

COMP 546

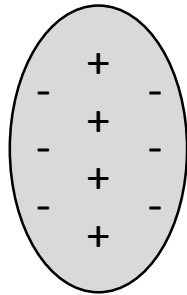
Lecture 6

orientation 2: complex cells  
binocular cells

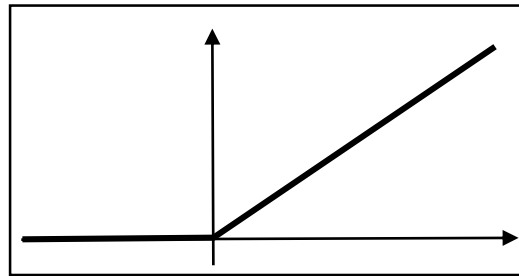
Tues. Jan. 30, 2018

# Recall last lecture: simple Cell

Linear response

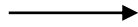
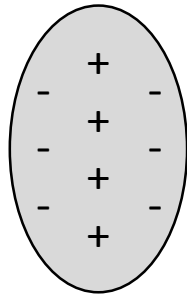


half wave rectification

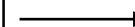
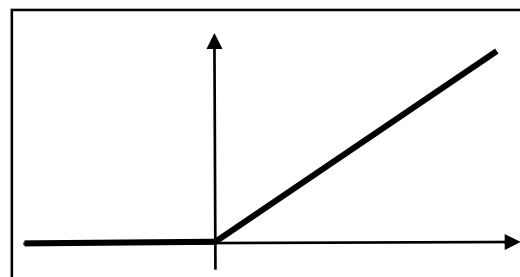
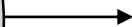
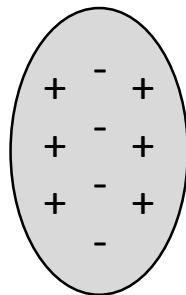
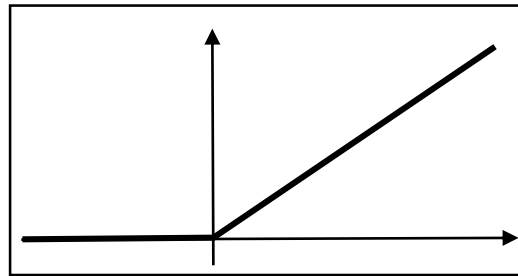


# Recall last lecture: simple Cell

Linear response

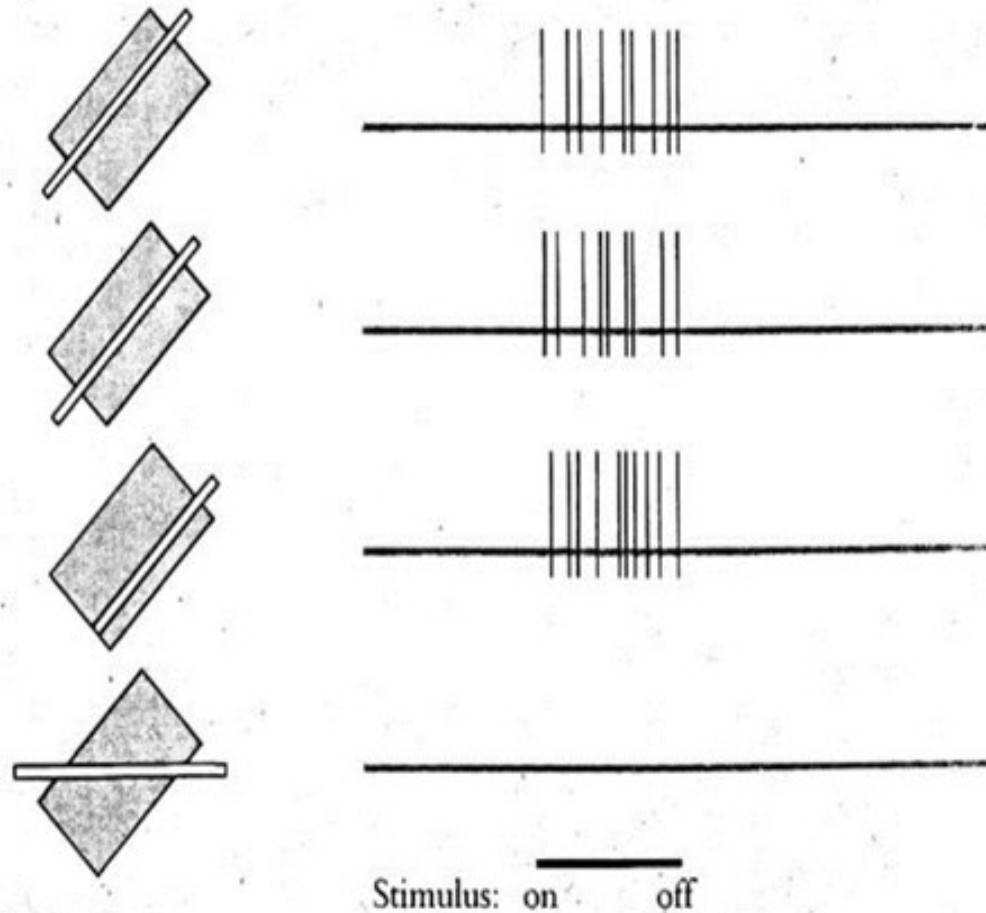


half wave rectification

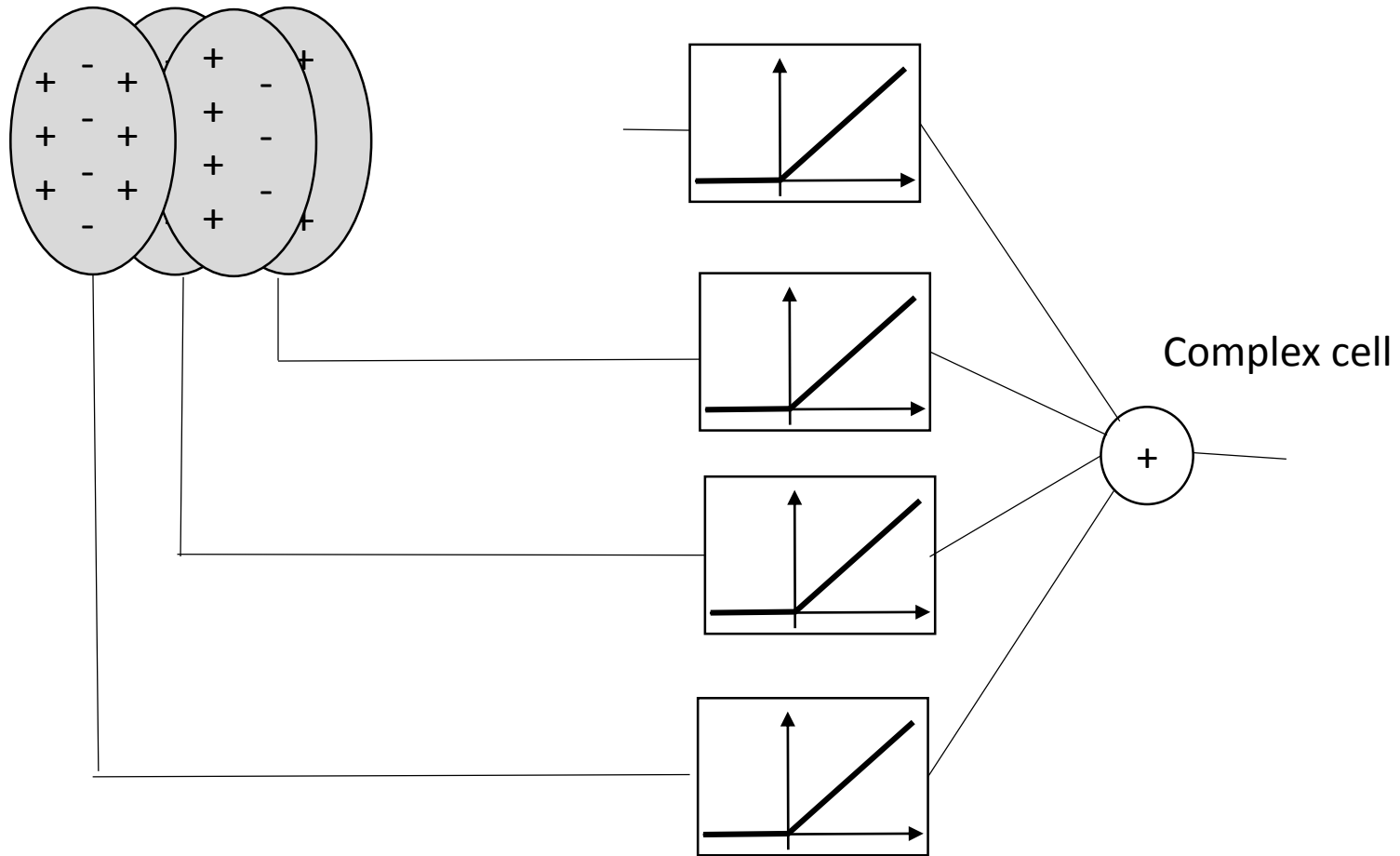


# “Complex Cell” (Hubel and Wiesel)

Responds to preferred orientation of line *anywhere* in receptive field.

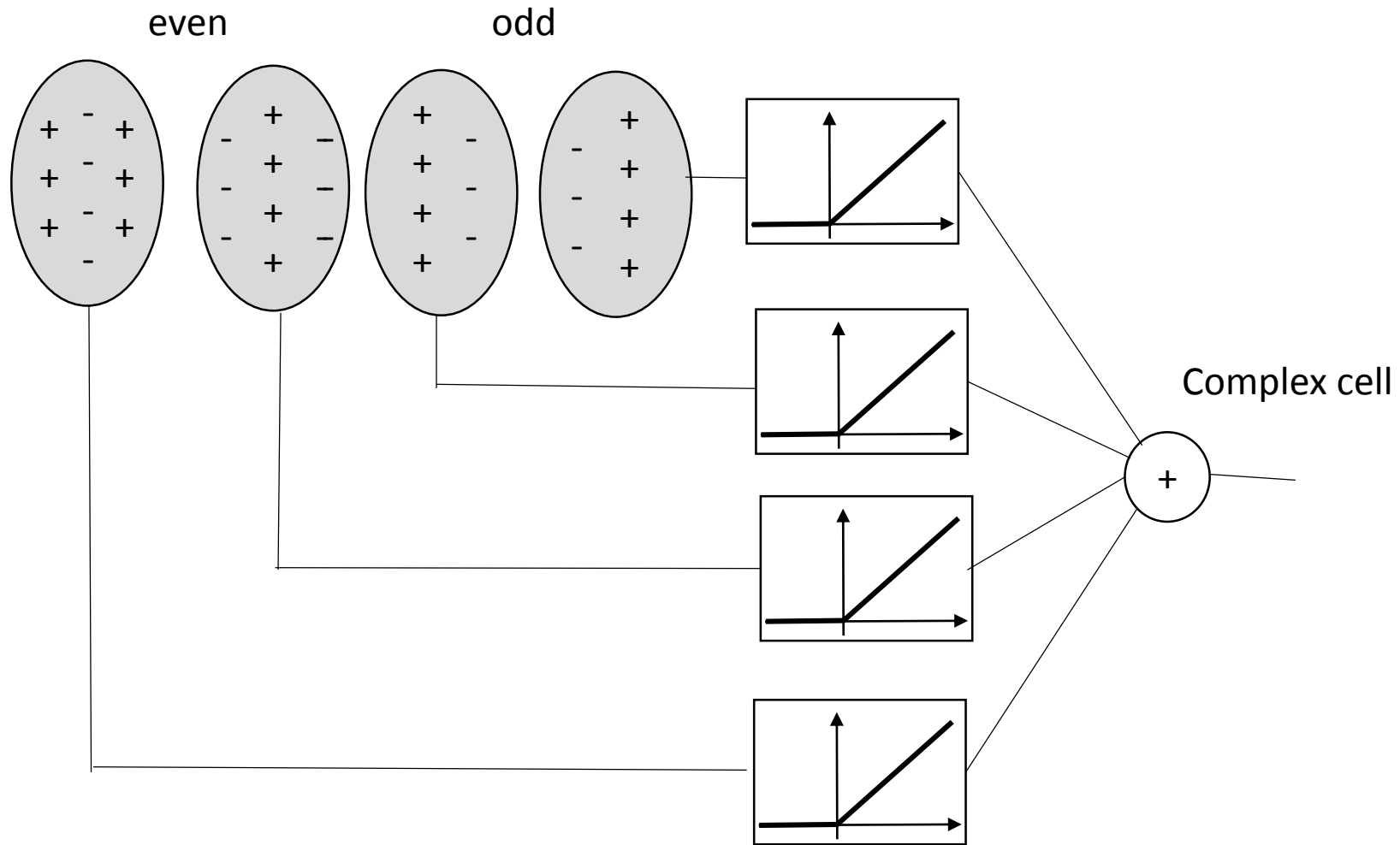


# How to construct a complex cell? (1)



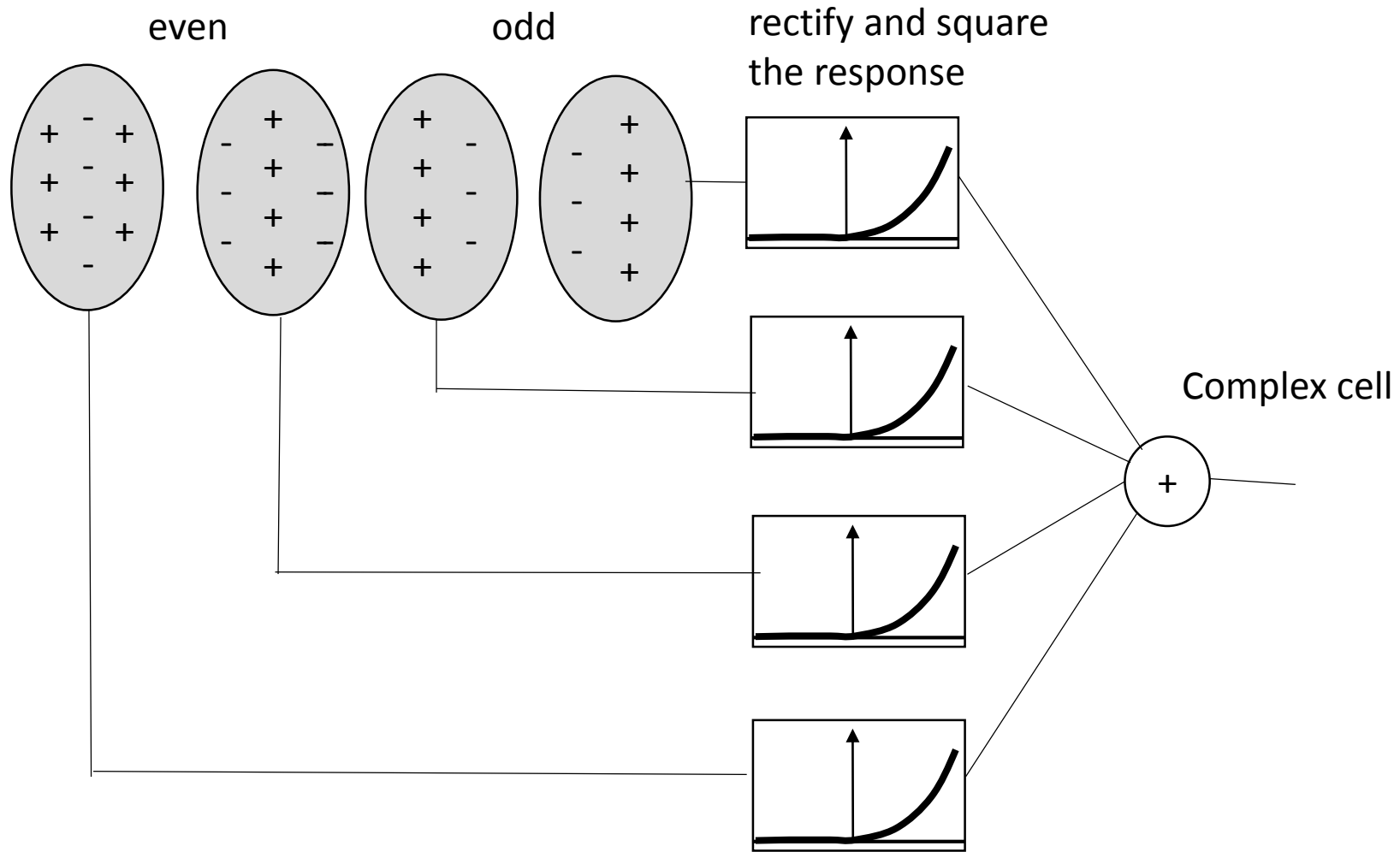
Use several simple cells with common orientation and *neighboring receptive field locations*. If we sum up their rectified responses then we get a response to image structure of that orientation anywhere in the overlapping receptive fields.

# How to construct a complex cell? (2)



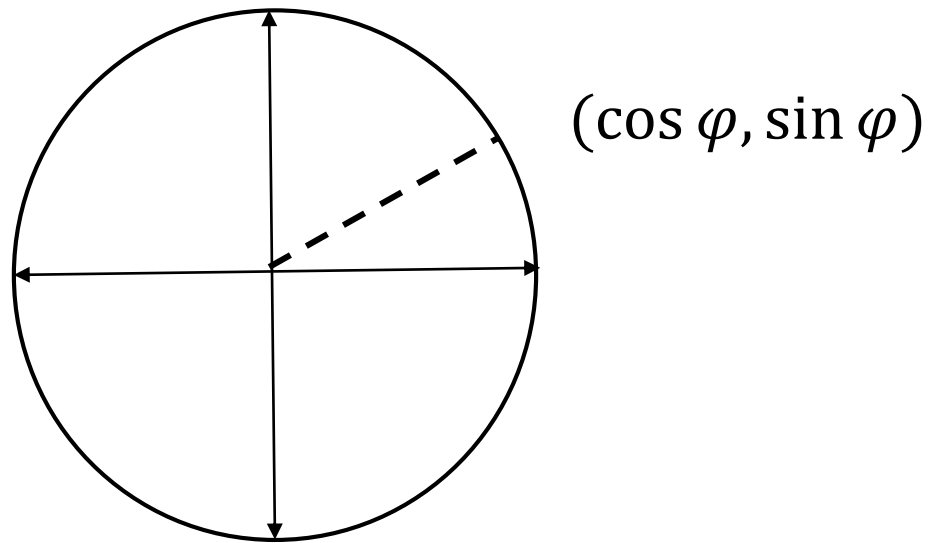
Now suppose these even cell and odd cells have *the same receptive field locations* (perfect overlap). Again sum up their rectified responses and the result is a response anywhere in the receptive field.

# How to construct a complex cell? (3)



This is the same as the last model but now we square the positive values. This model is more commonly used than model (2) and so we'll use this one.

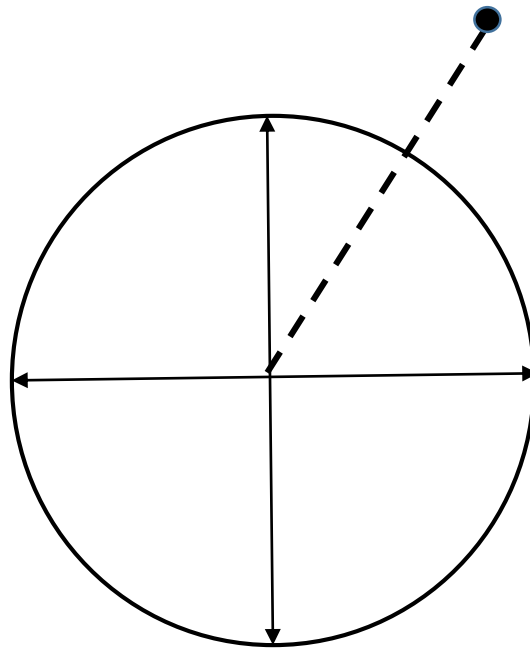
# Unit circle



$$\cos^2 \varphi + \sin^2 \varphi = 1$$



# Model of a Complex Cell (3)

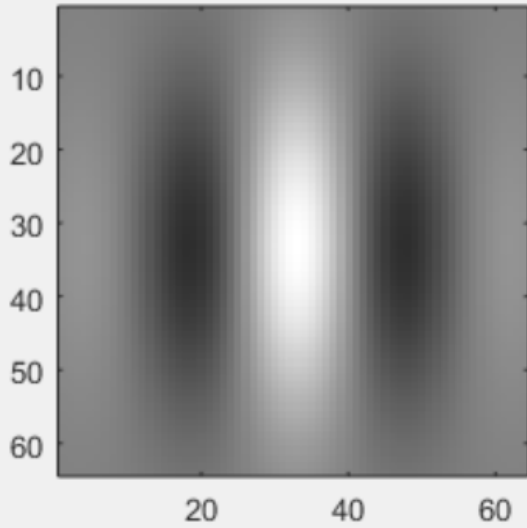


$$\begin{aligned} & ( \langle \cos Gabor(x, y), I(x, y) \rangle, \\ & \langle \sin Gabor(x, y), I(x, y) \rangle ) \end{aligned}$$

The response to an image  $I(x, y)$  is modelled as the Euclidean *length* of the vector, i.e. L2 norm

$$\| ( \langle \cos Gabor(x, y), I(x, y) \rangle, \langle \sin Gabor(x, y), I(x, y) \rangle ) \|_2$$

cosine Gabor



sine Gabor

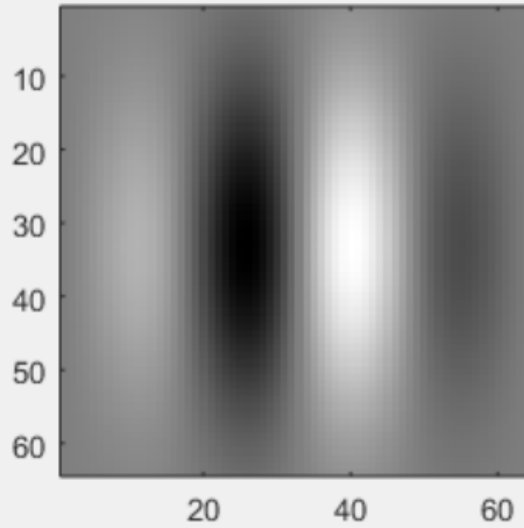
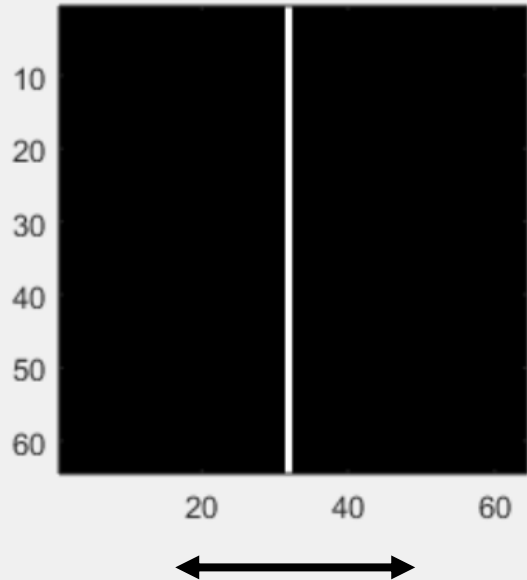
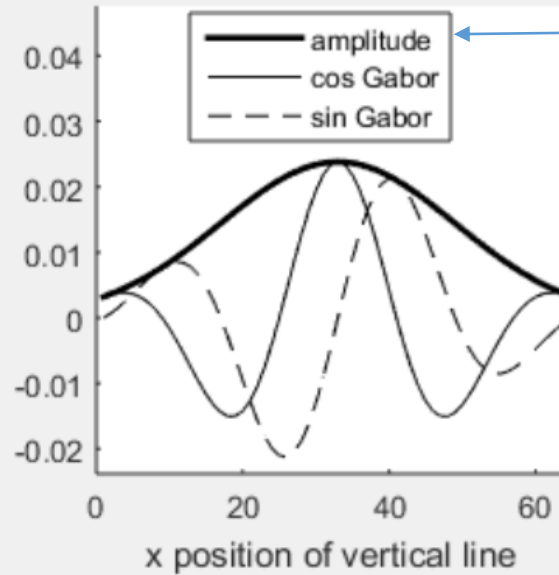


image of a vertical line



responses

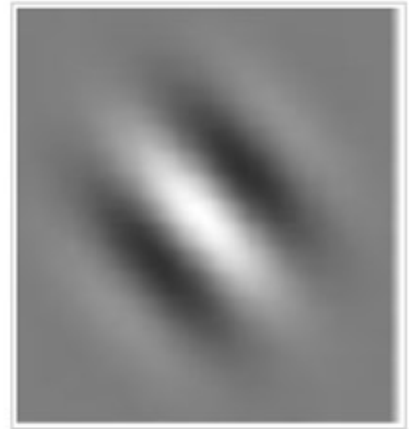
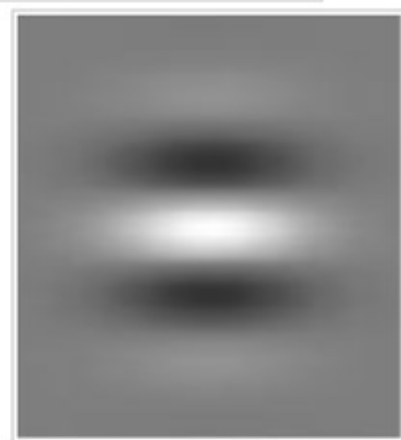
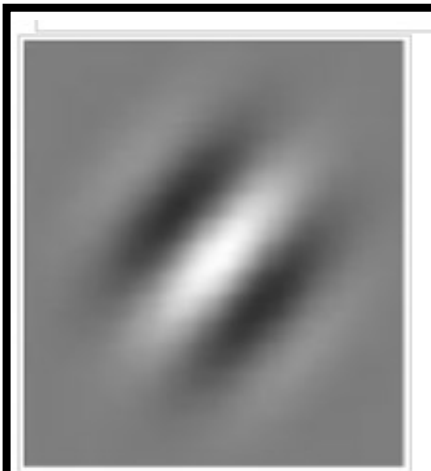
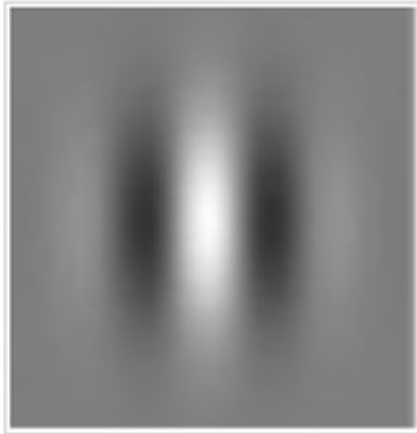


Complex cell response modelled as the length of 2D vector:

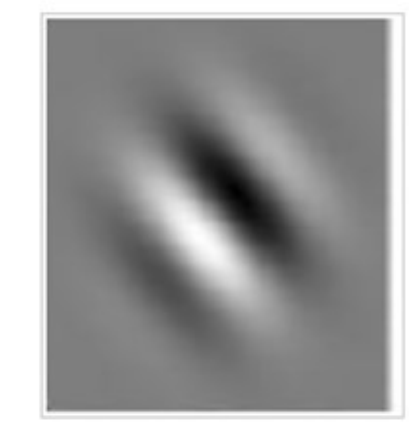
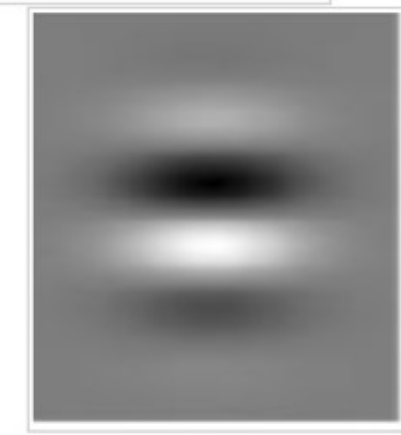
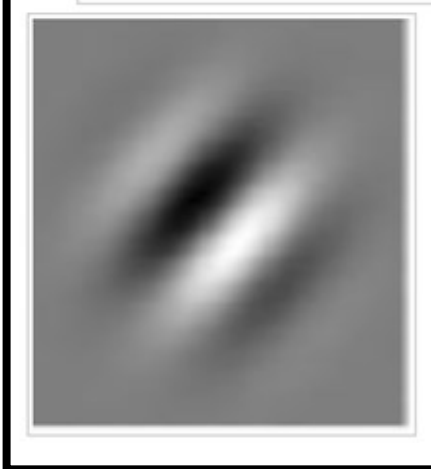
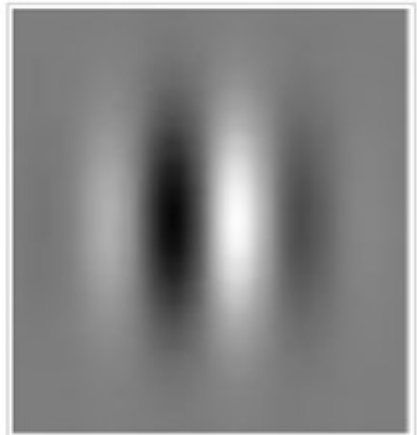
$$(\langle \cosGabor(x, y), I(x, y) \rangle, \langle \sinGabor(x, y), I(x, y) \rangle)$$

We can model complex cells of any orientation.

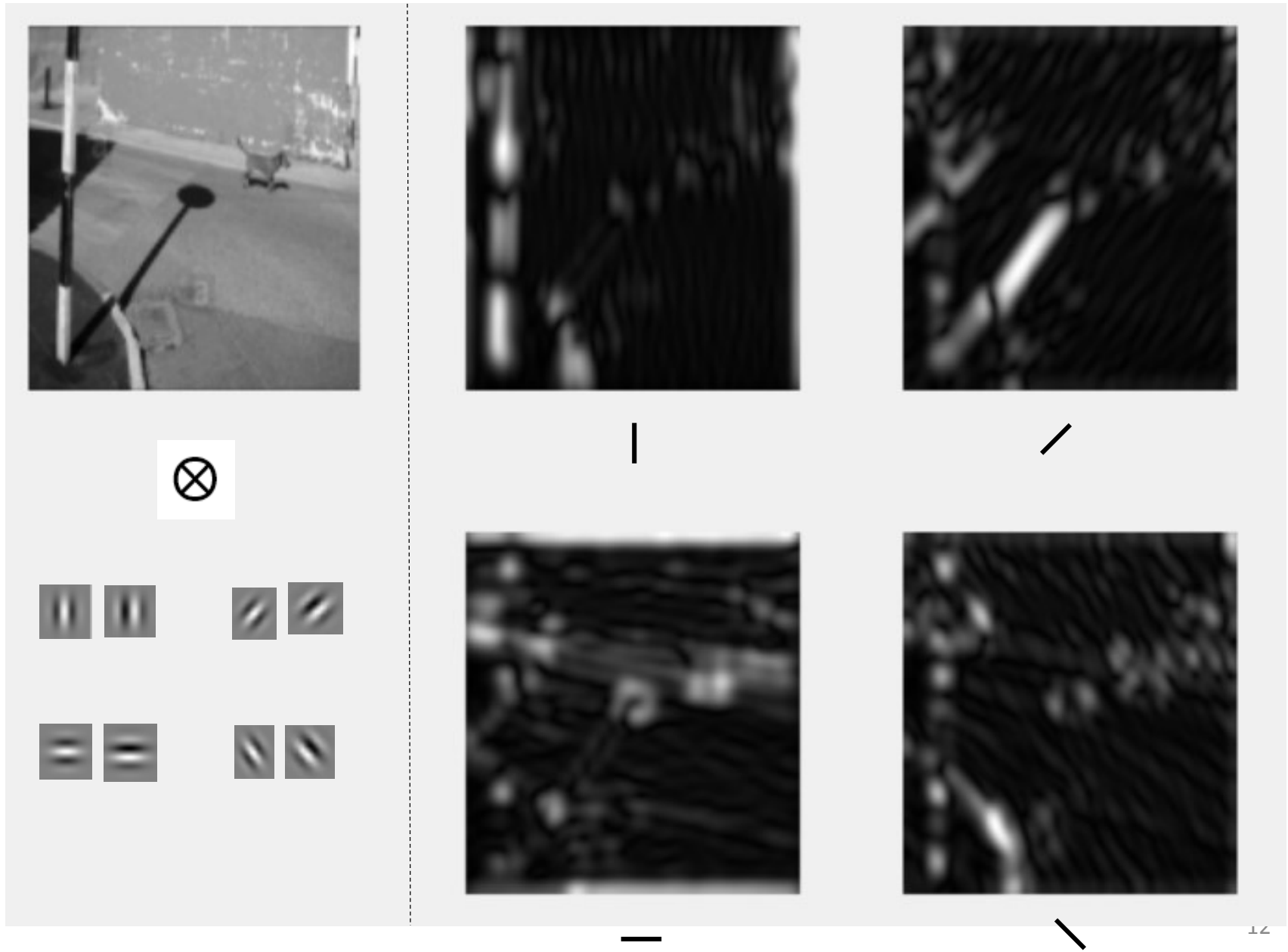
Cosine Gabor



Sine Gabor



# Example: image cross correlated with four complex cells



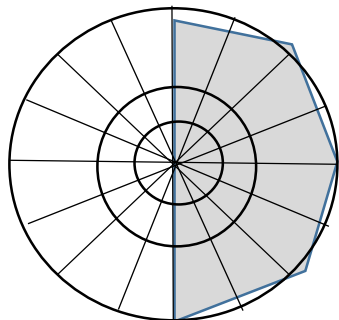
COMP 546

Lecture 6

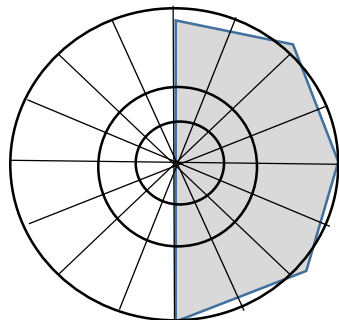
orientation 2: complex cells  
binocular cells

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$\phi = 90$



$\phi = 90$

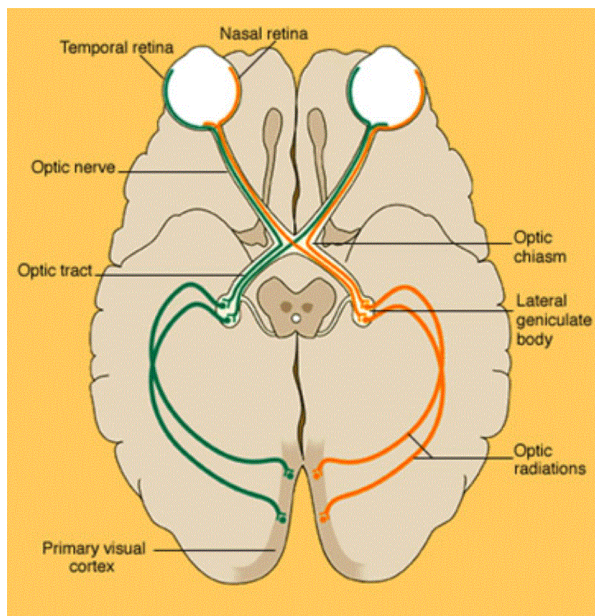


$\phi = -90$

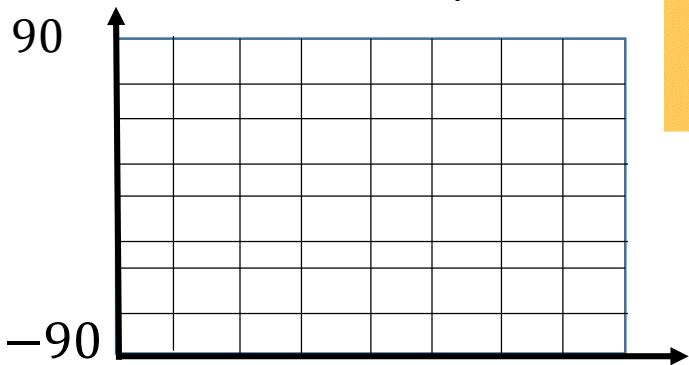
$\phi = -90$

Left halves of retina map to V1 in left hemisphere

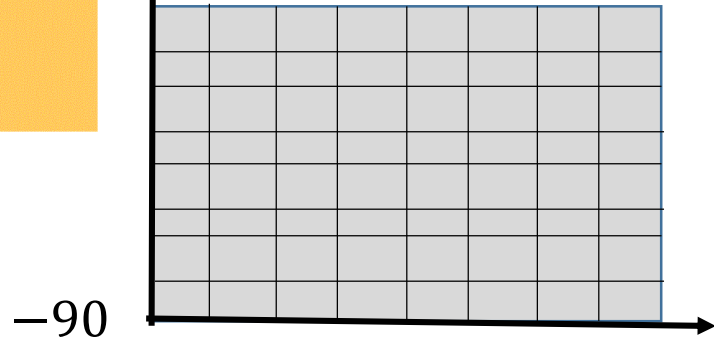
Right halves of retina map to V1 in right hemisphere



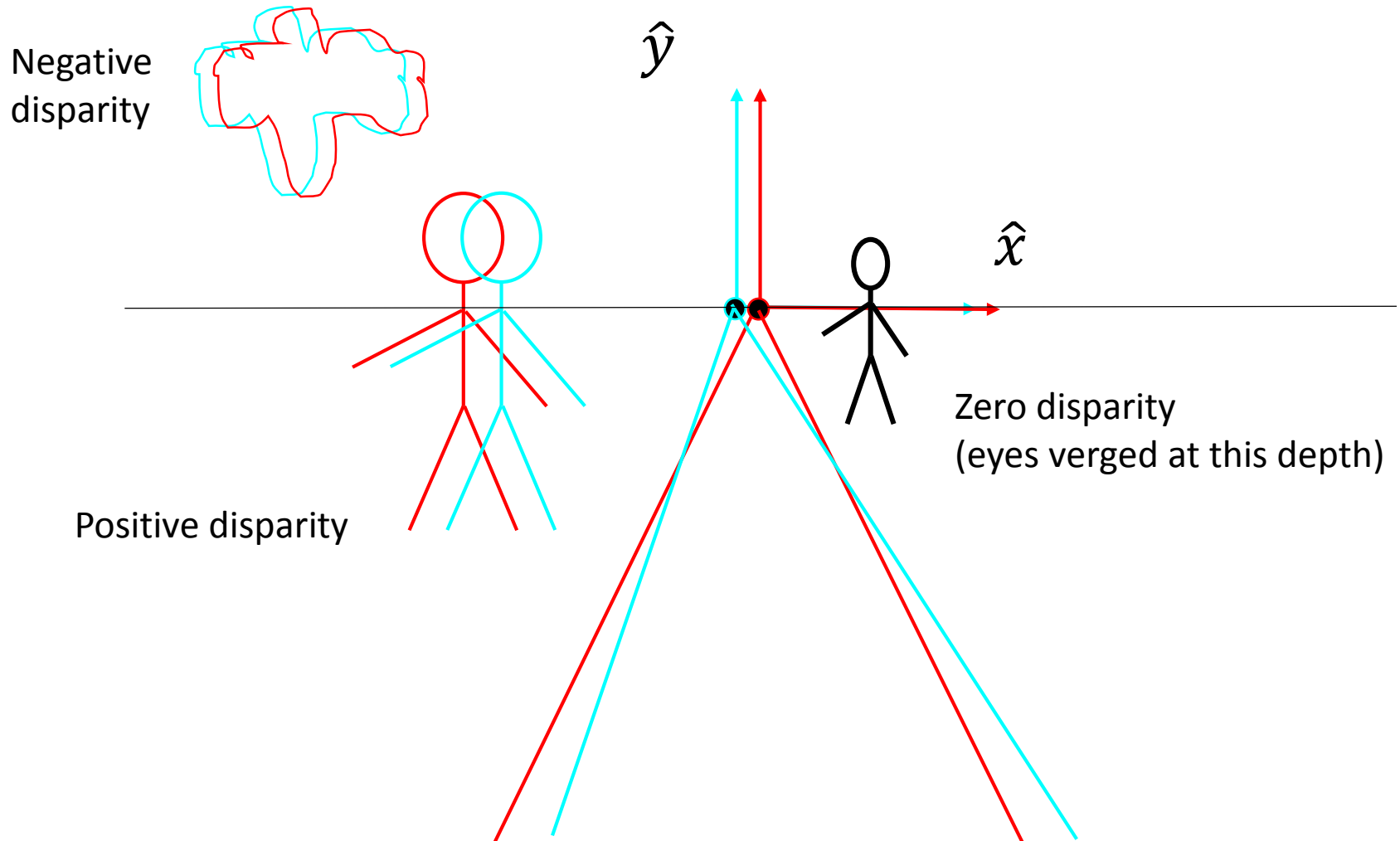
$\phi =$   
eccentricity



$\phi =$   
eccentricity



# Superimposed left and right eye images



# How to estimate binocular disparity ?

*Computer vision-ish* approach:

For each  $(x_0, y_0)$ , find disparity value  $d$  that minimizes:

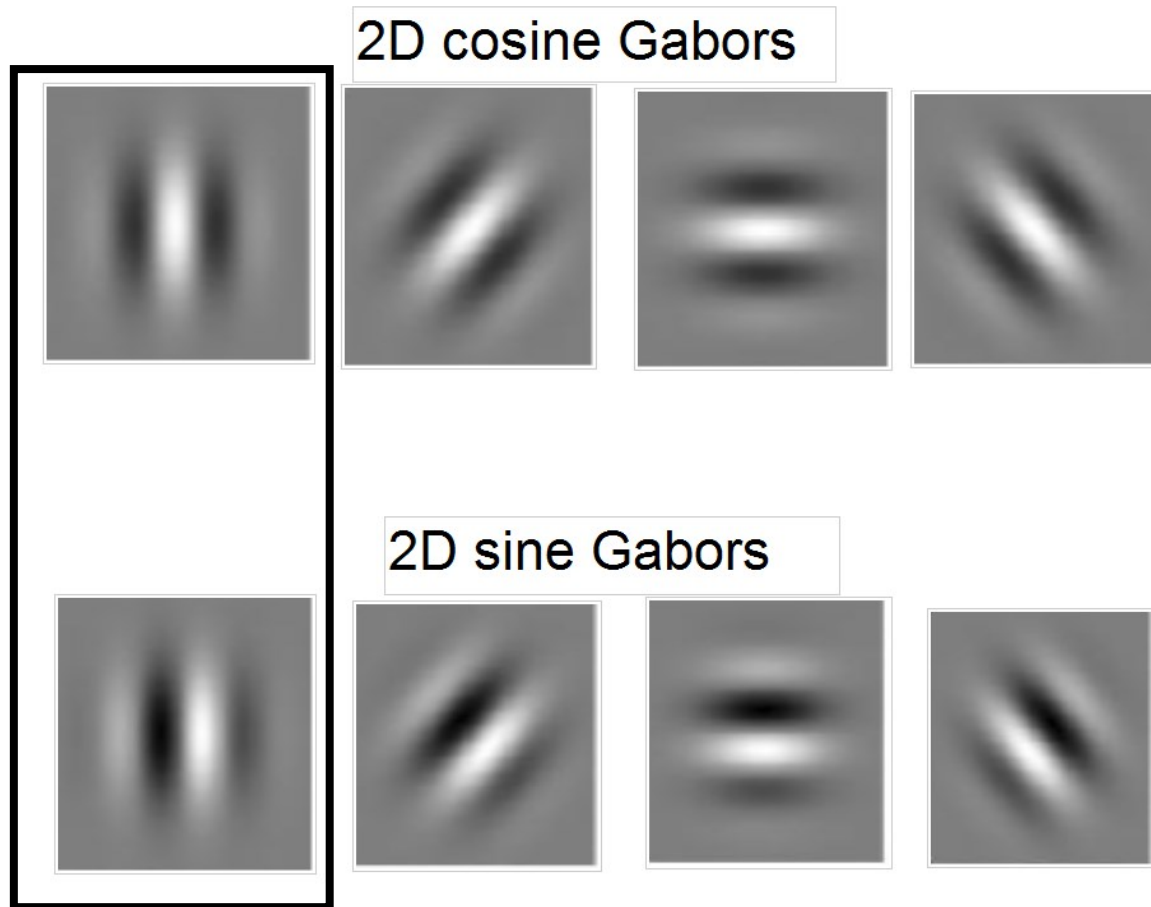
$$\sum_{x,y} ( I_{left}(x + d, y) - I_{right}(x, y) )^2$$

where sum is over a neighborhood of  $(x_0, y_0)$ .

i.e. Shift the left image to undo the disparity and register the left and right images.



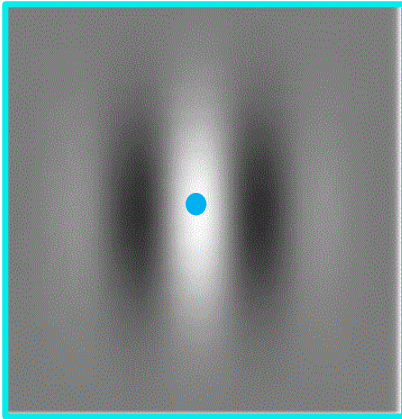
# How to build 'disparity tuned' binocular cells ?



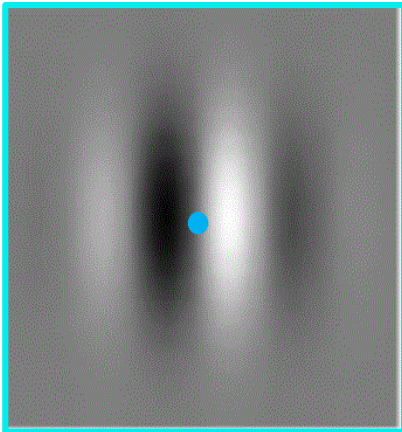
We use vertically oriented cells only.

Left eye

$$Gabor(x - x_0 - d, y - y_0)$$



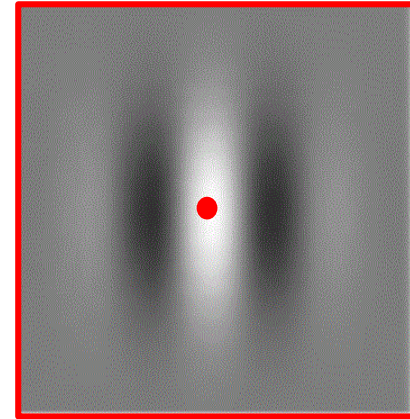
$$(x_0 + d, y_0)$$



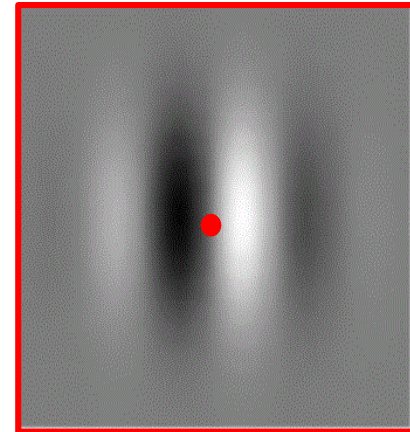
$$(x_0 + d, y_0)$$

Right eye

$$Gabor(x - x_0, y - y_0)$$



$$(x_0, y_0)$$



$$(x_0, y_0)$$

Idea 1: (analogous to computer vision)

To compute disparity at  $(x_0, y_0)$ , find the  $d$  that *minimizes*:

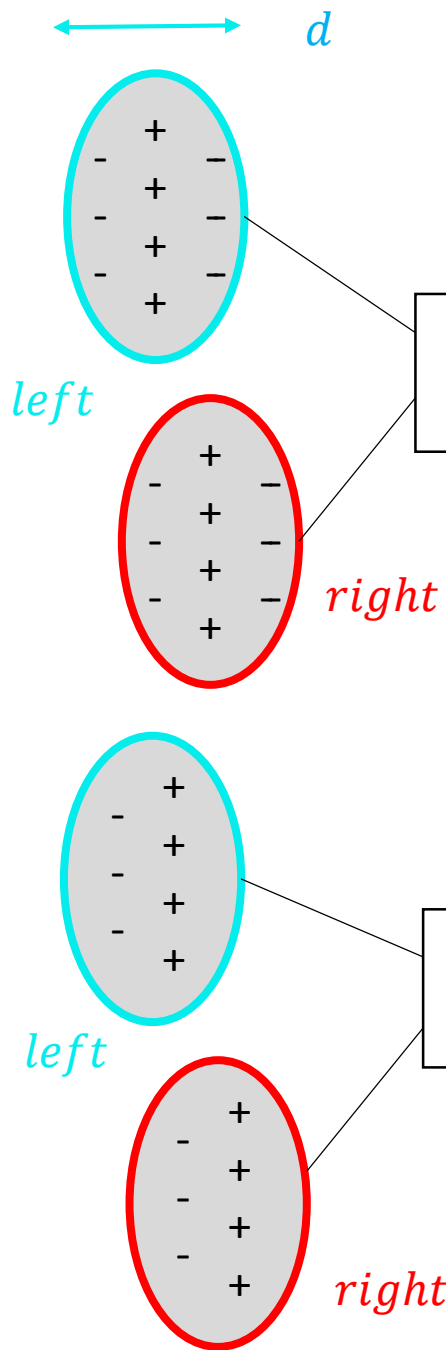
$$\begin{aligned} & \left( \langle \cos Gabor(x - x_0 - d, y - y_0), I_{left}(x, y) \rangle \right. \\ - & \left. \langle \cos Gabor(x - x_0, y - y_0), I_{right}(x, y) \rangle \right)^2 \\ & + \\ & \left( \langle \sin Gabor(x - x_0 - d, y - y_0), I_{left}(x, y) \rangle \right. \\ - & \left. \langle \sin Gabor(x - x_0, y - y_0), I_{right}(x, y) \rangle \right)^2 \end{aligned}$$

Idea 1: (analogous to computer vision)

To compute disparity at  $(x_0, y_0)$ , find the  $d$  that *minimizes*:

$$\begin{aligned} & \left( \langle \cos Gabor(x - x_0 - d, y - y_0), I_{left}(x, y) \rangle \right. \\ - & \left. \langle \cos Gabor(x - x_0, y - y_0), I_{right}(x, y) \rangle \right)^2 \\ & + \\ & \left( \langle \sin Gabor(x - x_0 - d, y - y_0), I_{left}(x, y) \rangle \right. \\ - & \left. \langle \sin Gabor(x - x_0, y - y_0), I_{right}(x, y) \rangle \right)^2 \end{aligned}$$

If  $I_{left}(x + d, y) = I_{right}(x, y)$  for all  $(x, y)$  in receptive fields, then the minimum should be 0.



Let  $y$  position of these cells be the same and  $x$  positions of left and right be separated by  $d$ . This binocular cell is tuned to disparity  $d$ .

Idea 1 (computer vision):

find the disparity  $d$  that *minimizes the squared differences*:

$$(c_l - c_r)^2 + (s_l - s_r)^2$$

$$= c_l^2 + c_r^2 + s_l^2 + s_r^2 - 2(c_l c_r + s_l s_r)$$

where  $c_l$  and  $s_l$  depend on  $d$ .

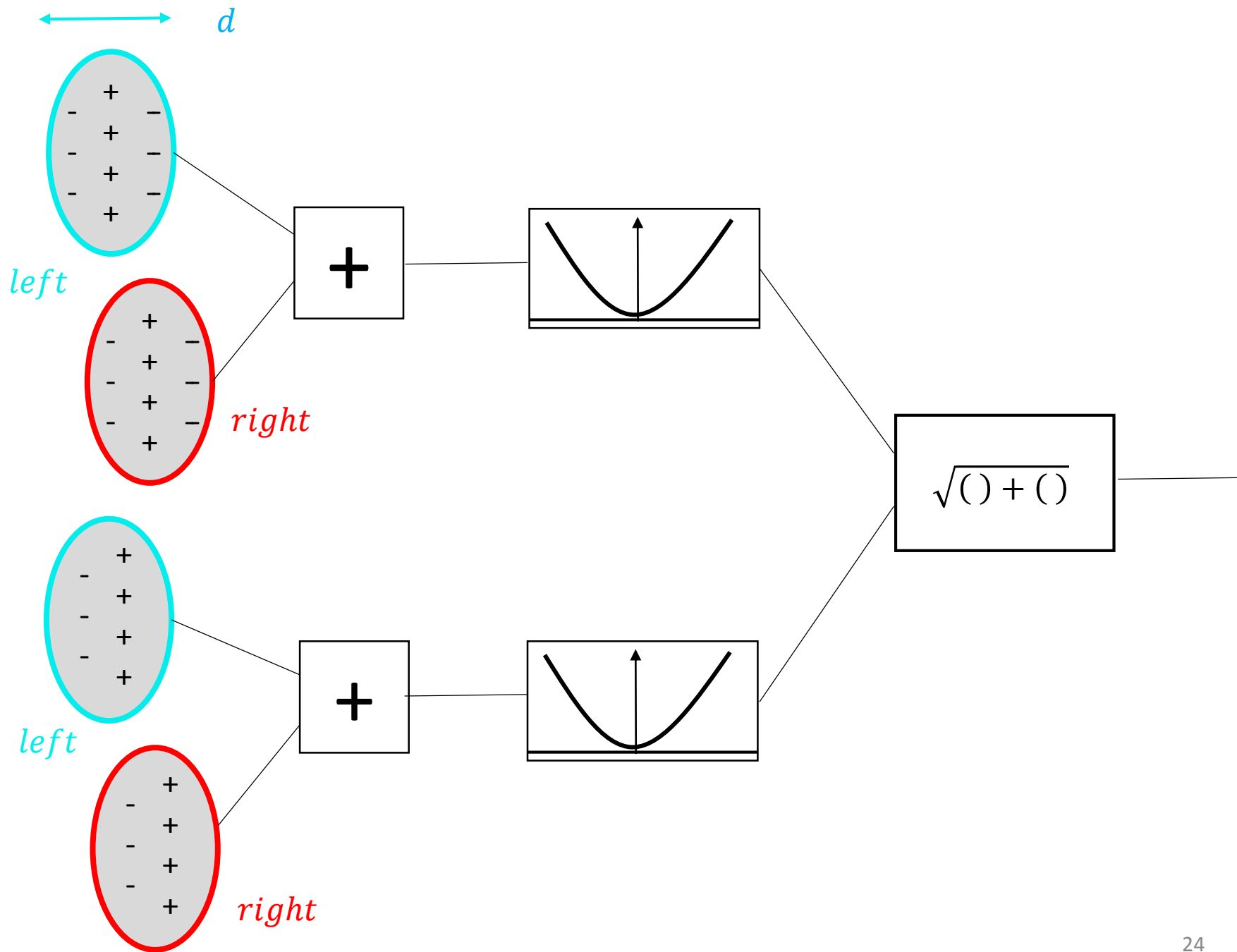
Idea 2 (biological vision):

find the shift  $d$  that *maximizes the squared sums* :

$$(c_l + c_r)^2 + (s_l + s_r)^2$$

$$= c_l^2 + c_r^2 + s_l^2 + s_r^2 + 2(c_l c_r + s_l s_r)$$

where  $c_l$  and  $s_l$  depend on  $d$ .

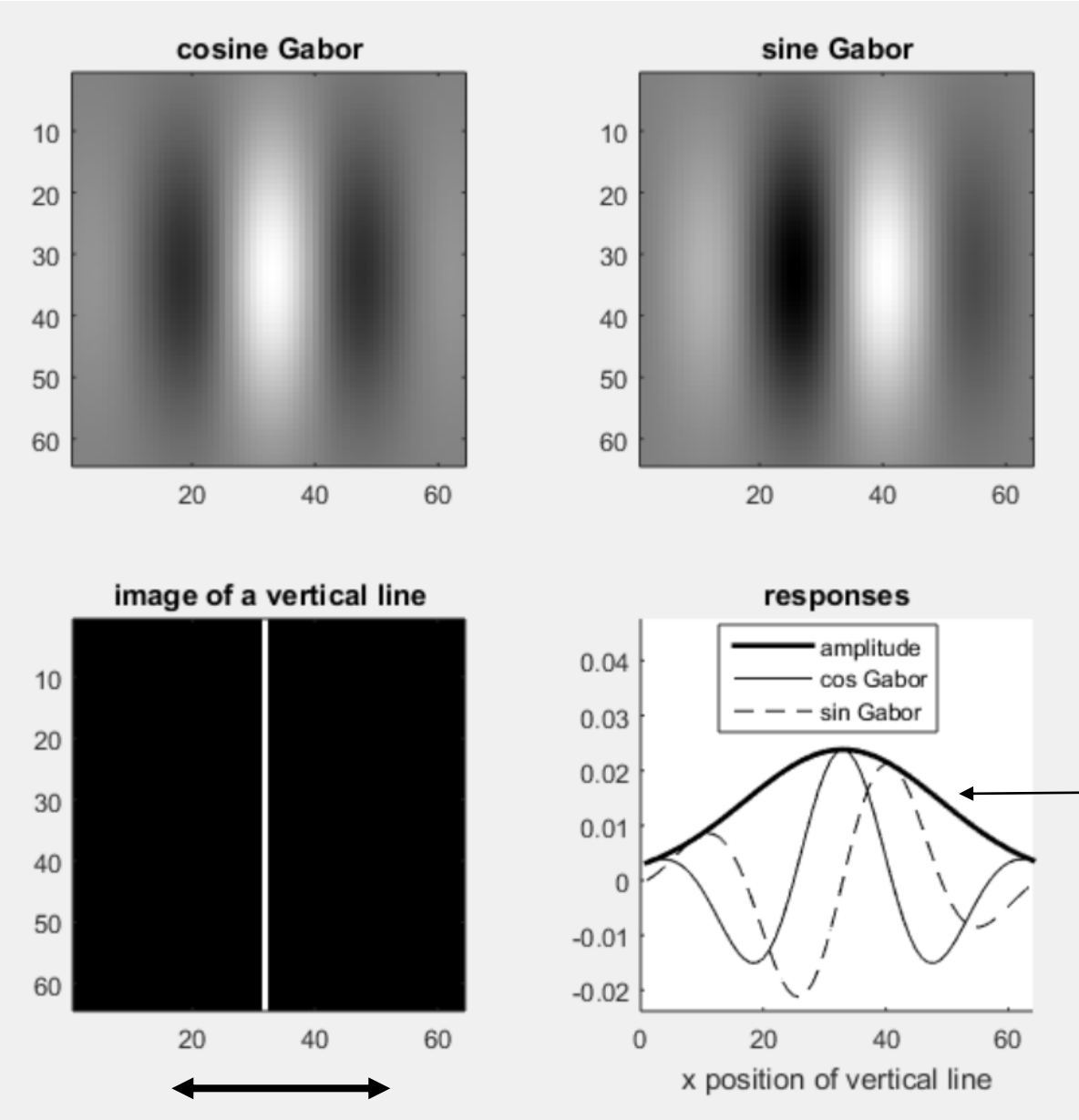




Q: What happens if you close an eye?

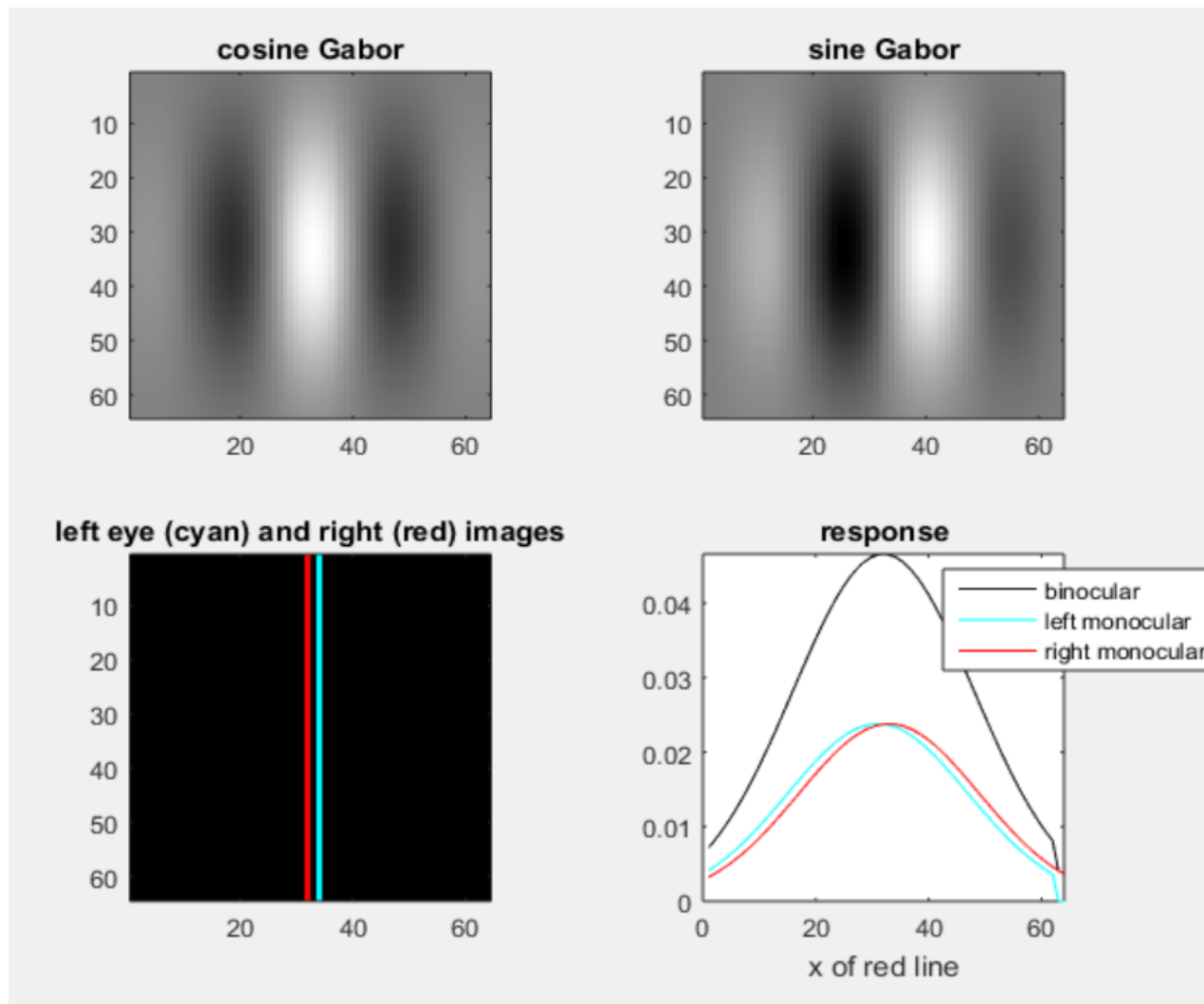
A: The cell behaves like a monocular complex cell.

# Recall (monocular) complex cell response to white line



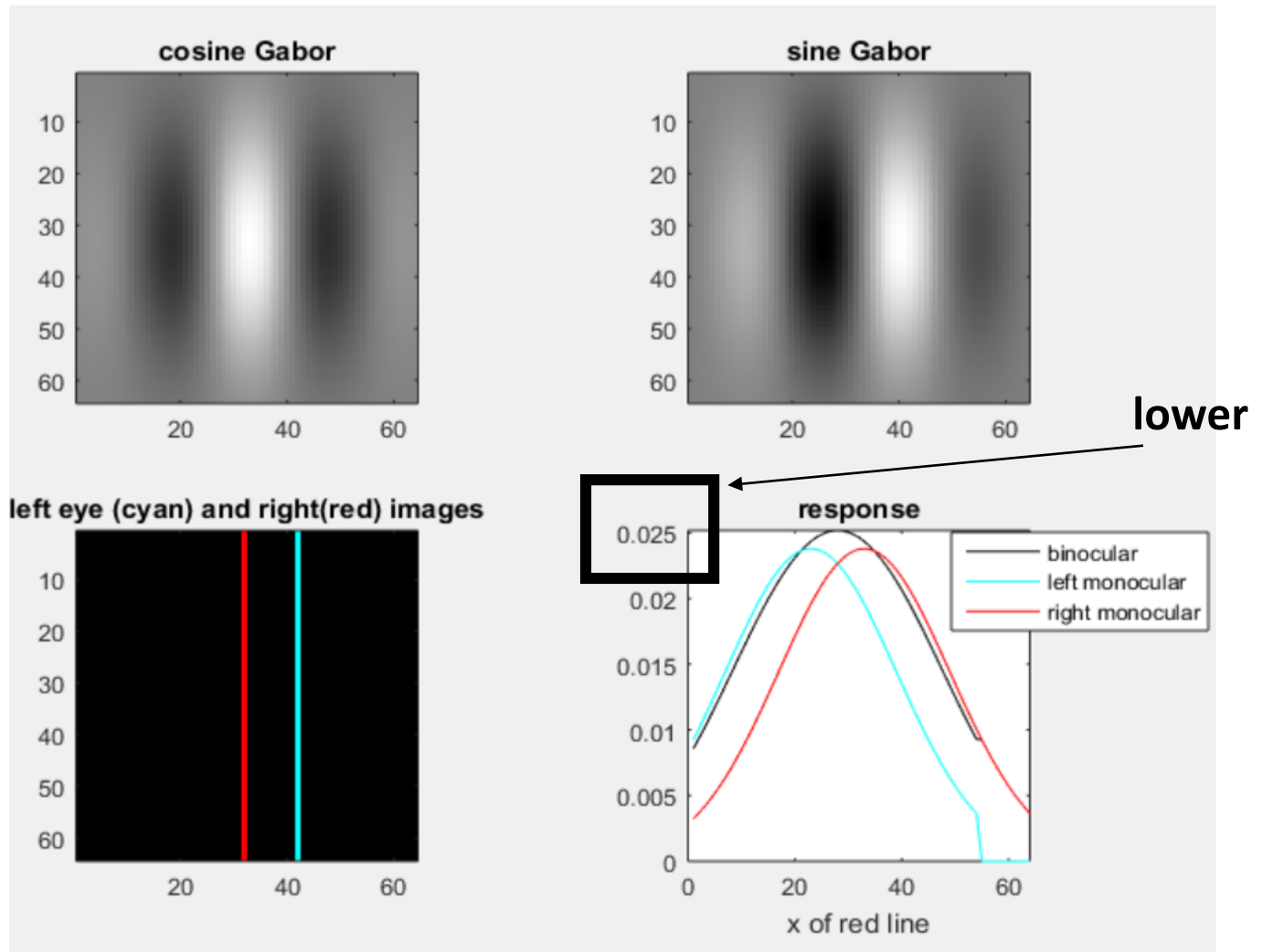
Response of complex cell

Response of binocular complex cell tuned to  $d = 0$  when disparity of white line is 2 pixels.



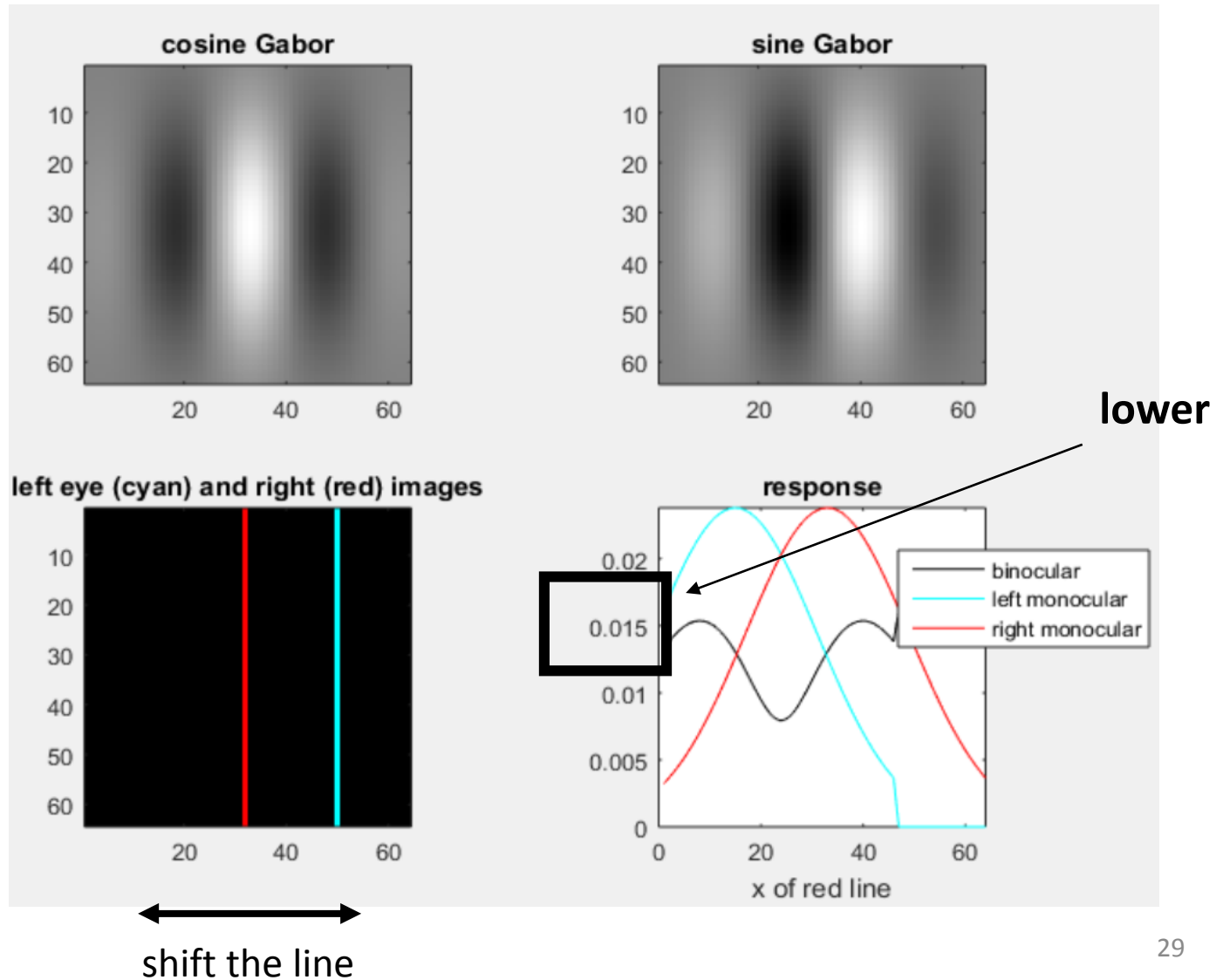
←→  
shift the line

Response of binocular complex cell tuned to  $d = 0$  when disparity of white line is 10 pixels.



←→  
shift the line

Response of binocular complex cell tuned to  $d = 0$  when disparity of white line is 18 pixels.



I will finish this next  
lecture.

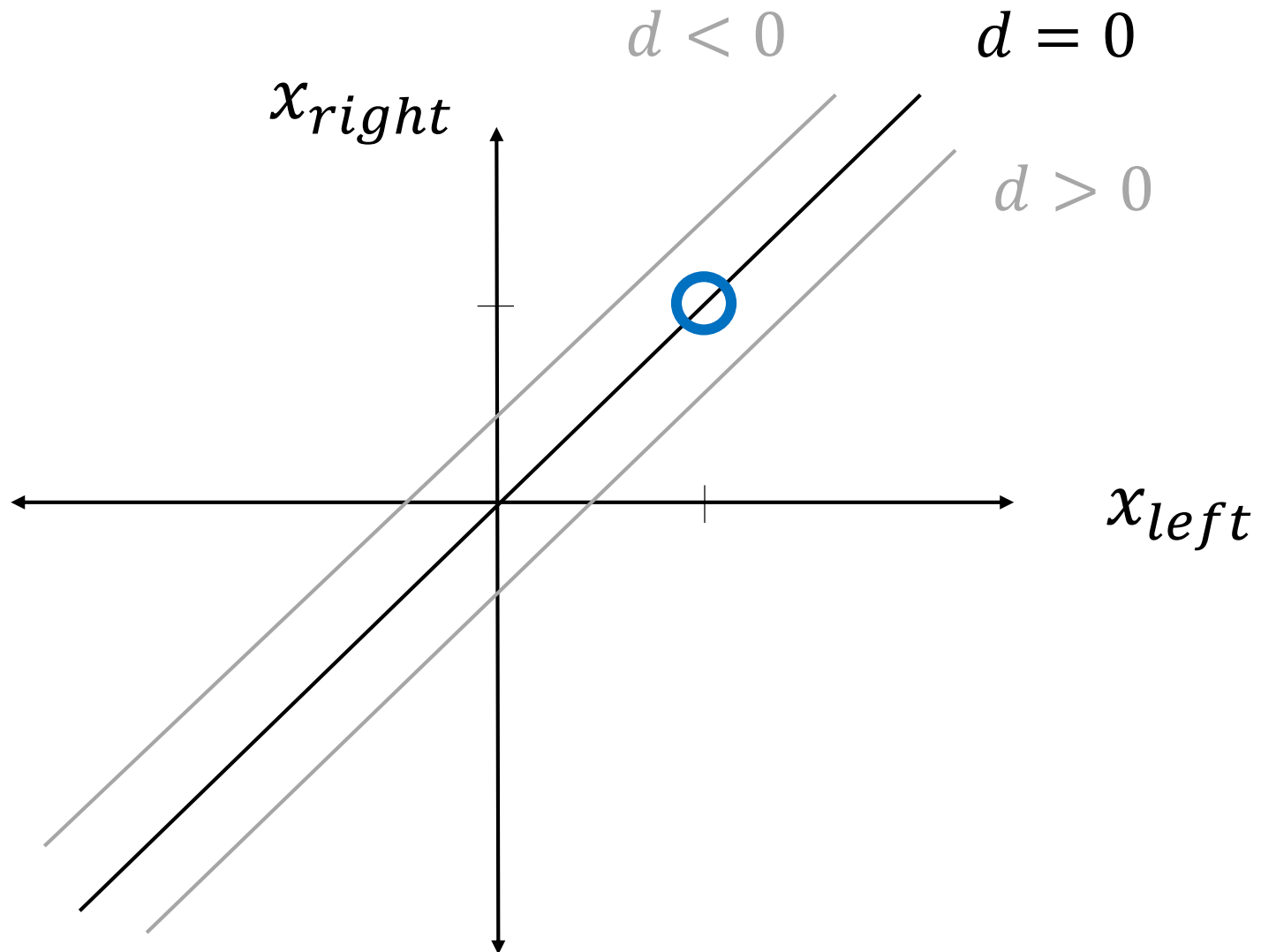
Each binocular cell has receptive field location centered at  $(x_l, y_l)$  and  $(x_r, y_r)$  in the two eyes.

Q: What disparity is each cell tuned for ?

A: We just discussed this.

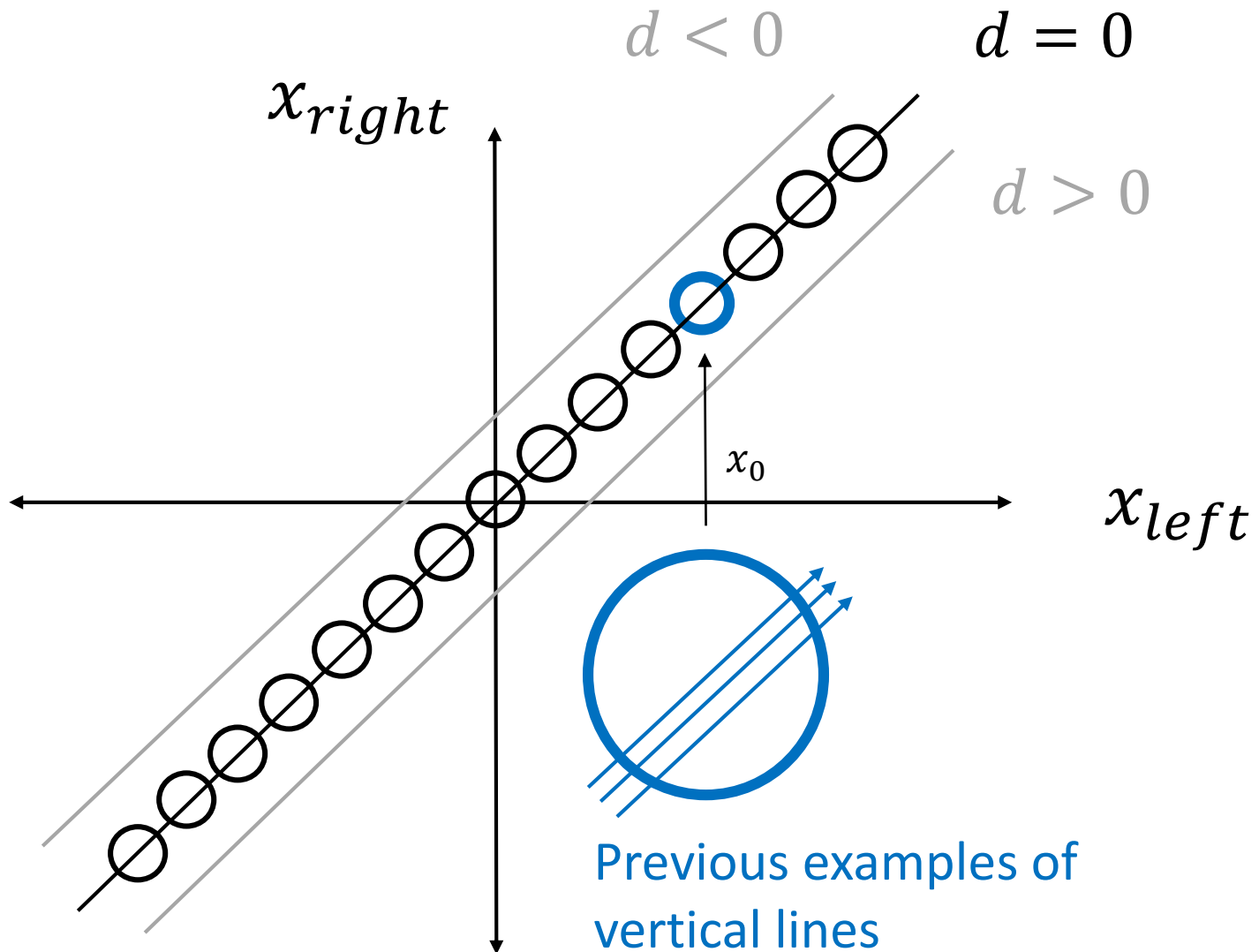
Q: How to visualize the set (“population”) of cells ?

# Disparity Space

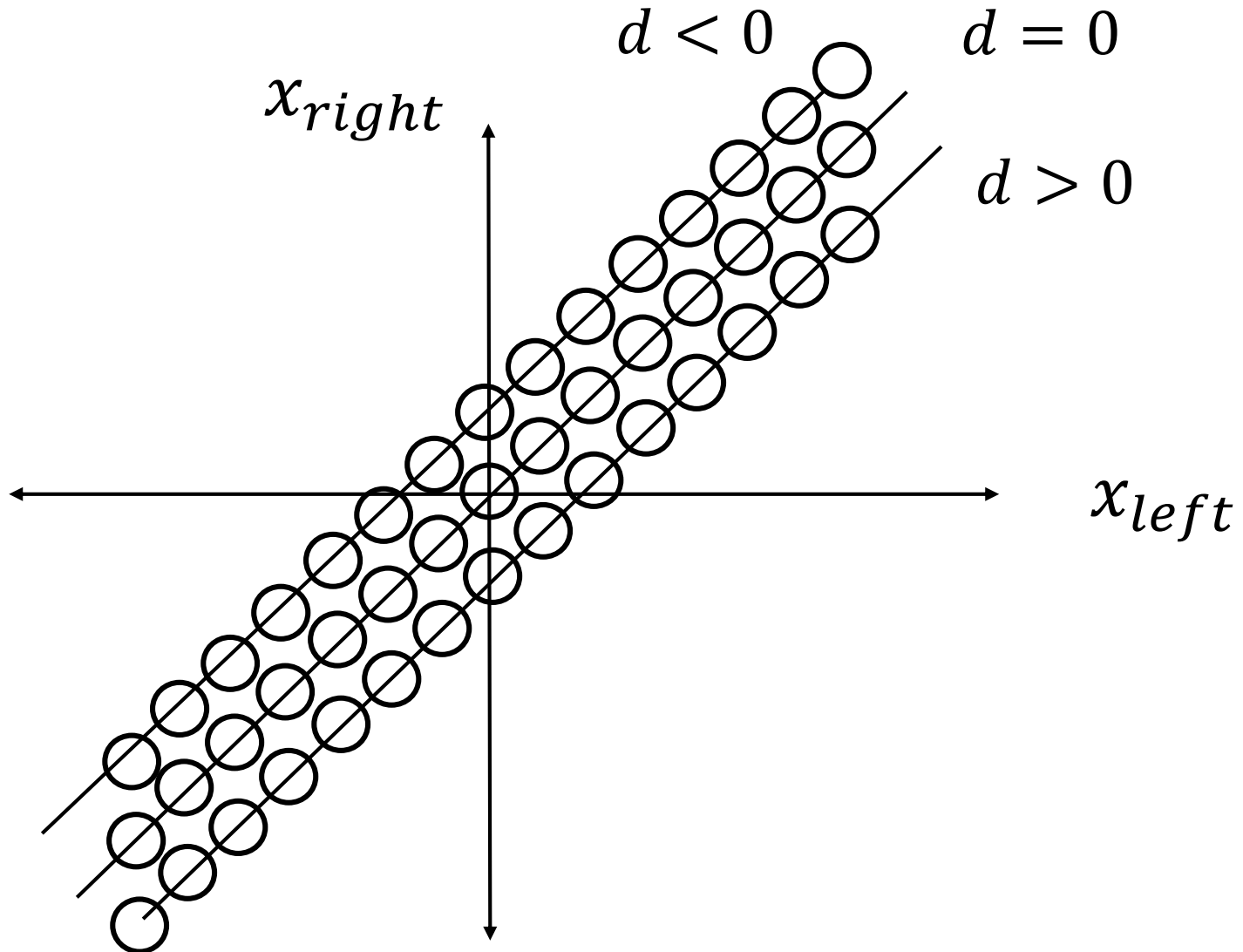




# Disparity Tuned Cells



# Disparity Tuned Cells



# Disparity Tuned Cells

