lecture 22

review of exercises

E 12 Q1, 3, 4, 6
E 16 Q2 a, c
E 17 Q1 a, b, 5
E 18 Q 1, 3, 4
E 19 Q1, 2, 4

Lighting, Material Shading

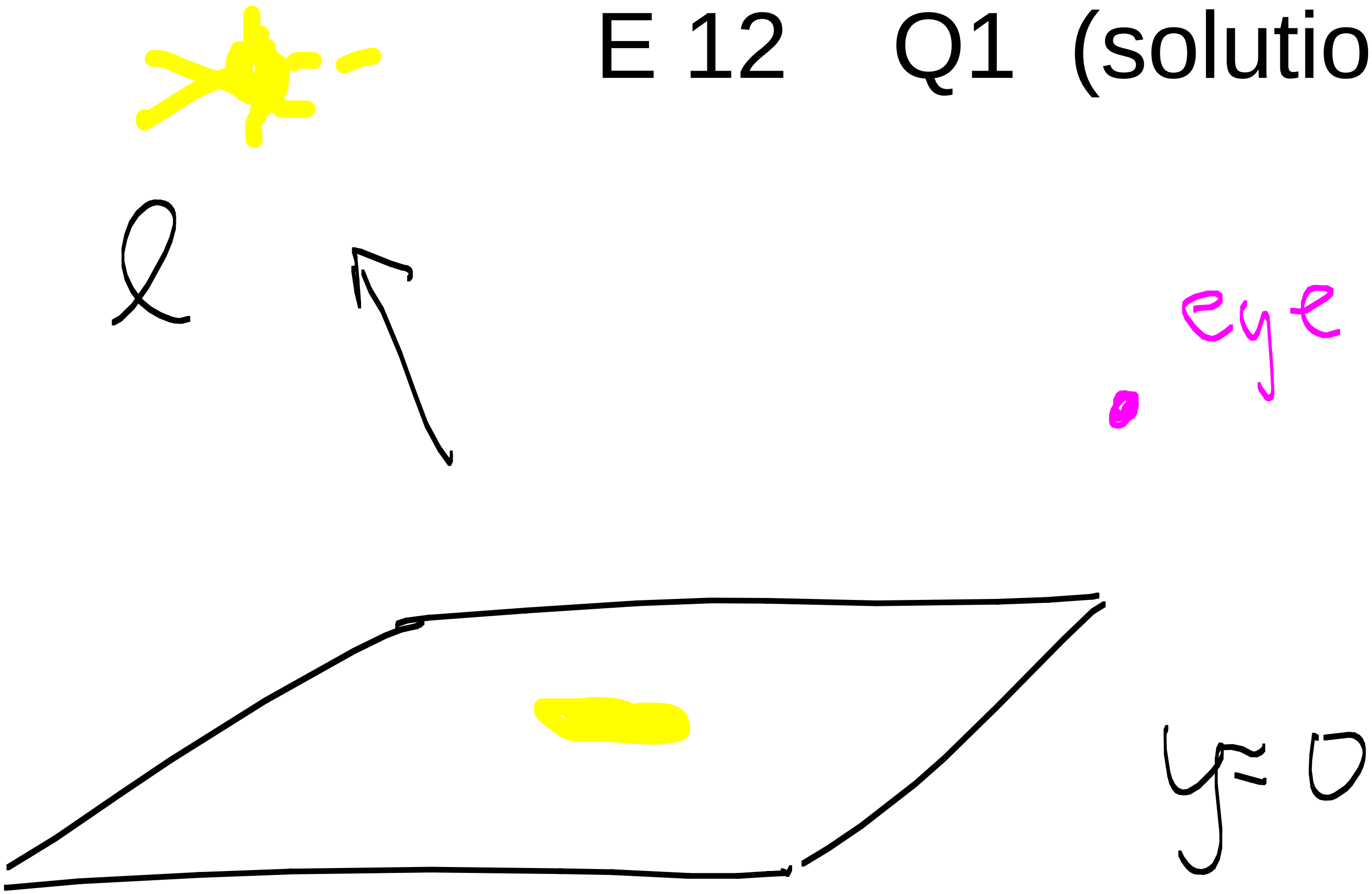
E 12 Q1

Suppose that a shiny ground plane y=0 is illuminated by sunlight. Let the direction of the sun be (1,2,2).

If a viewer is at (4,6,7), determine the position on the ground plane at which the peak of the highlight occurs.

Assume that the peak occurs where a mirror reflection takes a ray from the source and reflects it directly to the viewer position.

E 12 Q1 (solution)



Steps of the solution:

E 12 Q3

Consider a triangle in an image.

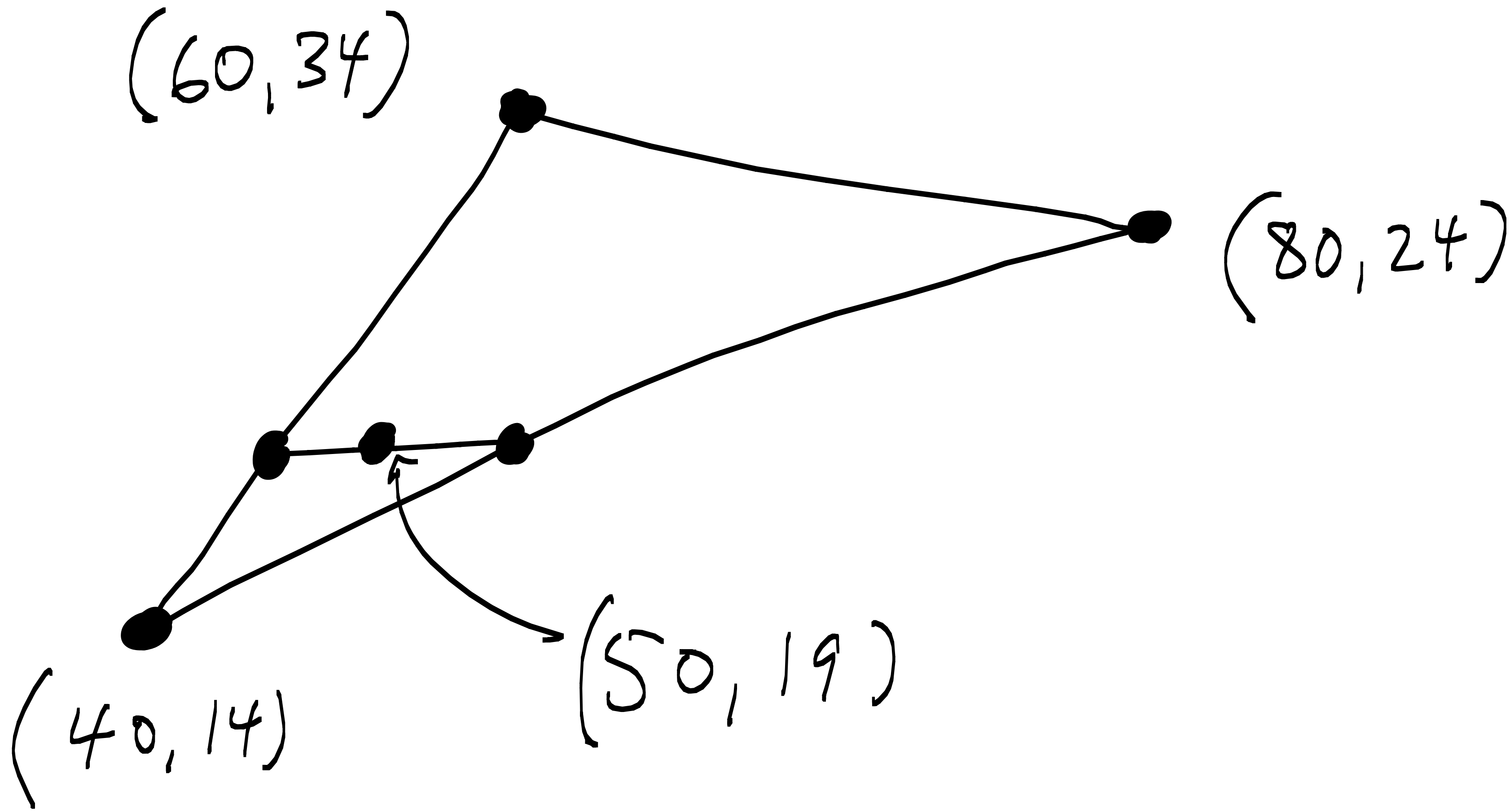
Suppose the intensities I(x,y) are:

I1 = I(40,14) = 10
I2 = I(60,34) = 90
I3 = I(80,24) = 140.

Using linear interpolation, calculate the intensity I(50,19).

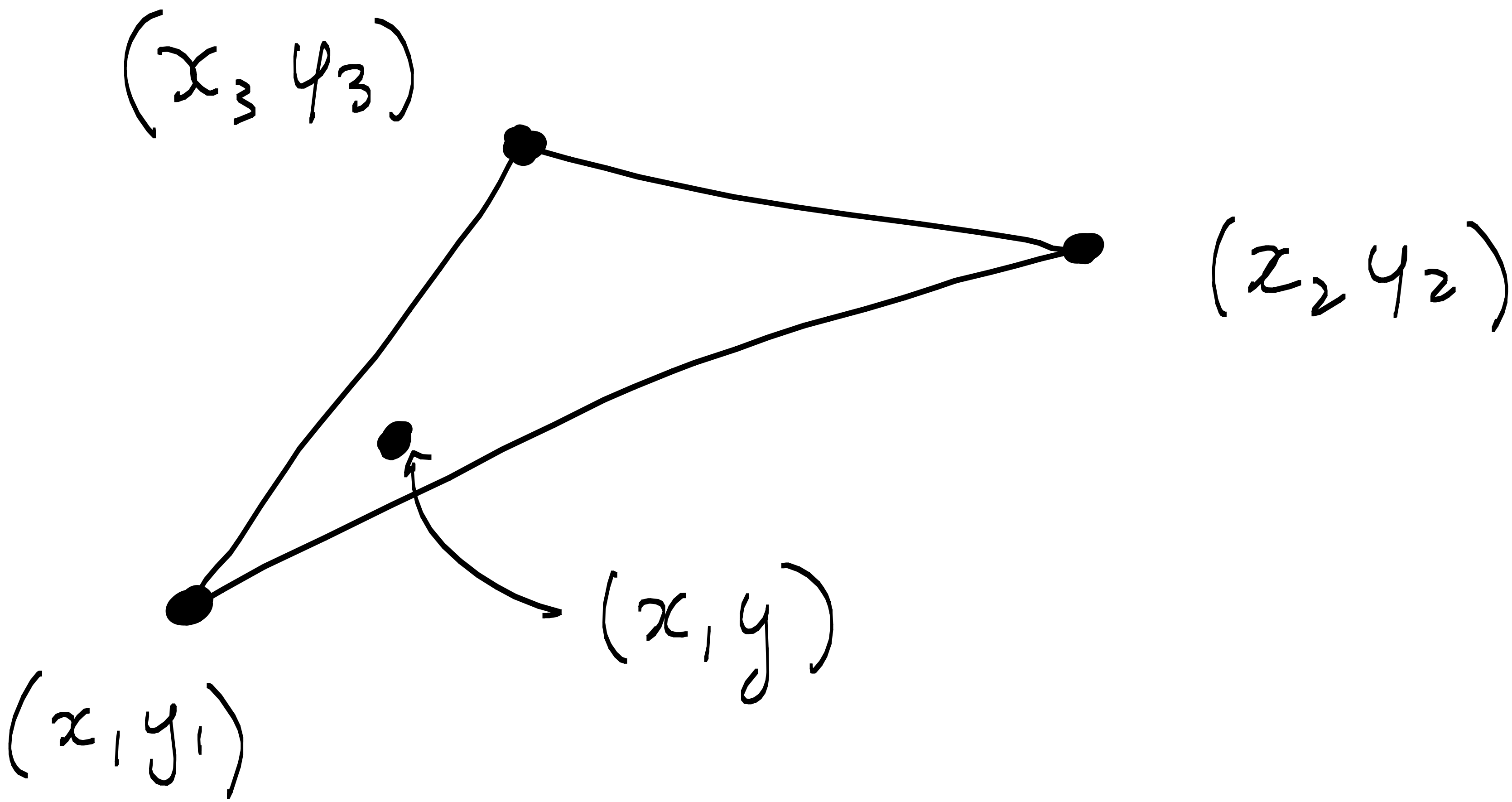
E 12 Q3 (solution)

$$I(x,y) = I(x_0,y_0) + (I(x_1,y_1) - I(x_0,y_0))(\frac{x - x_0}{x_1 - x_0})$$



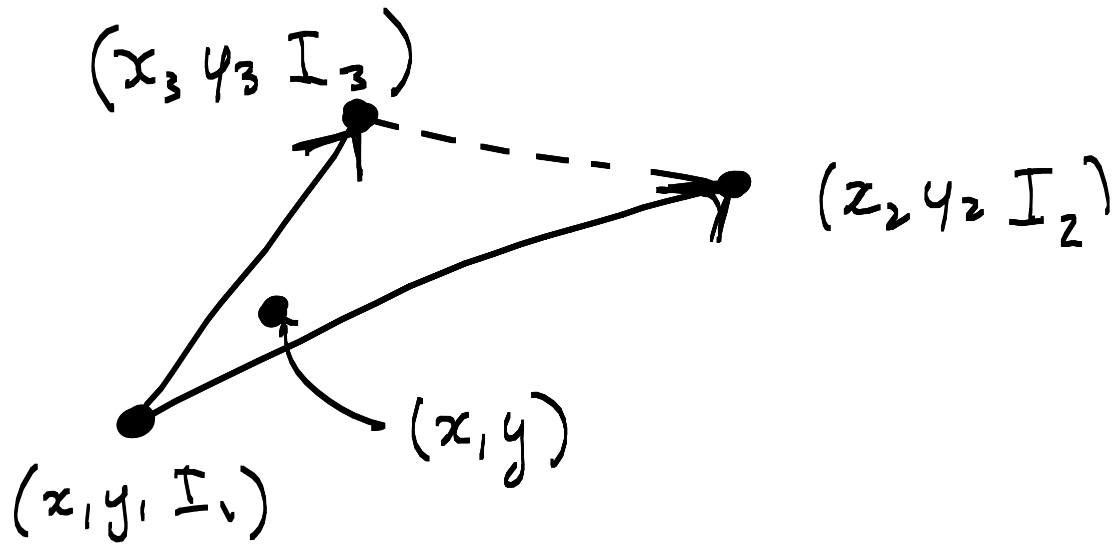
E 12 Q4 a

How would you use convex combinations of (x,y, I) to answer the previous question?



E 12 Q4a Solution

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ I \end{bmatrix}$$



$$\mathbf{p} = \mathbf{p}_1 + s(\mathbf{p}_2 - \mathbf{p}_1) + t(\mathbf{p}_3 - \mathbf{p}_2)$$

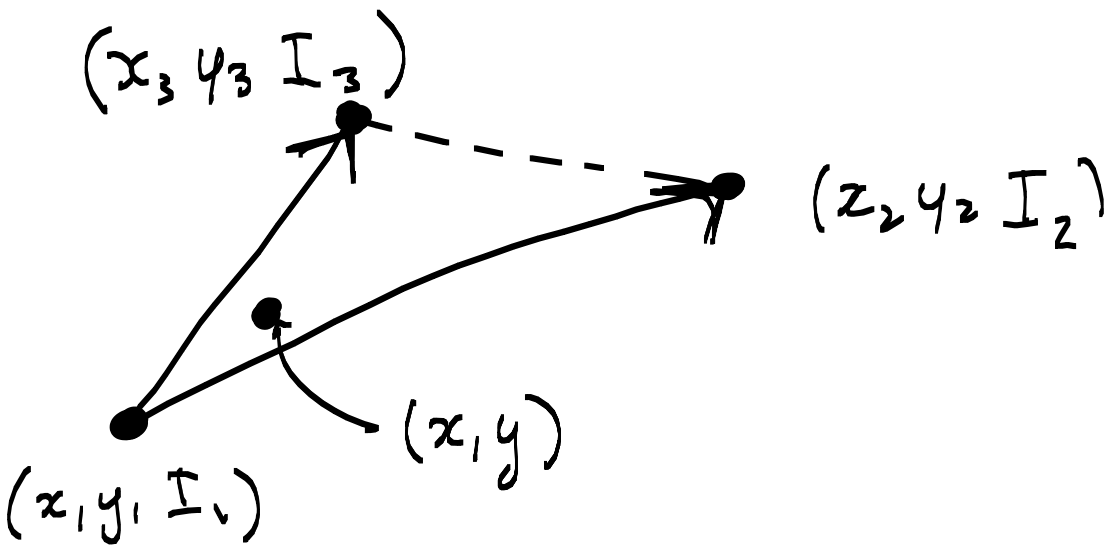
$$= (1 - s - t)\mathbf{p}_1 + s\mathbf{p}_2 + t\mathbf{p}_3$$

Three equations and two unknowns.
Given (x,y), solve for (s, t) and then solve for I.

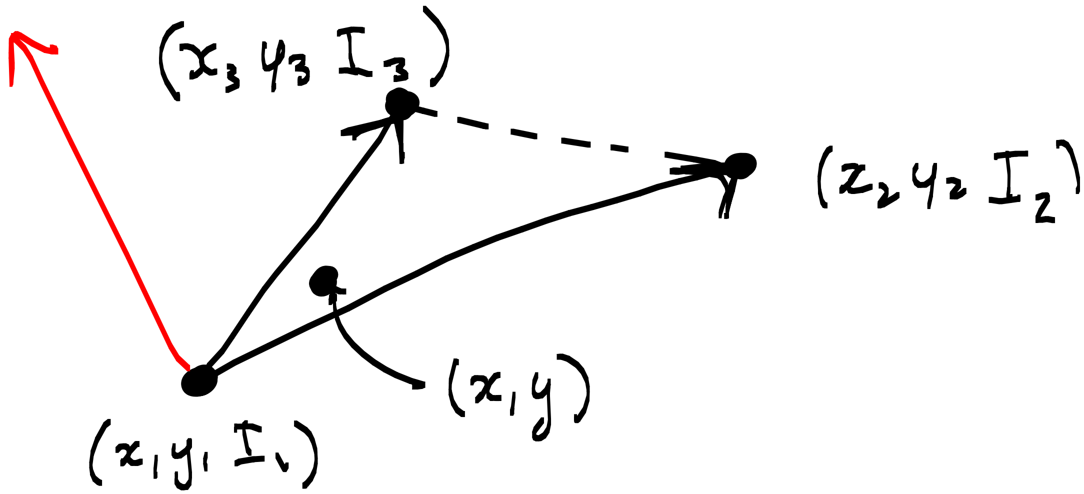
E 12 Q4 b

How would you solve the same problem by fitting a plane to the three points in (x,y,I) space?

(This is at the level of MATH 133.)



E 12 Q4 b solution



Take the cross product to get the normal of the triangle

$$n = (nx, ny, nI) = (p2 - p1) \times (p3 - p1)$$

E 12 Q6

It is often claimed that the Blinn-Phong model is cheaper than the Phong model when:

- the light source is at infinity, and

- the viewer is far from the polygon (in relation to the size of the polygon).

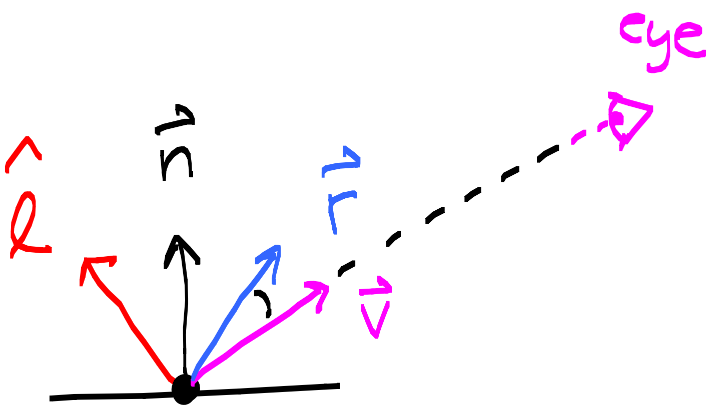
Why?

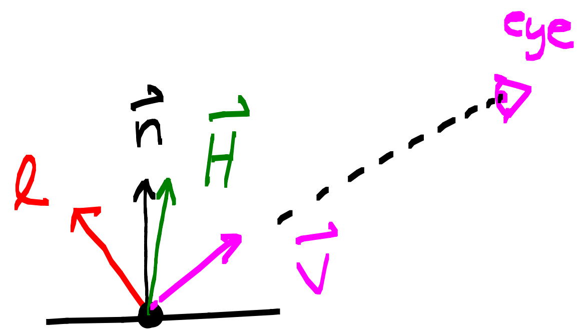
[The main purpose of this question is to get you thinking about what the models are. The claim itself is less of a concern for us.]

E 12 Q6 (solution)

$$\vec{r} = 2(\vec{n}\cdot\hat{\ell})\,\vec{n} - \hat{\ell}$$

$$\vec{H} = \frac{\hat{\ell} + \vec{v}}{\|\hat{\ell} + \vec{v}\|}$$





$$k_{specular}(\vec{r}\cdot\vec{v})^e$$

Phong model

$$k_{specular}(\vec{H}\cdot\vec{n})^e$$

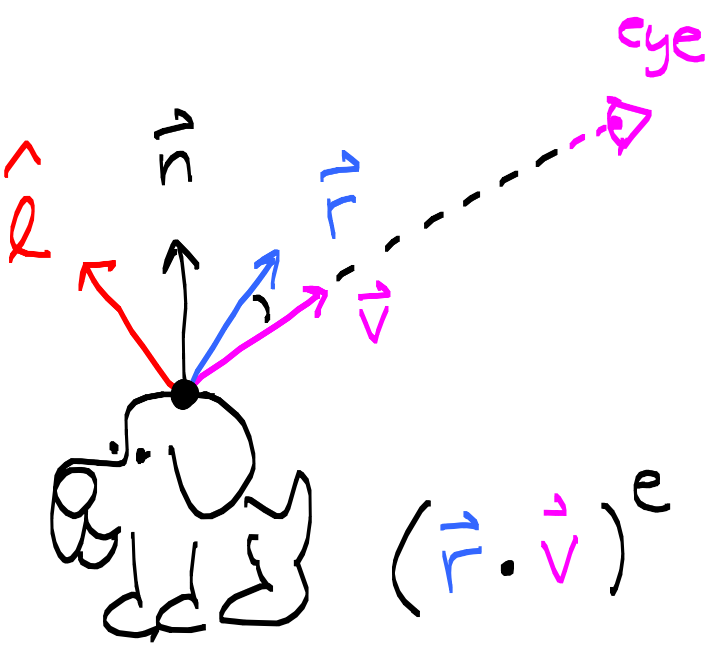
Blinn-Phong model

Phong: The reflection vector r depends strongly on the normal n and must be recomputed for each point.

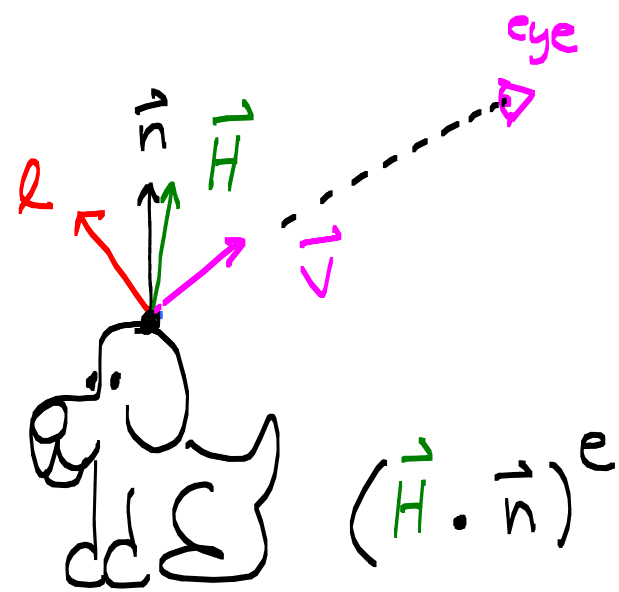
Blinn-Phong: If light source l is at infinity then it is constant.

And if the viewer is far from an object, then we can approximate v as constant over the object.

The half vector H is approximately constant over the object.



Phong model

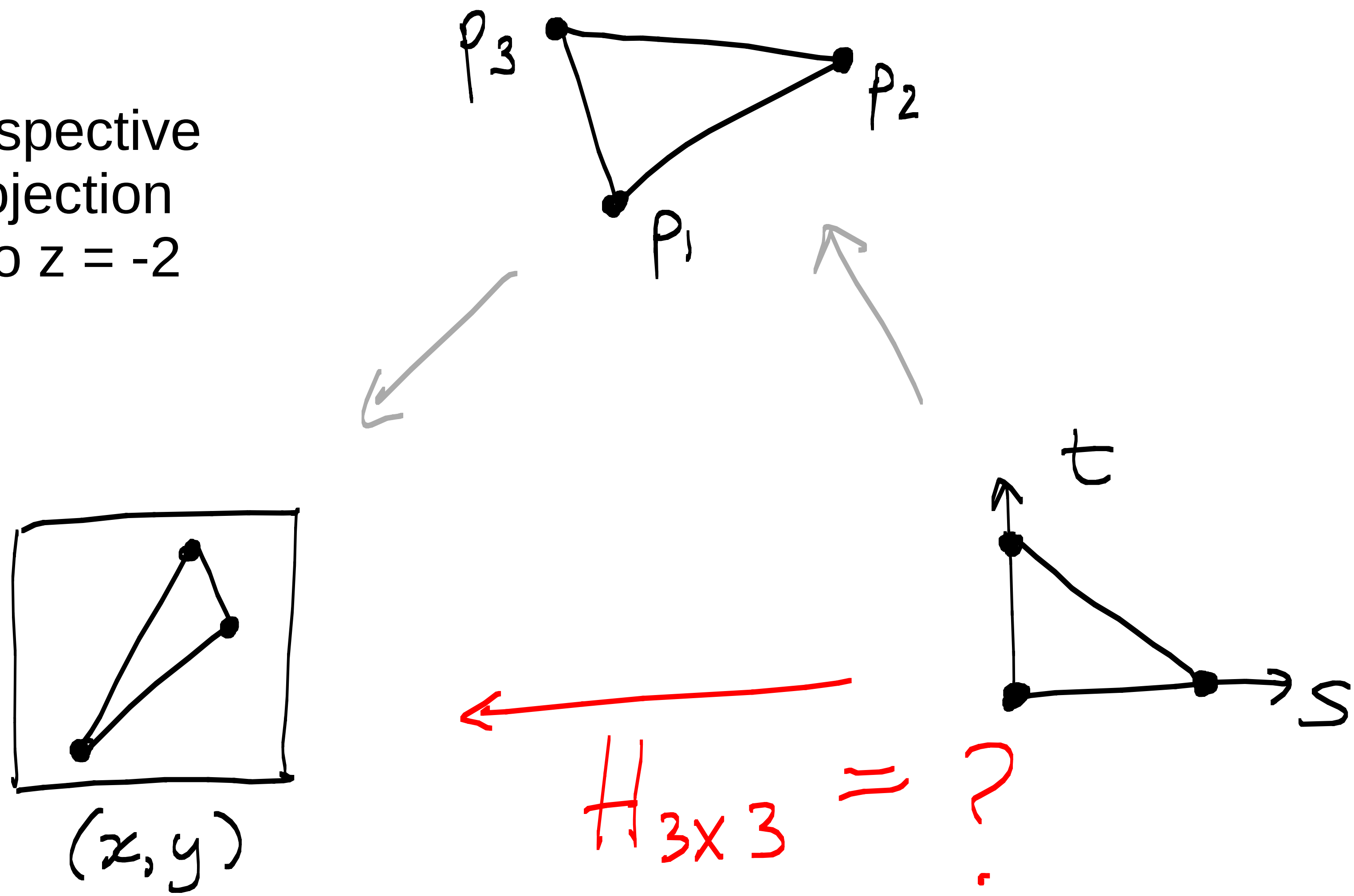


Blinn-Phong model

Texture Mapping

E 16 Q2a

perspective projection onto z = -2



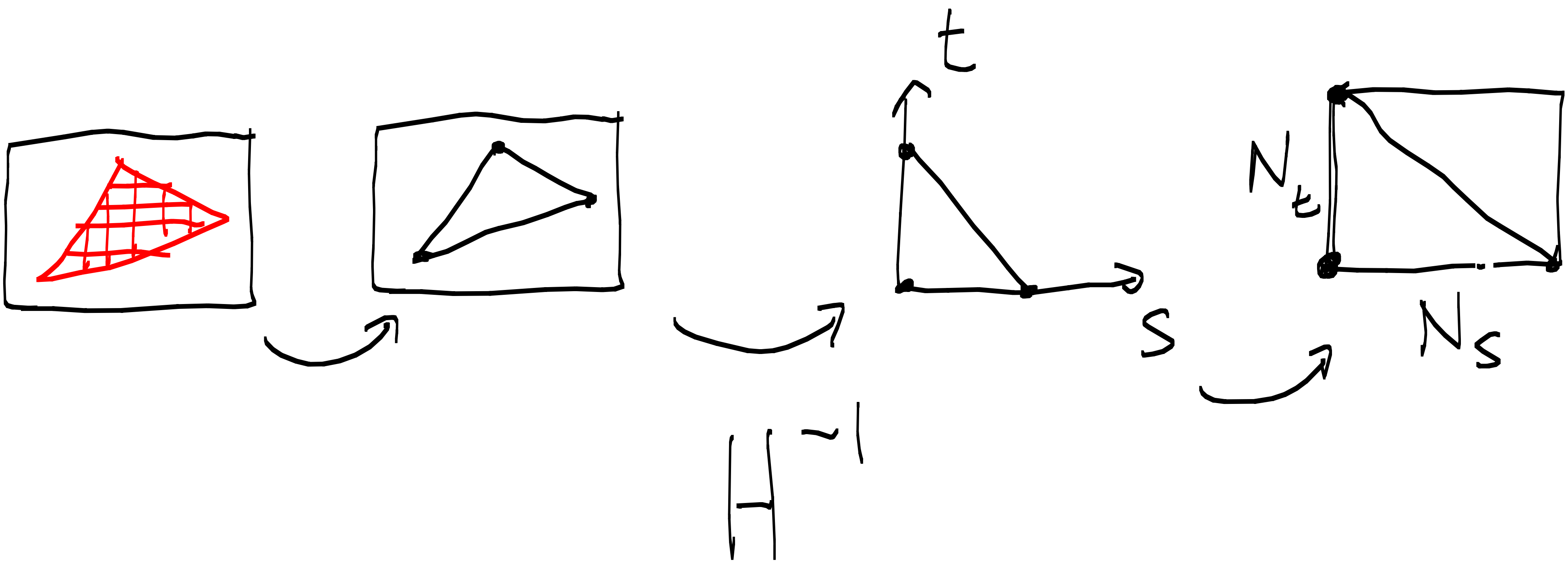
E 16 Q2a solution

$$\begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{3 \times 4} \underbrace{\left[\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ p_2 - p_1 & p_3 - p_1 & p_1 \\ \downarrow & \downarrow & \downarrow \\ \hline 0 & 0 & 1 \end{array}\right]}_{4 \times 3} \begin{bmatrix} s \\ t \\ 1 \end{bmatrix}$$

The posted solutions give more details on how to get H.

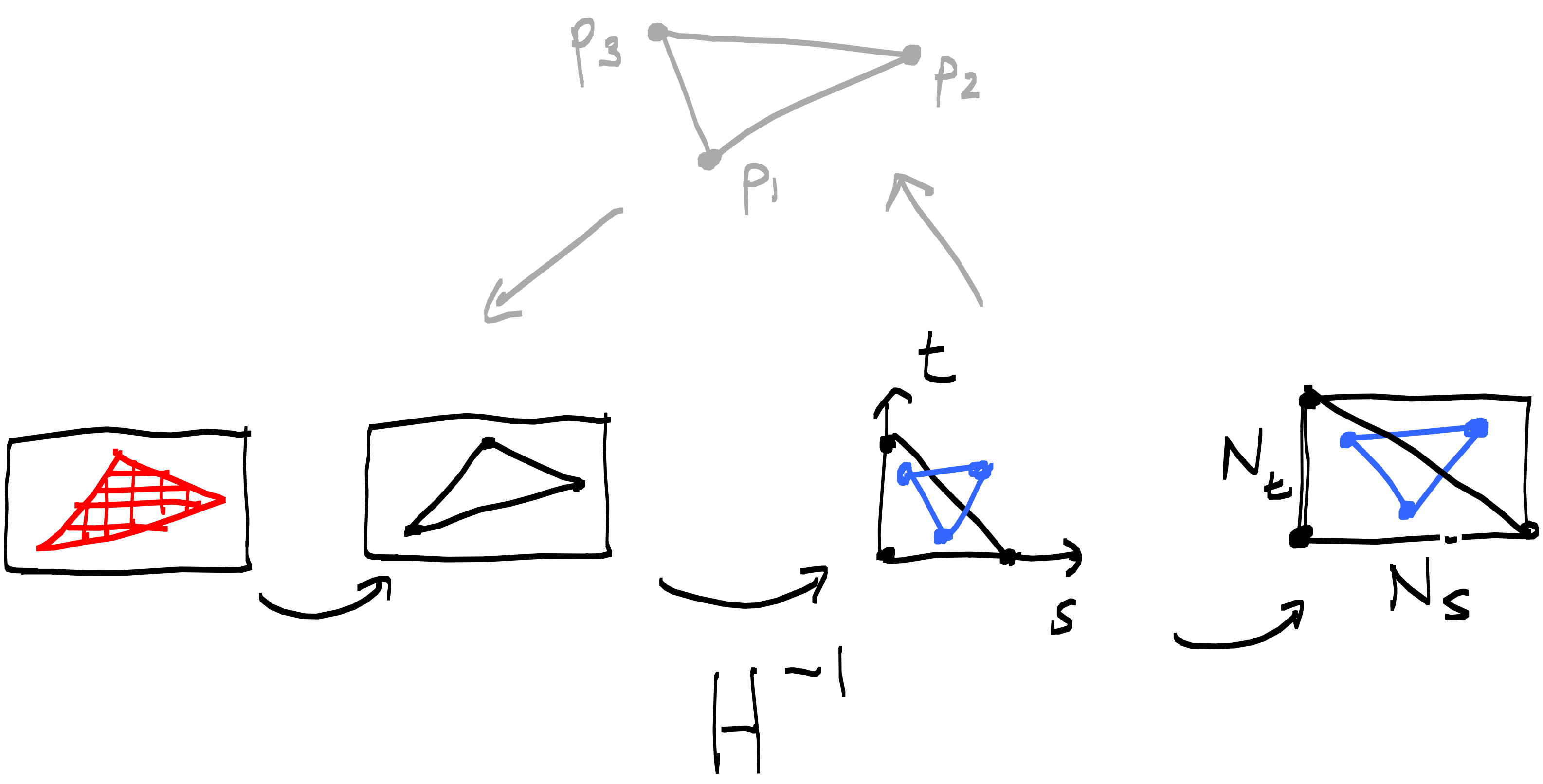
Recall why H needs to be computed.

In the case of texture mapping, whe polygon is rasterized, each fragment must be assigned texture coordinates so that the texture map can be indexed.

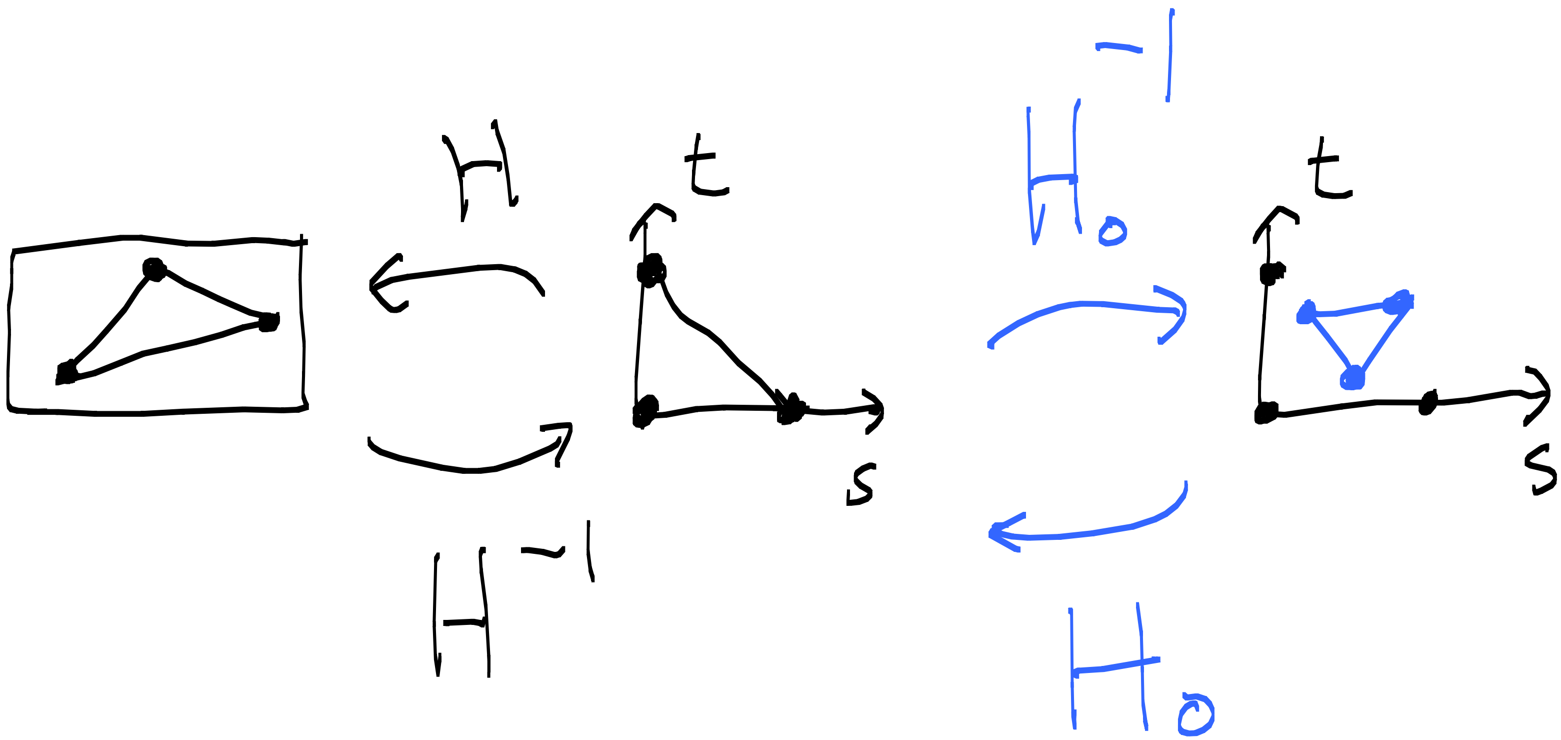


E 16 Q2c solution

Suppose we are given more general texture coordinates for the triangle. How would we compute a homography H in this case ?



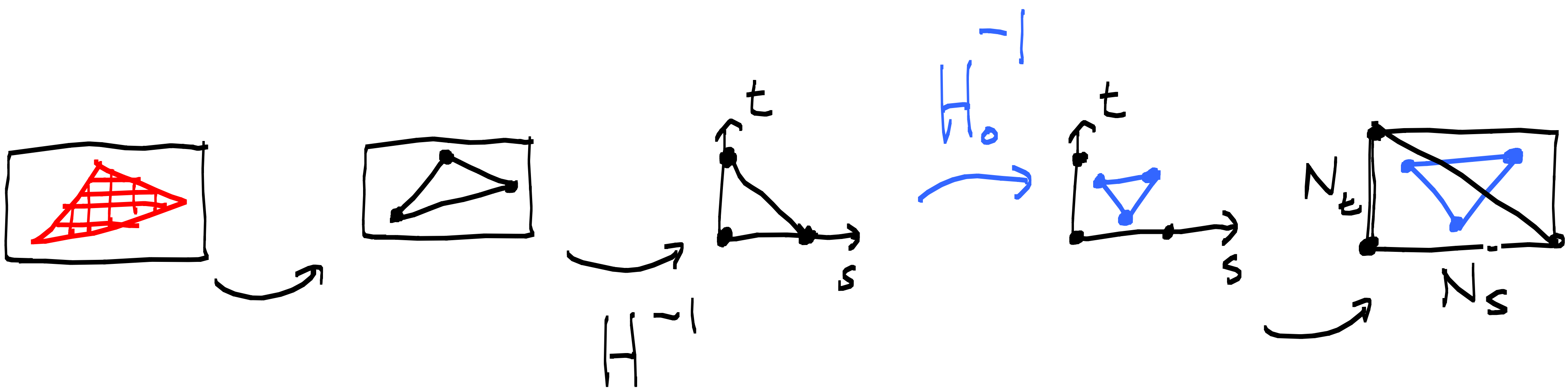
E 16 Q2c solution



For H_0^{-1}, see solution for E12 Q4 from earlier.

Again, why are these homographies computed?

In the case of texture mapping, whe polygon is rasterized, each fragment must be assigned texture coordinates so that the texture map can be indexed.



Procedural Shading

E 17 Q1 a

I claimed in the lecture that when mapping to a new coordinate system by the matrix $\mathbf{M}$, the surface normal should be mapped using $\mathbf{M}^{-T}$ i.e. the transpose of the inverse (or equivalently, the inverse of the transpose).

(a) What is the inverse of a 4×4 translation matrix $\mathbf{M}$?

E 17 Q1 a (Solution)

$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & -t_x \\ 0 & 1 & 0 & -t_y \\ 0 & 0 & 1 & -t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{translate} \qquad M_{translate}^{-1}$$

E 17 Q1 b

[**updated March 25**] Show that if $\mathbf{M}$ is defined by a rotation and/or translation, then for any surface normal vector $\mathbf{n}$ that is represented as a point at infinity,

$$\mathbf{n} = (n_x, n_y, n_z, 0)$$

$\mathbf{M}^{-T}\mathbf{n}$ and $\mathbf{Mn}$ represent the same *unit* surface normal. That is, when we normalize these two transformed vectors, we get the same unit length vector.

- M is 4x4, rotation and/or translation
- surface normal is point at infinity (4D)
- M^{-T} n and Mn represent the same (3D) unit normal.

E 17 Q1 b (Solution)

rotation

$$R^T = R^{-1} \quad \text{so} \quad (R^T)^{-1} = R$$

translation

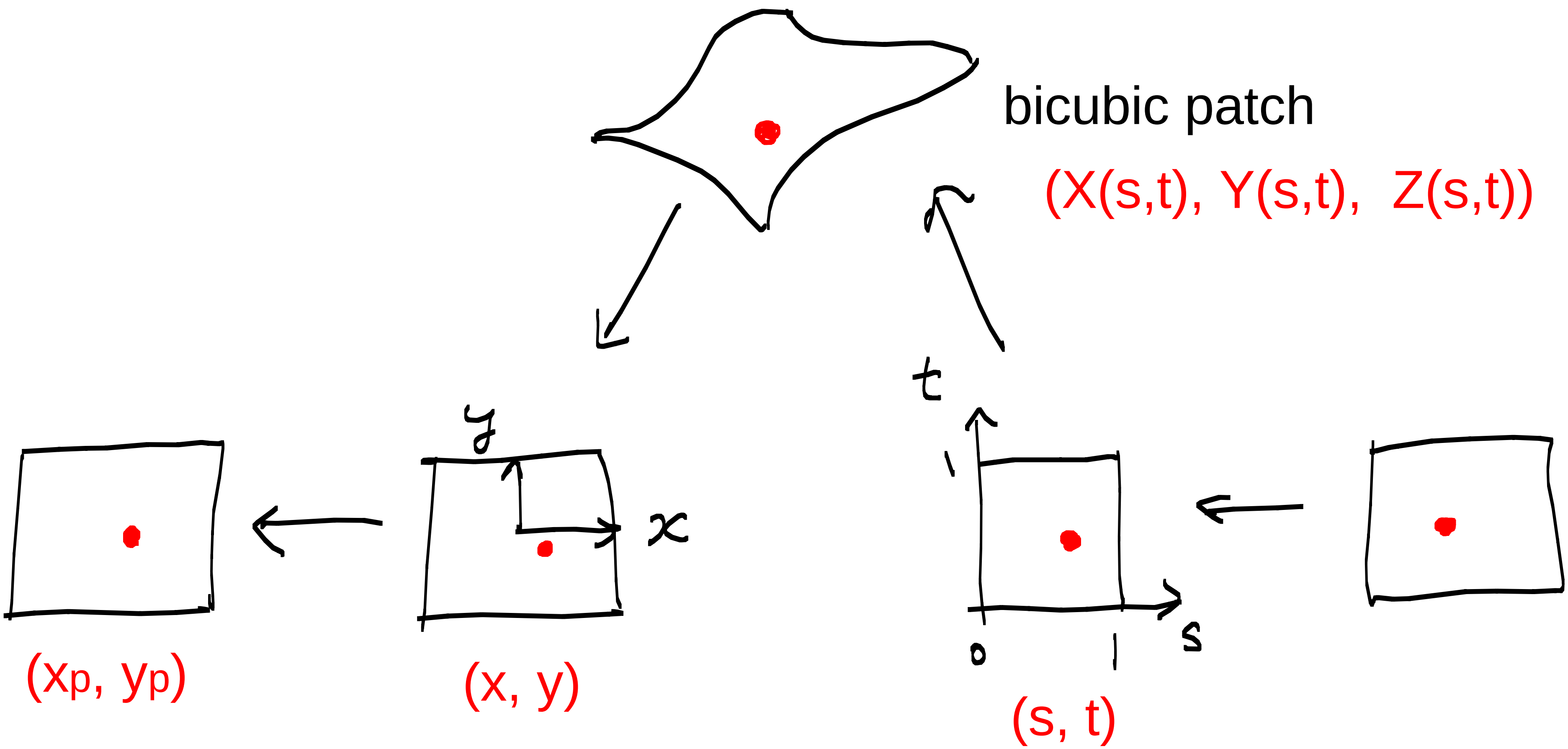
$$\mathbf{Mn} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}$$

$$\mathrm{M}^{-T}\mathbf{n} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -t_x & -t_y & -t_z & 1 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ -t_x n_x - t_y n_y - t_z n_z \end{bmatrix}$$

E 17   Q2 a

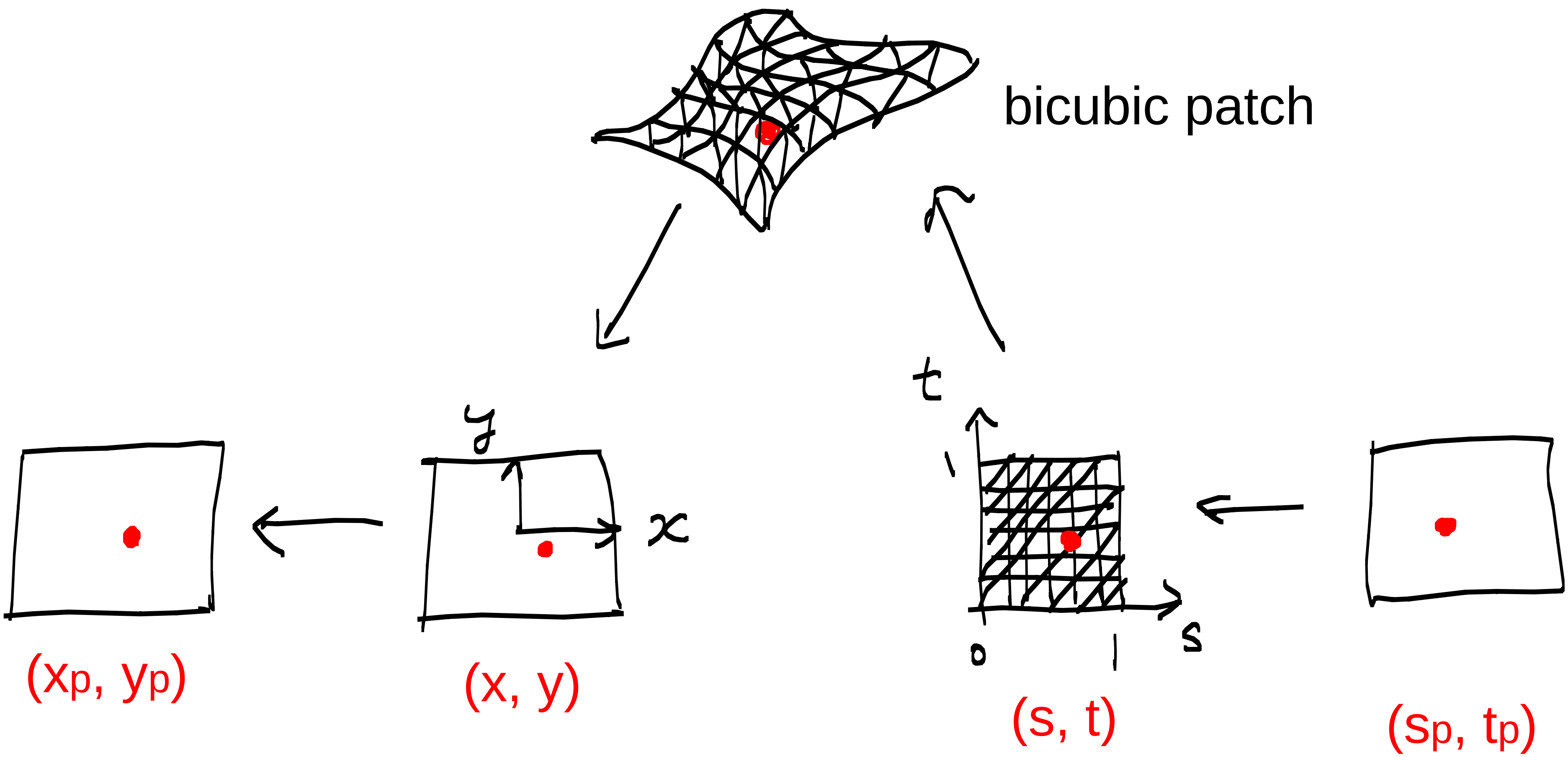
How would you do texture mapping with a bicubic?

What challenges arise that do not arise with a polygon or quadric?



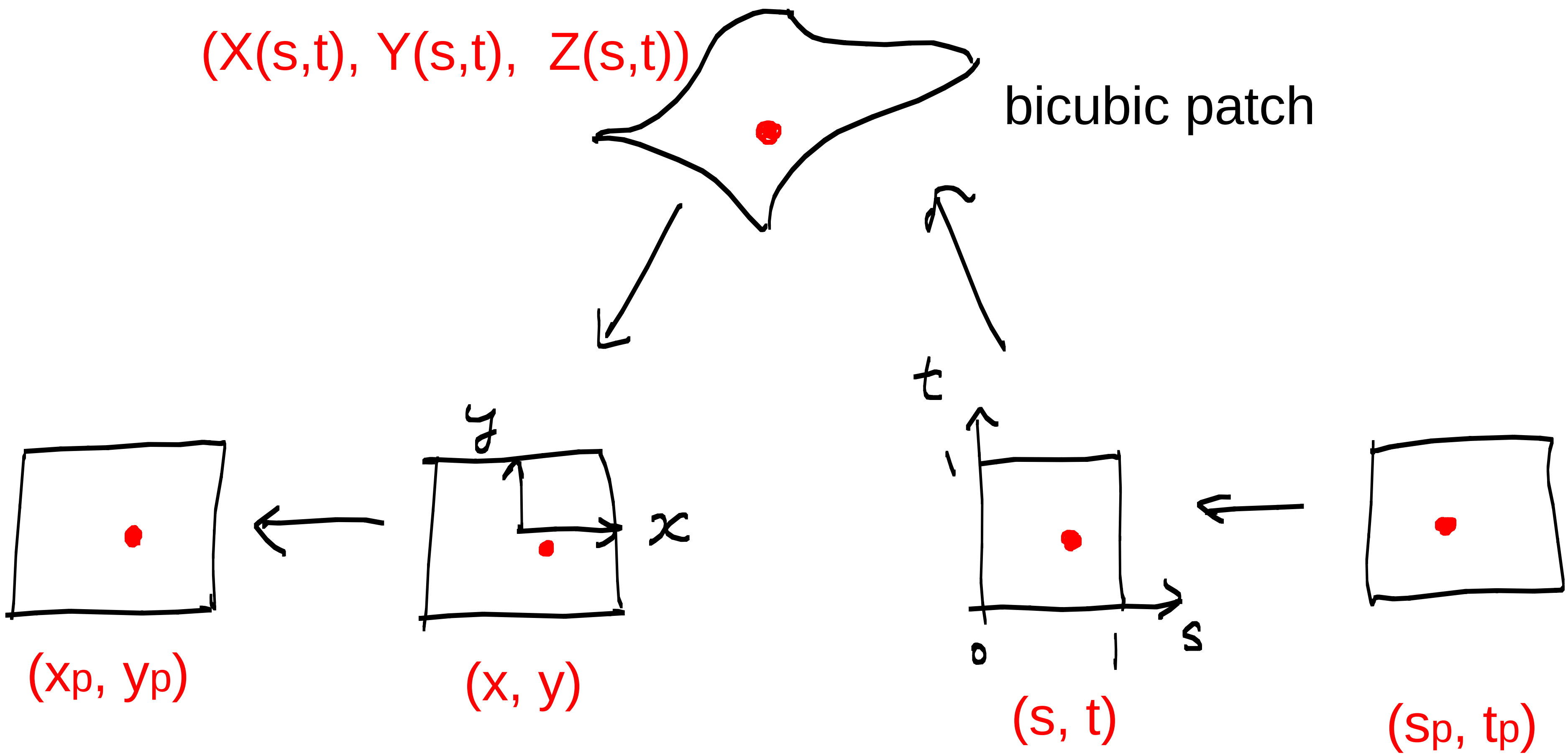
E 17 Q2 a (Solution)

One solution would be to triangulate.



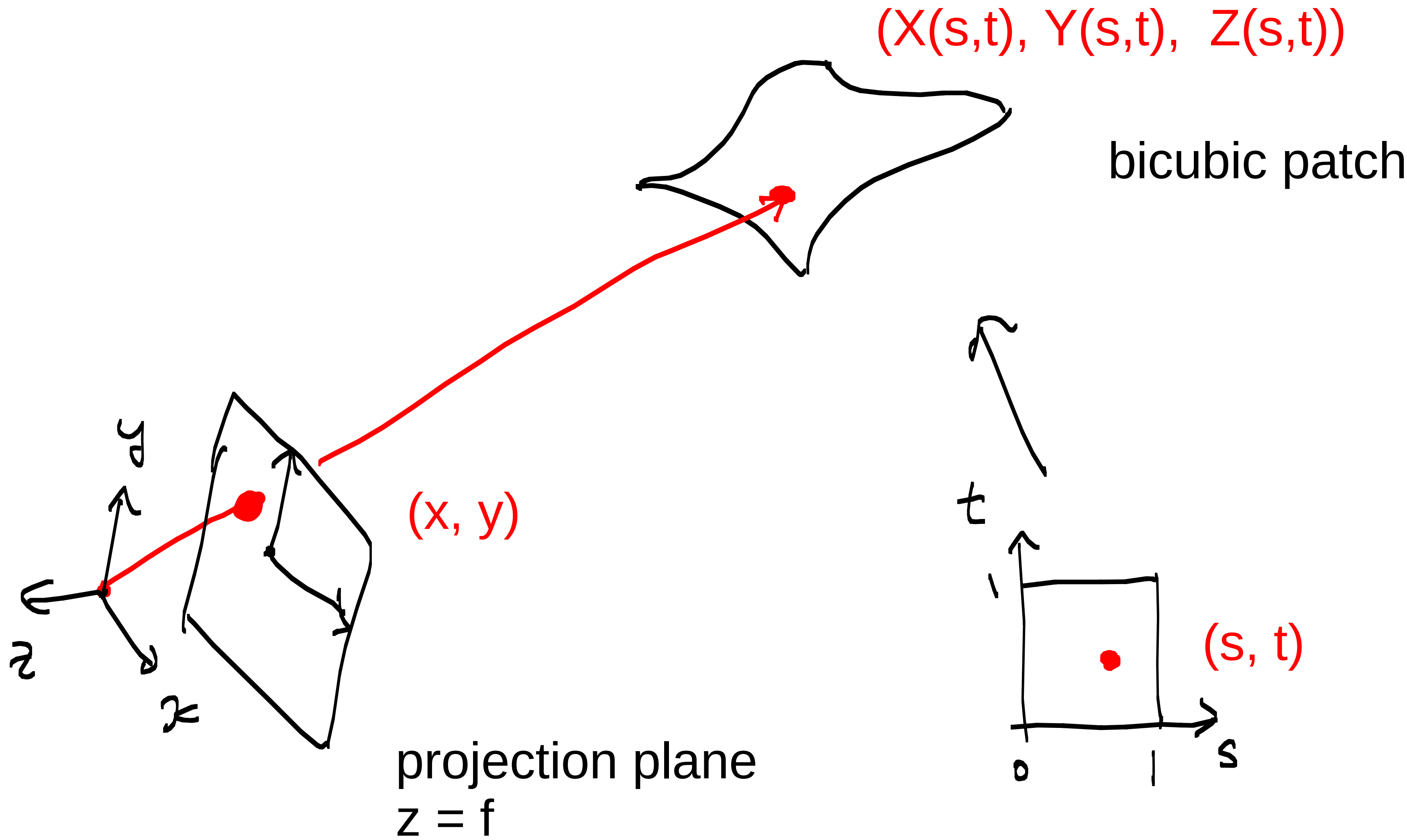
Another solution:

For any pixel (x_p, y_p), find texture coordinates (s, t) such that surface point $(X(s,t), Y(s,t), Z(s,t))$ projects to (x_p, y_p).



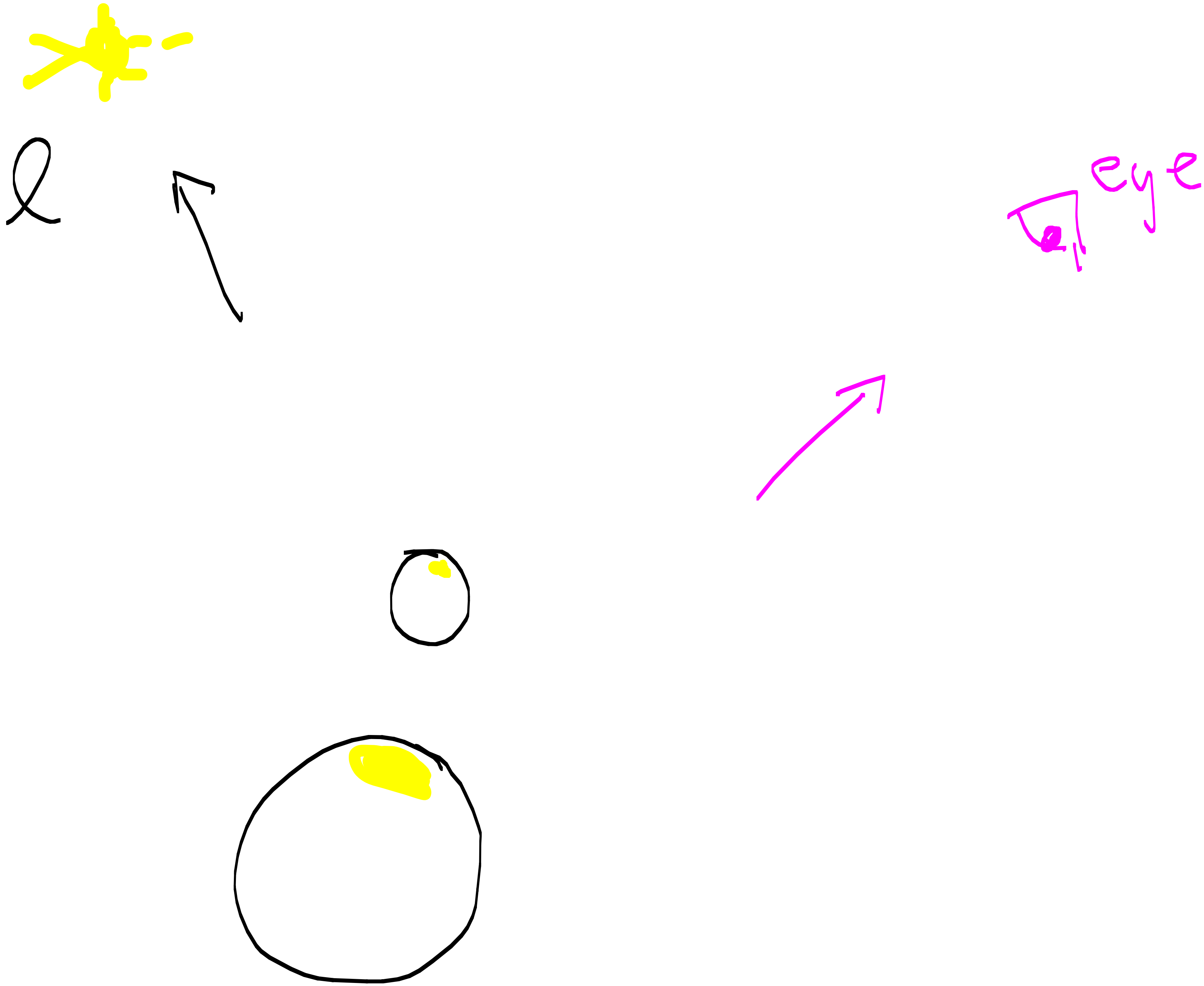
Find the intersection of a ray with a bicubic, namely find intersection point (X, Y, Z) and find (s, t).

Unlike with a quadric, you would need to use numerical methods for this. But it can be done.



E 17 Q 5

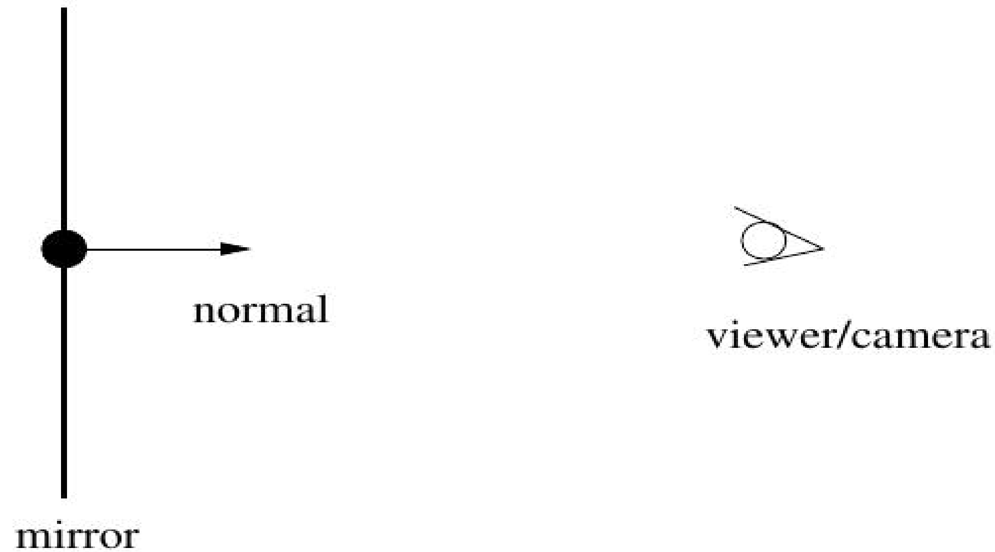
Highlights are not isolated points, but rather define small neighborhoods on the surface. The area of a highlight depends on the shininess. But it also depends on how curved the surface is. Why?



ℓ
eye

Ray tracing, environment mapping

E 18 Q 1

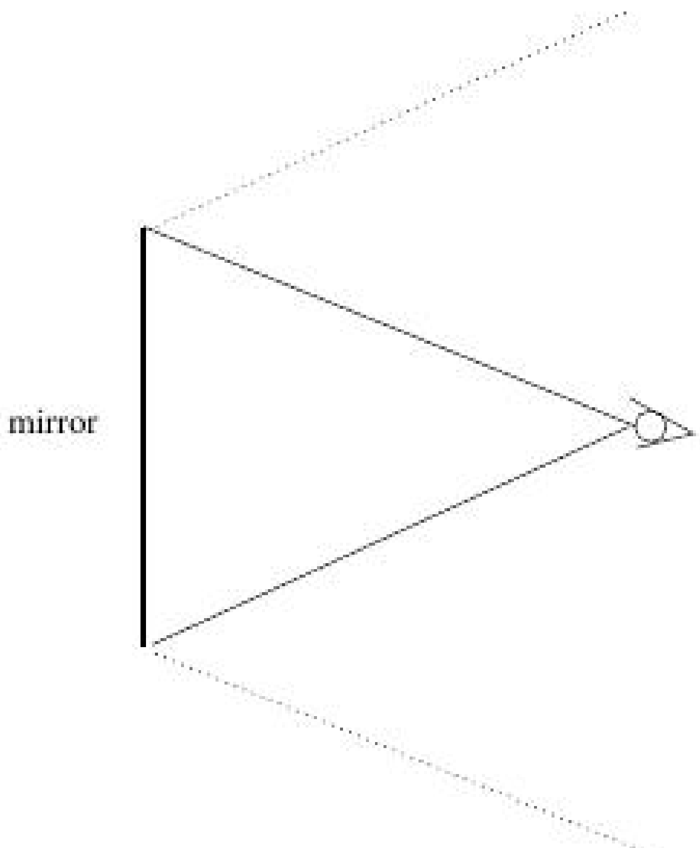


Indicate the scene volume that is visible to the viewer, as a reflection in the mirror. Compare two cases:

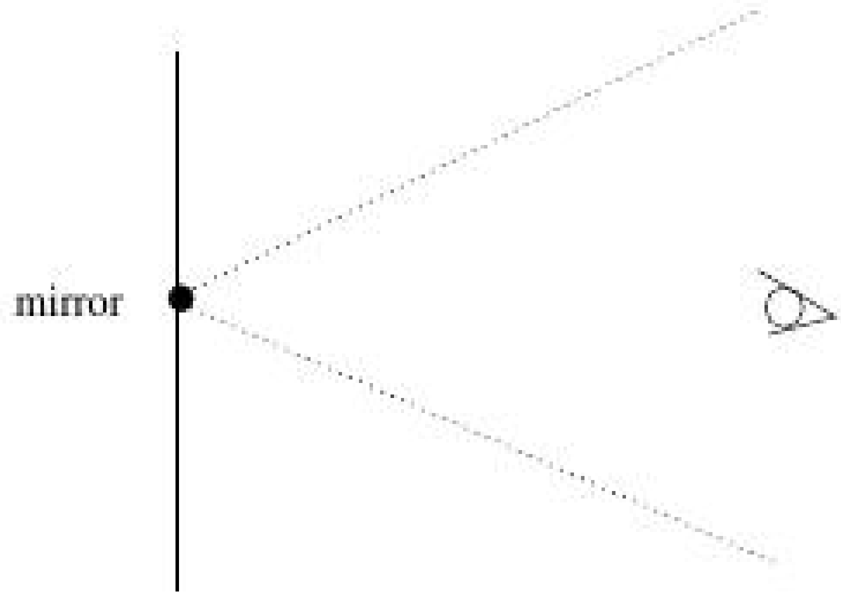
- if ray tracing were used;

- if environment mapping were used. (Assume that the environment map is computed from the black dot.)

E 18 Q 1 solution



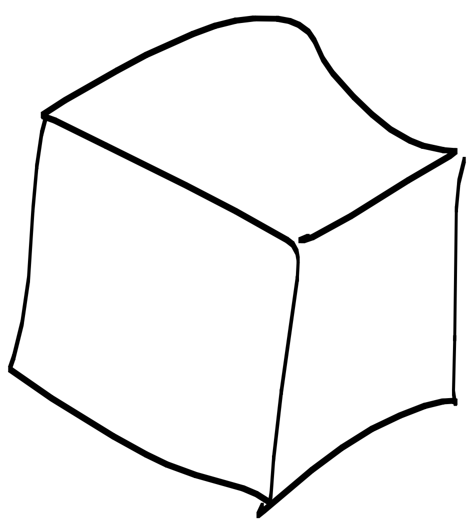
ray tracing

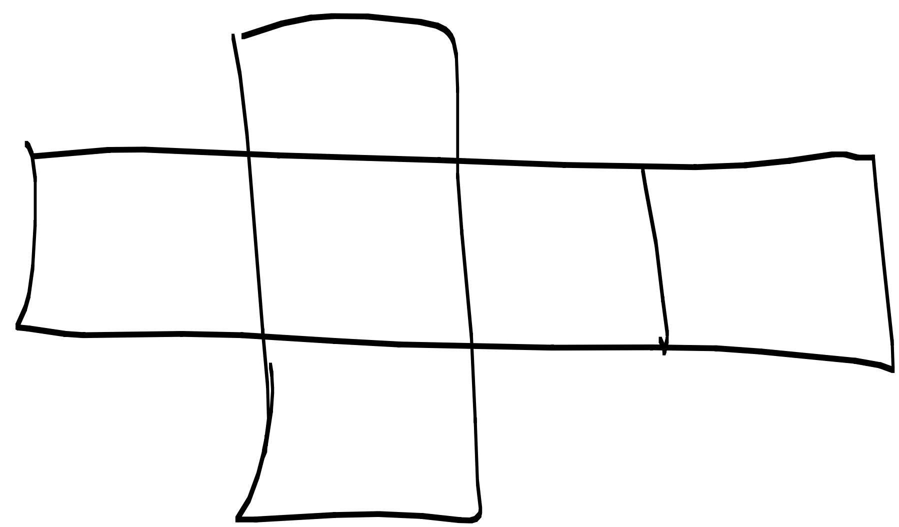


environment mapping

E 18 Q 3

Cube maps have been used in many clever ways in computer graphics. For example, they are used as a *fast lookup* to approximate a unit vector in the direction of some given 3D vector. How could that work?

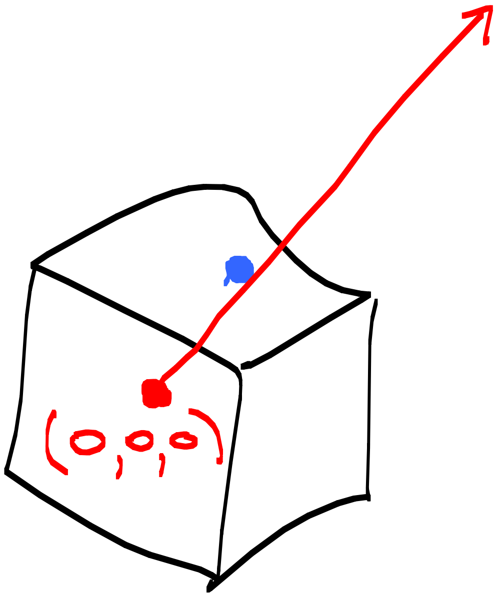




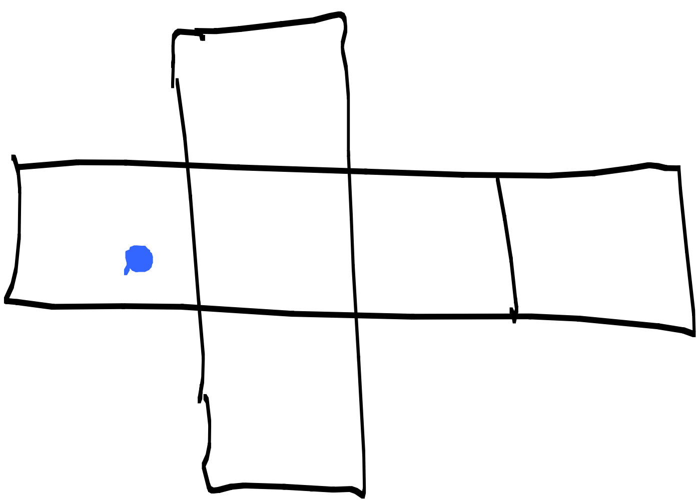
$[-1,1] \times [-1,1] \times [-1,1]$

E 18 Q 3 solution

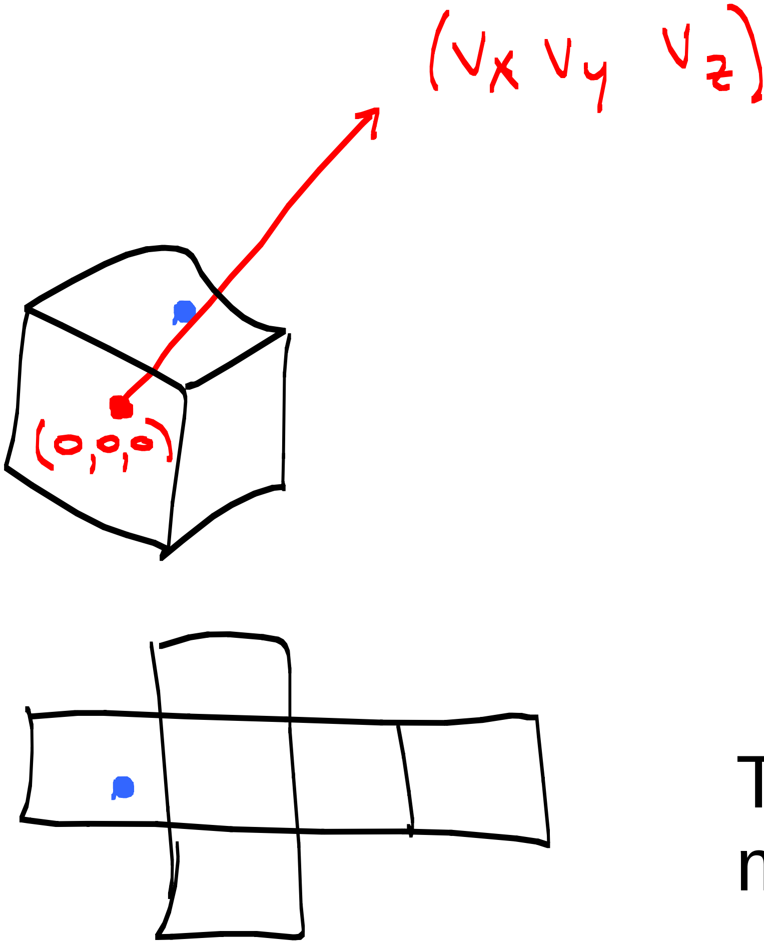
$(v_x \; v_y \; v_z)$



$$\text{round}\left\{ \frac{N(v_x, v_y, v_z)}{\max\{|v_x|\,|v_y|\,|v_z|\}} \right\}$$



What "RGB value" to store at this pixel ?



$$(\hat{V}_x, \hat{V}_y, \hat{V}_z) \equiv \text{round}\left\{ N \frac{(V_x, V_y, V_z)}{\max\{|V_x| |V_y| |V_z|\}} \right\}$$

The "RGB" pixels in the cube map store:

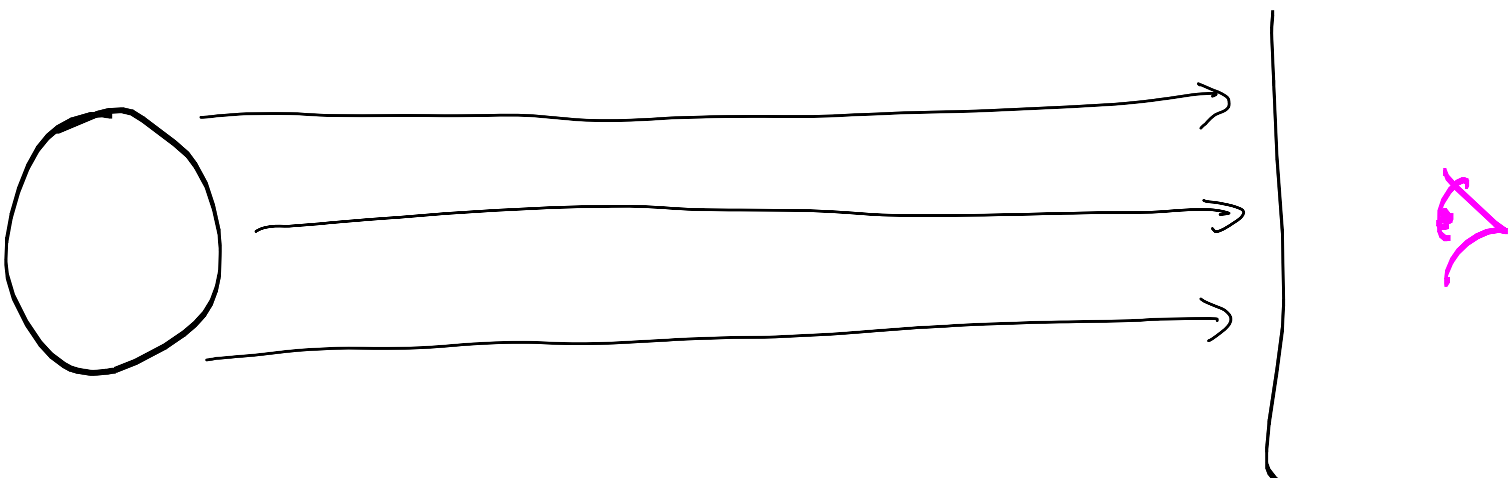
$$\frac{(\hat{V}_x, \hat{V}_y, \hat{V}_z)}{\sqrt{\hat{V}_x^2 + \hat{V}_y^2 + \hat{V}_z^2}}$$

which is the unit vector in the direction of the pixel

E 18 Q 4

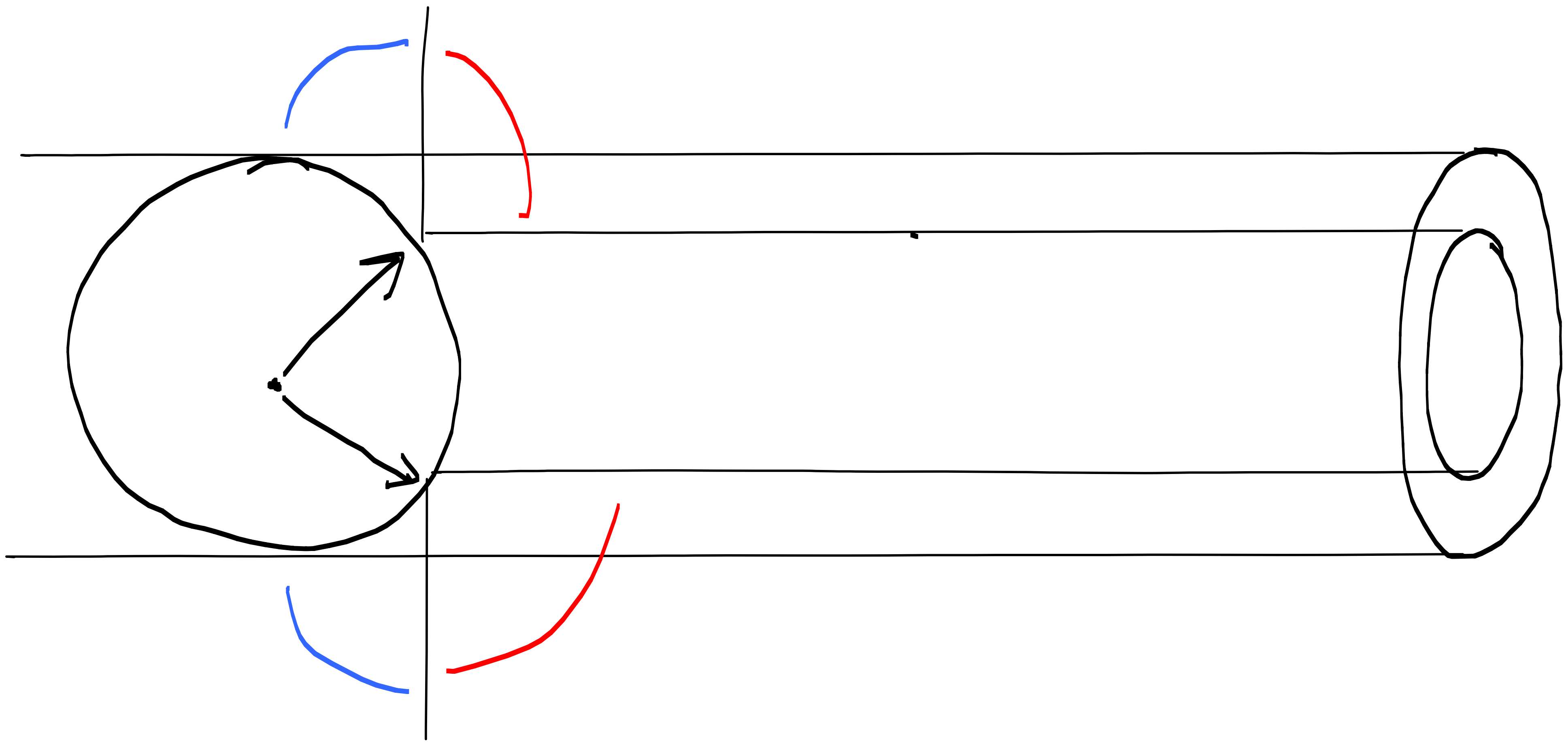
The environment mapping model uses orthographic projection of the mirror sphere onto an image plane.

Does it use more or fewer pixels to represent directions in the environment behind the sphere versus in front of the sphere?



mirror sphere

E 18 Q 4 solution



You can show (see PDF) that the 3D scene beyond the sphere takes as much area in the image as the scene in front of the sphere. However, the environement is severely distorted.

Shadows and Interreflections

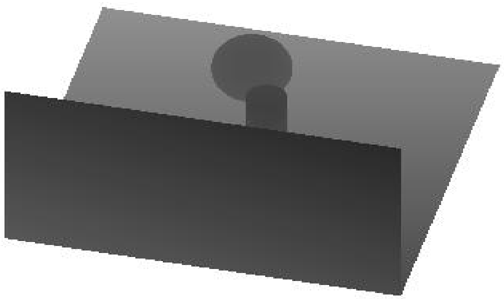
E 19   Q 1

A point light source can either be local (at a finite distance) or at infinity.

For each of these two possibilities, what projection model would be suitable for computing a **shadow map**?

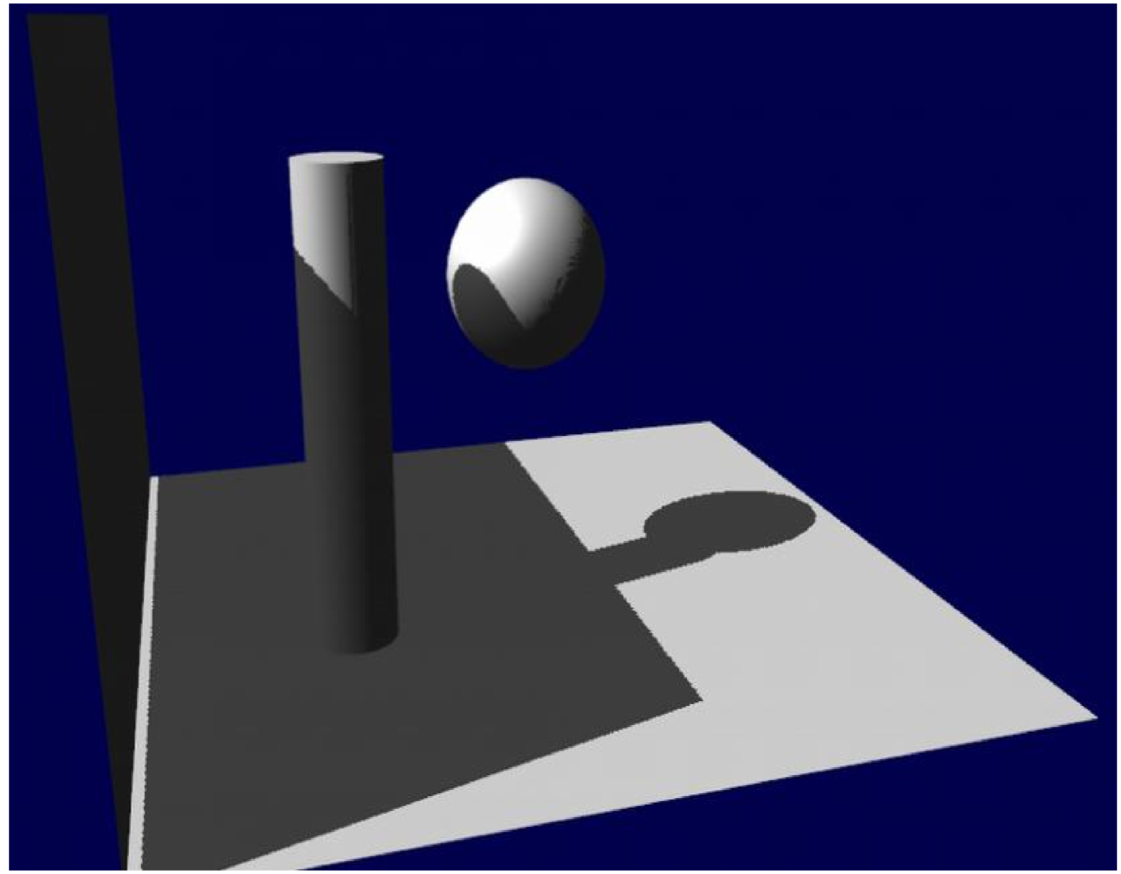
What can you say about the view volume used to compute the shadow map?

E 19   Q 1   (recall what is a shadow map)



$z_{shadow}(x_{shadow}, y_{shadow})$

"shadow map"
light source viewpoint

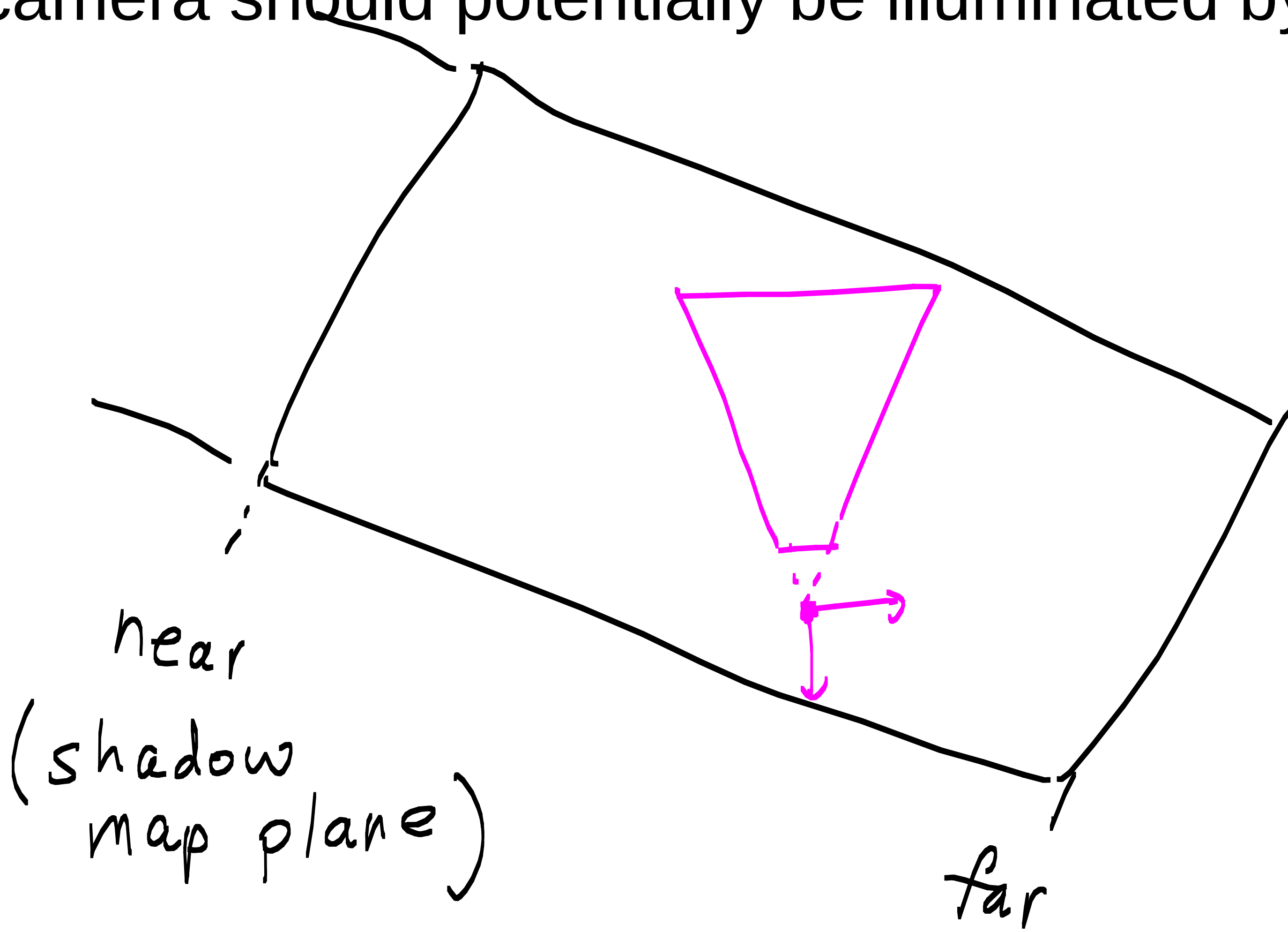


$I(x_p, y_p)$

RGB image (with shadows)
camera viewpoint

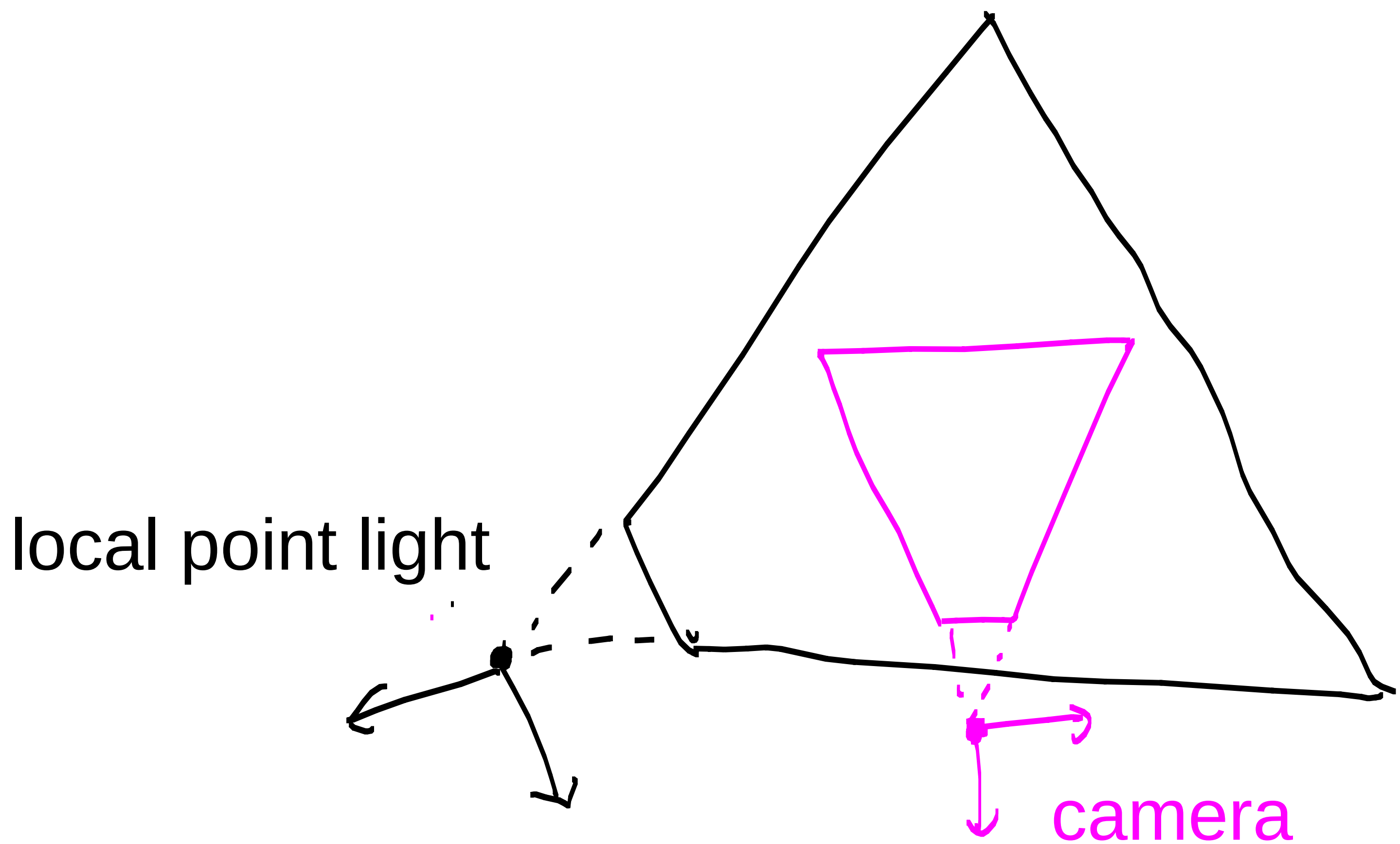
For a point light source at infinity, use an orthographic model.

If the light source is outside the camera's view volume, then the view volume of the shadow map (light source) should contain the camera's view volume since every point visible to the camera should potentially be illuminated by the light source.



For a local point light, use a perspective model.

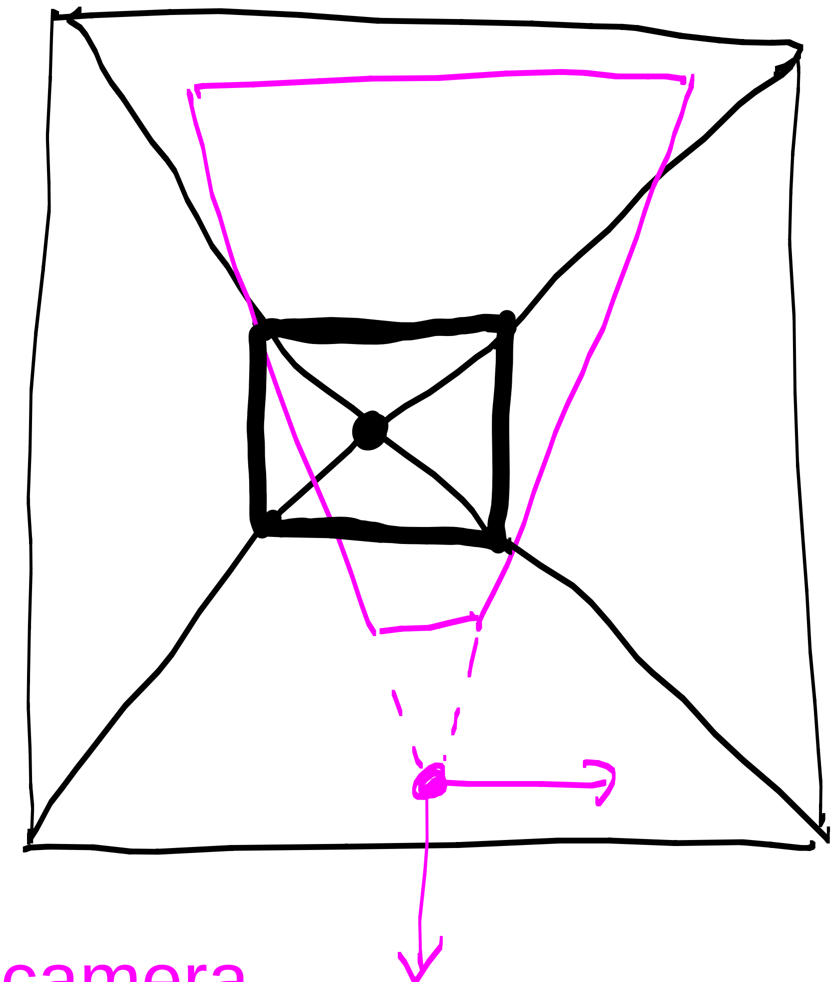
If the light source is outside the camera's view volume, then the view volume of the shadow map (light source) should contain the camera's view volume.
Why? Because every point visible to the camera should potentially be illuminated by the light source.



If, however, the point light is inside the view volume, then you will have to compute multiple shadow maps.

You can use a cube map for the shadow maps.

The (six) view volumes for the cube map should enclose the camera's view volume.



E 19   Q2

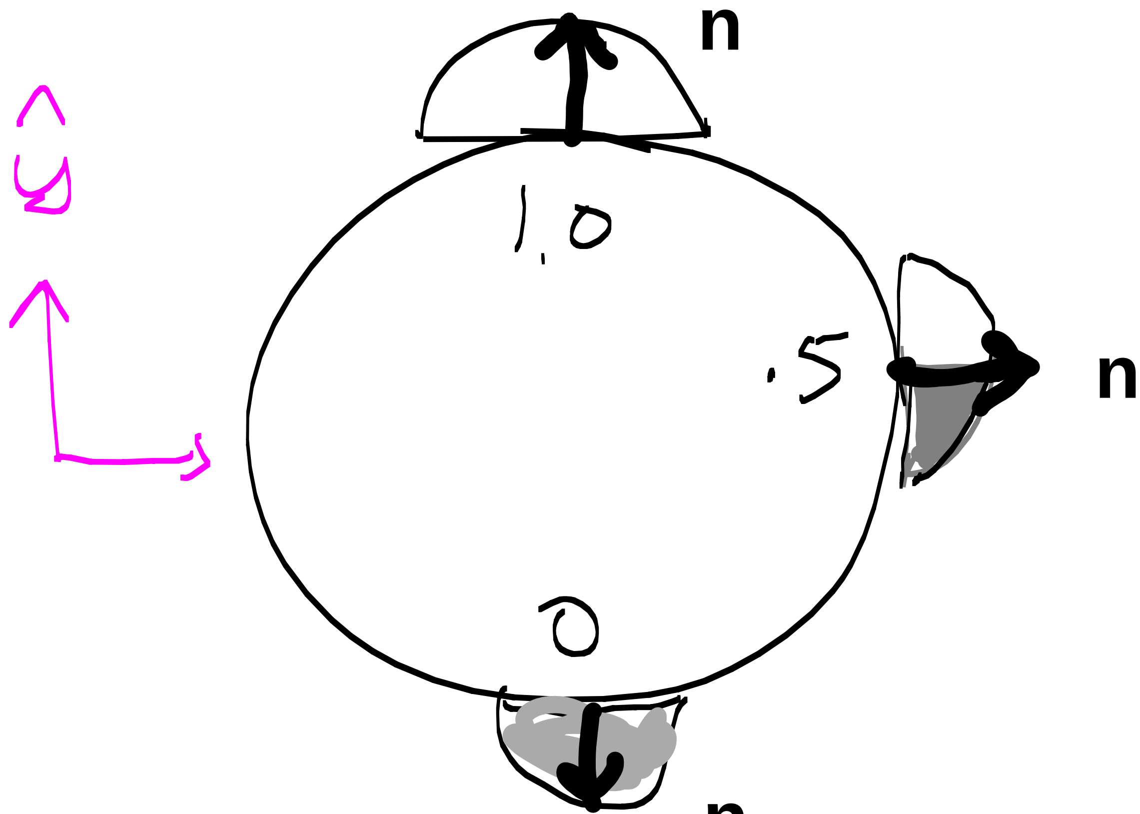
Suppose the scene is illuminated by a *hemispheric uniform* source which is centered in the y direction.   Give a qualitative formula for the diffuse component of shading that accounts for *attached shadow* effects, that is shadows that are due to the surface normal facing away from parts of the sky.

Recall what an attached shadow is for a point source.

E 19 Q2

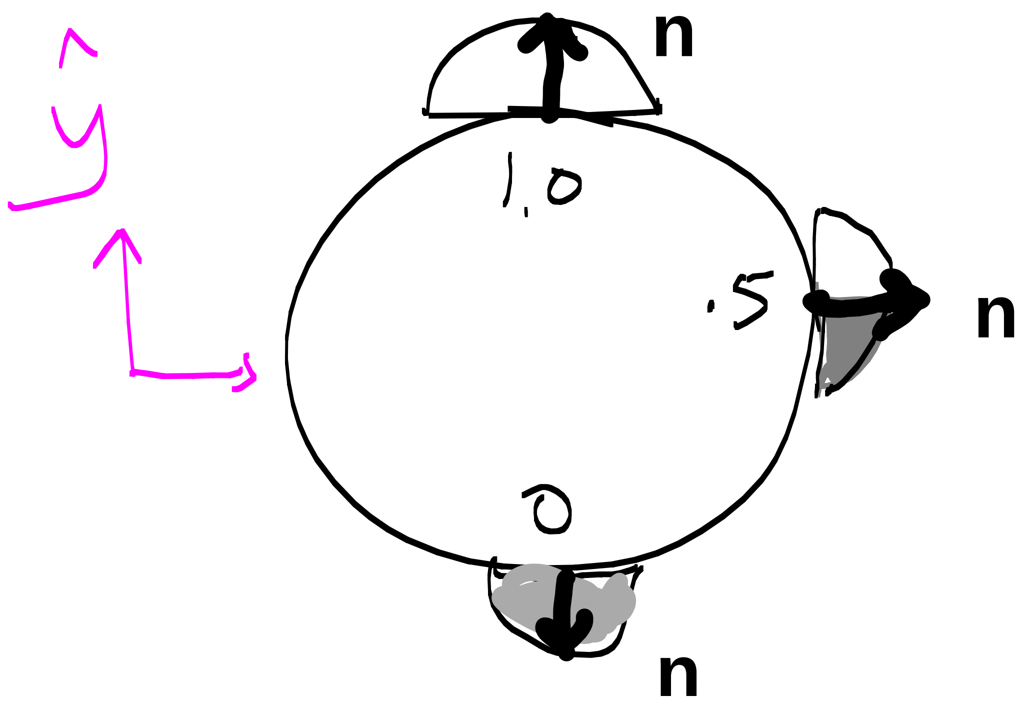
Assume :

- the light source is a uniform distant hemisphere *overhead*
- the fraction of the source is determined only by the surface normal n.



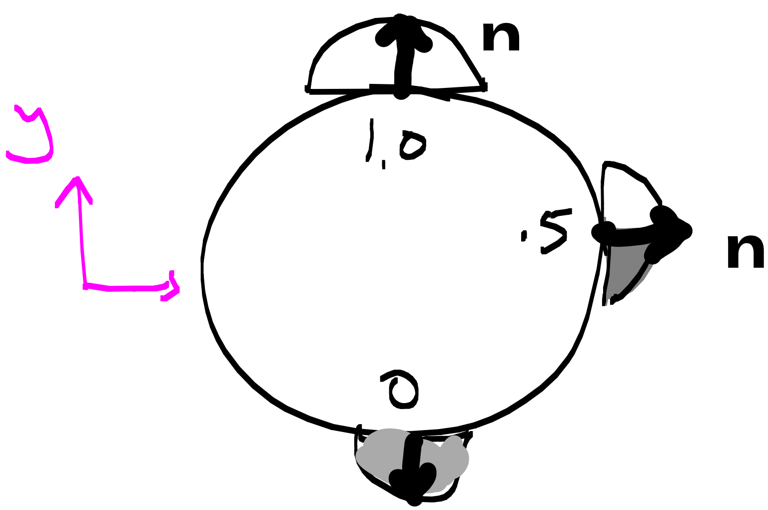
E 19 Q2 (Solution)

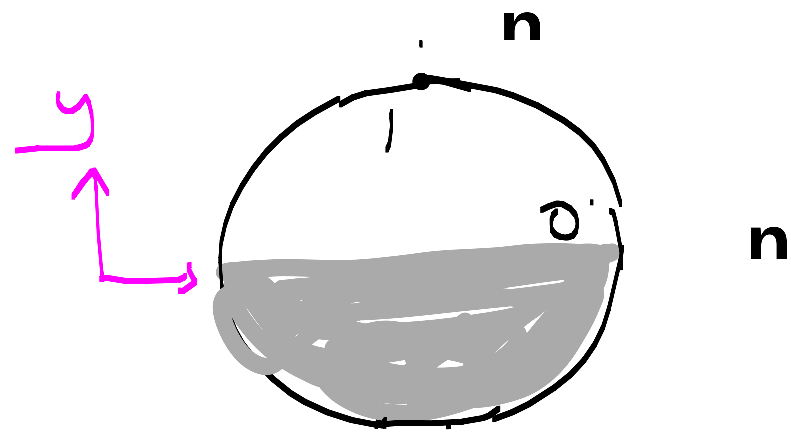
$$I_{diffuse}(x) = \frac{1 + n(x)\cdot \hat{y}}{2}$$



Q: What is the differences of this model and "sunny day" ?

A: Both depend on the surface normal. But the sunny day





$$\frac{1 + n(x) \cdot y}{2}$$

$$\max(n(x) \cdot y, 1)$$

E 19 Q4

Surfaces can reflect light to each other.

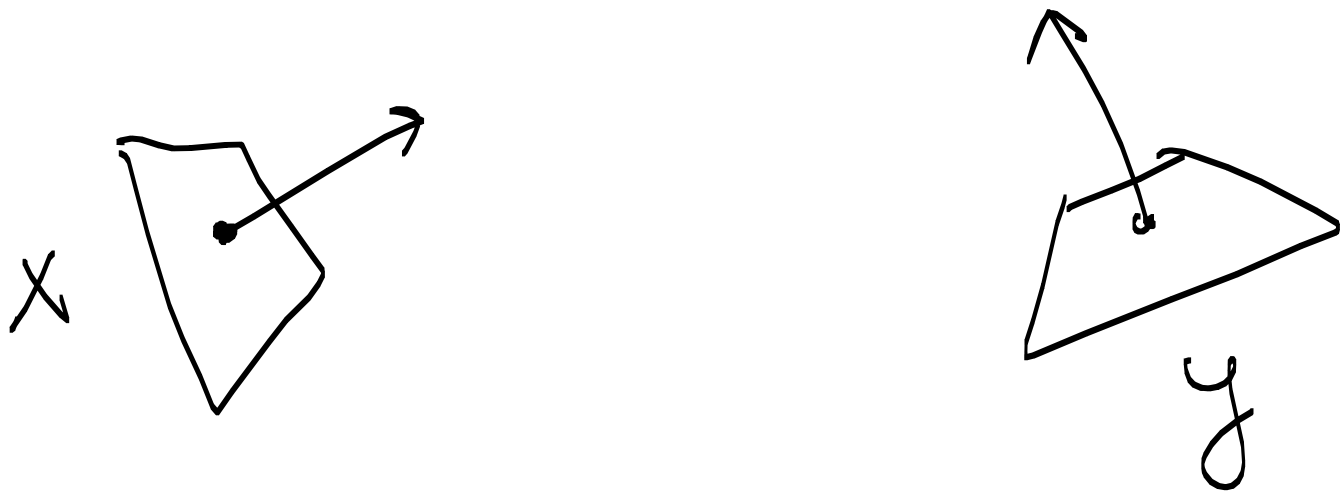
The amount can be represented by a function F(x,y) which serves a matrix element, where x and y are vertices in the scene.

How does F(x,y) depend on the properties at x and y ?

$$I(x) = I_{direct}(x) + \sum_{y} F(x,y)\, I(y)$$

E 19 Q4 (Solution)

The idea here is that y acts as a light source for x.
What determines how strong a light source it is ?



- How far is y from x ? (similar to point source distance effect)

- Is y in the direction of the surface normal of x ?

- Is the surface normal of y in the direction of the vector between x and y ? (how directly is y facing x ?)

- Are x and y visible from each other ?

Two more weeks to go

20. image compositing (slides)
    alpha, over, blending, pulling a matte
21. volume rendering (slides)
    blending N layers, fog, transfer functions
22. review of selected exercises (12, 16-19)

Part 4: Image Capture and Display

23. color
    trichromacy, display matching
24. image capture
    aperture, exposure, HDR, Weber's Law
25. image display
    gamma correction, tone mapping
26. review of selected exercises (20-21, 23-25)