricted problem and the unrestricted problem is $25.8\,kJ$ in $3218.3\,kJ$ which indicates that the restricted strategy is only 0.8% above the ideal lower-bound while its number of switchings is reduced from 182 gear changes (Fig. 4) to 59 switchings (Fig. 5). This restricted optimal shifting strategy improves the performance compared to a single run strategy by $100.5\,kJ$ or 3.1% compared to the first gear run and $45.1\,kJ$ or 1.4% compared to the second gear run.

IV. Concluding Remarks

In this paper the optimal gear selection problem is formulated for electric vehicles and the individual importance of gear selection on the energy consumption is demonstrated. It is concluded that although electric vehicles are able to run on a single gear, as is the case for the most of current EV's, they can perform more energy efficiently if equipped with gears.

With the introduction of gears to EV's, gear selection criteria need to be developed. While in the literature, gear selection strategies are rule-based or combined with the power distribution problem (for HEV's), the problem of optimal gear selection is considered in the current paper. It is shown that the minimum possible energy consumption results from several consecutive switchings in short periods of time which is undesirable due to physical limitations and performance efficiency. Hence, restrictions should be imposed on the switching commands. However, these restrictions, in general, result in the dependance of the gear selection decision to the whole time interval that makes them applicable only on offline control strategies.

In order to develop gear selection criteria that can be extended to online control strategies, it is mentioned that the optimality condition is favored to depend only on short subintervals, an example of which provided in the restriction of switching commands to at most one gear changing every Δt seconds. The choice of Δt depends on the horizon for which a precise prediction of the continuous control input F_{tr} and hence the maneuver of the car is available a priori or an adequate estimation can be provided. As demonstrated in the UDDS example, the proposed method for restricting the number of switchings eliminates numerous unnecessary switchings while the overall energy consumption remains close to the minimum possible value.

The feasible assumption that the continuous control input F_{tr} is known a priori restricts the pertinent optimal control methods to those for which this information may be derived at least for short periods of time. Hence, for applications like regular passenger vehicles, the estimation of F_{tr} for the purpose of optimal power delivery is of special interest. If this information can be constructed based on probability models then stochastic hybrid optimal control methods are applicable. In contrast, for applications including autonomous cruise control systems, intelligent speed adaptation (ISA) and autonomous (or self-driving) cars the input F_{tr} is a part of the controller's decision variables. Hence the determination of the optimal continuous control input F_{tr} and the discrete control input σ giving the gear sequence lies in the domain of the deteministic hybrid minimum principle (HMP) and hybrid dynamic programming (HDP). The reader is referred to [14], [15], [17] for relevant recent results on these topics.

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References

[1] S. Barsali, C. Miulli, and A. Possenti, A control strategy to minimize fuel consumption of series hybrid electric vehicles, IEEE Transactions on Energy Conversion, vol. 19, no. 1, pp. 187-195, 2004.

[2] Q. Gong, Y. Li, and Z. R. Peng, Trip-based optimal power management of plug-in hybrid electric vehicles, IEEE Transactions on Vehicular Technology, vol. 57, no. 6, pp. 3393-3401, 2008.

[3] S. I. Jeon, S. T. Jo, Y. I. Park et al., Multi-mode driving control of a parallel hybrid electric vehicle using driving pattern recognition, Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME, vol. 124, no. 1, pp. 141-149, 2002.

[4] L. V. Pérez, G. R. Bossio, D. Moitre et al., Optimization of power management in an hybrid electric vehicle using dynamic programming, Mathematics and Computers in Simulation, vol. 73, no. 1-4 SPEC. ISS., pp. 244-254, 2006.

[5] A. Sciarretta, M. Back, and L. Guzzella, Optimal control of parallel hybrid electric vehicles, IEEE Transactions on Control Systems Technology, vol. 12, no. 3, pp. 352-363, 2004.

[6] S. Stockar, V. Marano, M. Canova et al., Energy-optimal control of plug-in hybrid electric vehicles for real-world driving cycles, IEEE Transactions on Vehicular Technology, vol. 60, no. 7, pp. 2949-2962, 2011.

[7] H. D. Lee, S. K. Sul, H. S. Cho et al., Advanced gear-shifting and clutching strategy for a parallel-hybrid vehicle, IEEE Industry Applications Magazine, vol. 6, no. 6, pp. 26-32, 2000.

[8] P. Pisu, and G. Rizzoni, A comparative study of supervisory control strategies for hybrid electric vehicles, IEEE Transactions on Control Systems Technology, vol. 15, no. 3, pp. 506-518, 2007.

[9] P. Kock, H. J. Welfers, B. Passenberg et al., Saving energy through predictive control of longitudinal dynamics of heavy trucks, VDI Berichte, no. 2033, pp. 53–67, 2008.

[10] B. Passenberg, P. Kock, and O. Stursberg, Combined Time and Fuel Optimal Driving of Trucks based on a Hybrid Model, Proceedings of the European Control Conference ECC 2009. pp. 4955-4960.

[11] Fu, J., and Bortolinb G., Gear Shift Optimization for Off-road Construction Vehicles, Procedia - Social and Behavioral Sciences, vol. 54, pp. 989 – 998, 2012.

[12] I. A. Blagojević, G. S. Vorotović, G. S. Ivanović et al., Energy efficiency improvement by gear shifting optimization, Thermal Science, vol. 17, no. 1, pp. 91-10, 2013.

[13] G. Xiong, J. Xi, Y. Zhang et al., Optimal design and simulation evaluation of economical gear-shifting schedule for AMT in pure electronic bus, Proceedings of the IEEE International Conference on Industrial Technology ICIT 2010. pp. 1131-1135.

[14] M. S. Shaikh, and P. E. Caines, On the hybrid optimal control problem: Theory and algorithms, IEEE Transactions on Automatic Control, vol. 52, no. 9, pp. 1587-1603, 2007, Corrigendum: vol. 54, no. 6, pp 1428, 2009.

[15] A. Pakniyat, and P. E. Caines, On the Minimum Principle and Dynamic Programming for Hybrid Systems, Proceedings of the 19th IFAC World Congress, 2014.

[16] A. Pakniyat, and P. E. Caines, The Hybrid Minimum Principle in the Presence of Switching Cost, Proceedings of the 52nd IEEE Conference on Decision and Control, CDC 2013, pp. 3831-3836.

[17] A. Pakniyat, and P. E. Caines, On the Minimum Principle and Dynamic Programming for Hybrid Systems, Research Report, Department of Electrical and Computer Engineering (ECE), McGill University, July 2014.