COMP 558: Assignment 1
Available: Sunday, January 25th, 2015
Due Date: Sunday, February 8th, 2015 (before midnight) via mycourses.

Notes: You are encouraged to become familiar with the **MATLAB** environment which is currently installed on Unix machines in the CS labs and to use it for the experimental parts of the assignment. Morteza has already had a tutorial on its use and some helpful code is posted on mycourses. Hints on assignment 1 will be provided once you get going and have further questions. I EXPECT EVERYONE TO SUBMIT ORIGINAL WORK FOR THIS ASSIGMENT. This means that if you have consulted anyone or any sources (including source code), you must disclose this explicitly. Anything you submit reflects your own work. Your submission should be in the form of an electronic report (PDF), which includes a description of what you did, answers to the specific questions, and a presentation and discussion of your results. Submit code that you have written to generate your results as a separate .zip file.

Question 1: Edge Detection - Marr and Hildreth (12 marks)

1. **(3 marks)** In no more than 3 paragraphs describe the main steps of the Marr and Hildreth approach to edge detection. Your answer should be concise but complete.

2. **(3 marks)** Consider a 1D Gaussian function given by $G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$. Show that the Fourier Transform of this Gaussian is also a Gaussian (hint: use an expansion of the complex exponential first and then try to simplify). For this you can treat the frequency domain variable as $\omega = 2\pi f$. What is the relationship between the standard deviation of the Gaussian σ and the standard deviation of its Fourier Transform, σ_ω? Based on this, and the property that these ideas extend to higher dimensions what is the effect in the frequency domain of convolving a 2D image with a 2D Gaussian function?

3. **(3 marks)** Consider an image $I(x, y)$ which is a scalar function of the 2 spatial variables x, y, i.e., $I : \mathbb{R}^2 \rightarrow \mathbb{R}^+$. Prove that the Laplacian of the image (or for that matter any scalar function) is invariant to rotation. To do this define the Laplacian as $\Delta I = I_{rr} + I_{r'r'}$, where r, r' comprise an orthogonal basis for I. (A special case is where r is associated with the direction of the x-axis and r' with the direction of the y-axis.) Now, using a suitable coordinate system, show that the value of ΔI does not depend on the specific choice of basis directions.

4. **(3 marks)** Marr and Hildreth argue that to detect edges the zero-crossings of a second derivative taken in the direction perpendicular to the loal direction of the edge should be used. They then use a non-oriented operator, the Laplacian, to detect these zero-crossings, arguing that under a certain condition this is mathematically sound. What is that condition (state it and right it down mathematically)? Prove that their argument is mathematically sound.

Question 2: Edge Detection - Experiments (8 marks)

For this question you will test your implementations of the Marr and Hildreth algorithm on the two images for edge detection located at:
`http://www.cim.mcgill.ca/~morteza/COMP558-Winter2015.php`.

1. **(2 marks)** First write down a mathematical expression for the Laplacian of a 2D Gaussian at a particular scale σ. Use a convenient choice of coordinate system to represent this expression. Then create plots of this function for various scales $\sigma = 1, 2, 4, 8$ and visualize these plots using suitable surface plotting functions in Matlab.

2. **(6 marks)** Now convolve the two images located at the webpage above with the above 2D Gaussians at different scales, locate the zero-crossings of each result, and find a suitable way to display the results either as binary plots or as overlays on the original images. Discuss the effect of scale σ on the nature of edges detected on each image. Also consider the underlying condition mentioned in Question 1., upon which this edge detection scheme is based. In light of this, comment on when the method works and when it does poorly with regard to the features of the two images.