Design and Multi-Objective Optimization of a Linkage for a Haptic Interface

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Abstract – A method to carry out the design of linkage for a haptic interface is described. Factors such as size, workspace, intrusion, inertia, response and structural properties are considered in this process. The dependencies of the various criteria are examined and a hierarchical method is applied. The result is a compact device which is easy to manufacture and which fulfills the requirements demanded by its application. Several quantitative measures designed to capture its principal properties are at the heart the process.

I. Introduction and Problem Statement

The design of a haptic interface is driven by many requirements. Because a haptic device is essentially a human-machine interface, it must have the general features of an ergonomic design. In particular, it should be compact and the operating workspace should be large in relation to the size of the device itself. It should also cause minimal spatial intrusion in the work area of the user. Thus, the size relations are the first general indicators of performance.

The frequency response must be wide since humans are known to perceive force stimuli well above 300 Hz. The device must also be accurate since the amplitude of force signals are sensed by most operators over many orders of magnitude [16]. This, ideally, requires the complete absence of backlash, friction, and other disturbing dynamics; in particular structural dynamics. Note however that precision is of no particular importance due to reasons including postural persistence and various phenomena in the human perception and control of the position and motion of limbs.

The response of the device must also be uniform throughout the usable workspace so the perceived signals will not be distorted as humans are sensitive to minute differences in the amplitude and nature of mechanical signals. Ideally, the inertia of the device should be much less than the inertia of the finger tissue displaced by the device in order to establish a robust causal relationship between an input force signal and perceived motions. This requirement is, to date, the most difficult to achieve.

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Although the response can be improved by feedback or feedforward compensation techniques, these techniques have only limited applicability. It is well known that apparent inertia is not easily reduced by feedback [3]. Moreover, compensation, whether it is applied via feedback or feedforward, by principle will make up for the lack of uniformity of the transfer function from the actuators to the end-effector. We will propose a device which is based on linkages. Thus, its transfer function will vary with its posture from a best case to a worst case. Since ultimately, the performance is limited by the actuators, it follows that a superior design will result from the minimization of the "distance" between the worst and the best case. In this condition, the actuators can optimally be used.

There are many haptic devices which were designed with various goals in mind. Cadoz and colleagues describe an electromechanical system for the simulation of musical instruments which consists of a collection of one degree of freedom mechanisms to address the question of precise rendition of mechanical phenomena [4]. Millman, Stanley and Colgate report on the design of four degree-of-freedom haptic interface which can deliver a large output force [5]. Minsky and co-workers describe possible applications of such devices [13]. Howe built a high-fidelity two degree of freedom device for the study of tactile sensing in precise manipulation [7]. Kelley and Salcudean also designed a two degree of freedom device which avoids the use of linkages [8]. Matshuhira and colleagues used linkages to achieve large workspace [12]. Yokokoji and Yoshikawa looked at interaction of operator dynamics with hand controllers [22].

Previous work in dynamics optimization of linkages can be found in [1, 9].

II. Method

The device, because of its function [15], must have two degrees of freedom, and permit the displacement of a knob inside a rectangular area.

A design method derived from the pioneering work of Vertut and colleagues is illustrated [21, 20]. We will proceed similarly. From Section I, we have:

Size: The device should fit on a table top. A guideline for its size, in terms of its footprint, is provided by the size of an ordinary book.

Workspace: Translational motions should occur in the largest possible area within the footprint.

Intrusion: This is difficult to quantify. However, qualitatively we seek a low profile compatible with the human hand.

Inertia: For all practical purposes, given the state of actuator technology, an ideal inertia of a fraction of gram throughout a workspace of the order 0.01 m^2 is deemed unachieveable. Therefore out of several candidate designs, we will select those for which inertia is the smallest.

Response: Below its first natural frequency, the response of the device will be governed by its multi-body dynamics. In was noted that the Weber fraction $\Delta I/I$ of vibrotactile sensations is better than 0.4 for many kinds of stimuli [2]. For design purposes it was decided that the open loop response

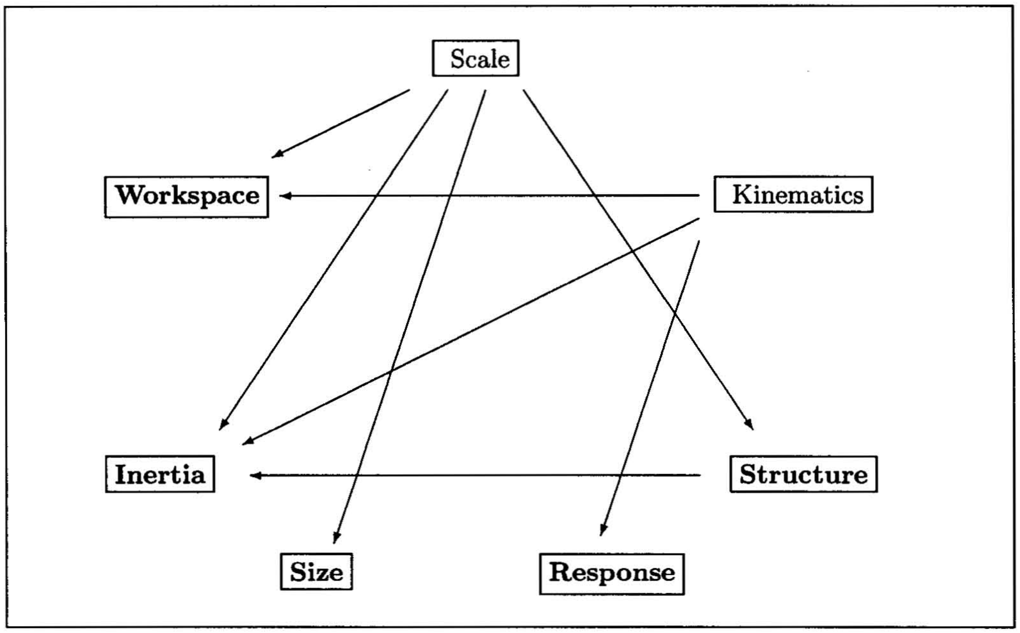

Figure 1: As seen above, many of the properties of the device depend on its scale and on the kinematic parameters. See text.

of the device in terms of torque-to-acceleration should not vary by more than a factor 0.5 which is a commonly accepted figure (3db) used for display devices.

Structure: The lowest natural frequency should be higher than the needed frequency response. Since the prototype is to be manufactured from metallic links, its response in the vicinity of the first natural frequency will be extremely undamped. Using the actuators to extend the frequency response beyond this point would require large amounts of actuator energy, which we saw before is in short supply. The use of fibrous or composite materials could provide the necessary damping (as for loudspeakers which are routinely used beyond their natural frequency), but this is beyond the scope of this paper. Strength is also an issue, since the device is likely to be exposed to abuse.

For any mechanism, the displacement workspace will increase with the square of its scale in the planar case, and by the cube in the spatial case; while the orientation workspace remains invariant in both cases. Except for the structural properties, all the criteria are governed by the kinematic parameters. Except for the change in acceleration response, all the criteria, depend on the scale. The Figure 1 summarizes the design dependencies.

When the scale is increased, the size (footprint) is obviously increased by its square and so is the workspace, since we are dealing with a planar mechanism. All other things being equal, the inertia will grow rapidly: with the cube of the scale. Evidently, the inertia will be a major limiting factor. Hence, the scale and everything that depend on it will be limited.

When the kinematics is changed, both in structure (the type of mechanism) and geometry (the various normalized link lengths), many things are affected. A parallel kinematic chain will almost certainly have better structural properties than a serial one. This is not represented on the Figure 1 because there is only a discrete number of choices. However, when the kinematic parameters are continuously changed for a given mechanism all of the criteria will vary: The inertial properties will change both in magnitude (Inertia) and smoothness (Response) and the workspace shape will be affected. Finally, for a given design, changes in materials and link shapes for the improvement of the structural properties, both in strength and resonance frequency may result in a rapidly rising inertia.

Inspection of Figure 1 suggest two methods of design: Search through the space of kinematic parameters that will satisfy the Response, Inertia, and Workspace requirements, and then adjust the scale, attempting to raise it as much as possible, or vice versa. Here we selected to fix the scale and then to search for kinematic parameters.

III. Carrying Out the Optimization

A five-bar mechanism was selected as a candidate kinematic chain to achieved the goals stated in Section I. When it is symmetrical with respect to its ground link, it is described by three lengths, or two ratios and the scale. The ground link length d is taken as a fixed parameter and the optimization search will be performed on the link lengths a (inner link) and b (outer link).

A. Multi-Objective Search.

The well known concept of singular values decomposition is applied to the optimization of the link lengths. Any matrix $\boldsymbol{A}$ can be factored into $\boldsymbol{A} = \boldsymbol{Q}_1 \boldsymbol{\Sigma} \boldsymbol{Q}_2$, where $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ are orthogonal ($\boldsymbol{Q}^{-1} = \boldsymbol{Q}^T$) and $\boldsymbol{\Sigma}$ is diagonal with the n singular values of $\boldsymbol{A}$ on its diagonal. The columns of $\boldsymbol{Q}_1$ are the eigenvectors of $\boldsymbol{A}\boldsymbol{A}^T$, and the columns of $\boldsymbol{Q}_2$ are the eigenvalues of $\boldsymbol{A}^T\boldsymbol{A}$ [18].

Here, because velocities are low in the desired operating conditions,[1] the transformation from actuator torques to tip acceleration can be locally approximated by a linear map $\ddot{x} = \boldsymbol{R}\,\tau$, where is x designates the coordinates of the tip and τ the vector of actuator torques. The determination of $\boldsymbol{R}$ is straightforward: the inertia tensor was derived using Lagrange's method and the Jacobian matrix was derived from the statics. This mechanism also admits a closed-form position analysis in the two directions.

This transformation $\boldsymbol{R}$ can be regarded as a *deformation.* The singular decomposition separates it into a stretching component, the Σ matrix and a rotation component, the $\boldsymbol{Q} = \boldsymbol{Q}_1 = \boldsymbol{Q}_2$ matrix with immediate physical interpretation: the ratio of singular

[1] Note that this observation was made by also made by Vertut and Liégeois who, like us, were concerned by the acceleration capabilities of manipulators [20].

values will measure the skew of the response. It is often called the dexterity [17] and its value is 1 when there is no skew. Their product will measure the overall gain of the system as a function of the operating point and is often called the manipulability [23].

Size: This is driven by two factors: the shape and size of the workspace, a connected 10 X 16 cm rectangular region; and its location with respect to the ground link. The distance of the region to the ground link should be minimized to improve the footprint.

Workspace: The workspace will be optimized using the hierarchical approach outlined in [10] with three criteria described in the following two paragraphs.

Inertia: The inertial properties of the device are rather intricate to describe. However, they have the general form $\sum_V dm \, . \, d^2$. Because of its dependence on the structural properties (Figure 1), they will grow at least with the cube of the scale, if not faster. This criterion is consequently very sensitive and is selected as the primary objective.

Response: There are two secondary objectives which together capture this criterion. The ratio σ_2/σ_1 of the singular values of the acceleration map which should never be lower than 0.5. The geometric mean of the singular value has the same dimension as ~~the dexterity~~ a gain: define $(\sigma_2\sigma_1)^{1/2}$ as the manipulability measure.

Structure: Since no finite element code was available in this project, this criterion was determined experimentally. The link shapes were approximated by simple shapes for the purpose of calculating an approximation of their moment of inertia as a function of scale. After determination of the link sizes, several models were built and their structural properties investigated. It can be readily observed that the outer links undergo in-plane stress in two directions: one due to the action of the actuators (principal direction), and the other due to the operator's hand resting on the knob. A triangular shape was selected. The inner links undergo bending on two direction, as well as torsion. A tapering H crossection beam structure was selected for this link.

B. Results.

It is obvious that any mechanism of this kind can be made as isotropic as desired (less skew) as the link lengths go to infinity. Of course, this is at the expense of a rapidly growing inertia. Consequently, of all the acceptable designs, we shall choose the smallest and this will improve inertia, size (footprint), and structural properties.

Both the dexterity and the manipulability of the acceleration map were calculated for rectangular ranges of the tip positions as a function of the link lengths a and b. Since they are related, the dexterity was examined first (higher in the hierarchy). The manipulability should be as large as possible, but should also remain constant, thus, it was normalized to its with respect to its lowest value within the workspace of interest.

The central observation made before searching for a and b is that the ground link length d must not be zero. The reason for this is twofold. For practical reasons, space must be provided for the actuators. But from a kinematic optimization view point, a zero length would causes the design to only depend on one parameter (besides the scale) which would considerably reduce the space of possible designs! One other way to see that is to observe that a zero ground link length would cause all kinematic and dynamic properties to depend on the radial extension of the linkage with their profile in a given direction dependent only on the ratio a/b.

The range of possible combinations of a and b is vast but this range could be reduced considerably by an early pruning. It was observed with the help of computer simulations that the performance severely deteriorated in two broad cases: when $a > b$, and when $a << b$. As a result, the search was narrowed down to regions where a and b were close and $b \geq a$, and several locally optimal designs were found.

For example, with $d = 4$, Figure 2(a) for $a = 13$ and $b = 15$ shows a wide vertical range, but it is a conservative design because the allowable range exceeds the specifications. Figure 2(b) ($a = 15$ and $b = 13$: violation of condition $a < b$) and 2(c) ($a = 10$ and $b = 18$: violation of condition a close to b) are examples of unacceptable performance. Figure 2(c) with $a = 10$ and $b = 18$, shows that the range of allowable dexterity is considerably reduced.

Since the example of Figure 2(a) does not fully exploit the allowable range of dexterity, the link lengths should be made smaller. A search in this direction was performed. With $a = 9$ and $b = 11$, Figure 2(d) reveals that this choice no longer yields a sufficient range. The final result of the search is shown on Figure 2(e) with $a = 10$ and $b = 12$. There are other combinations of link lengths which also achieve a sufficient vertical range, for example $a = b = 14$ on Figure 2(f), but they all correspond to longer link lengths and therefore a larger inertia. Figure 2(g) is the same as Figure 2(e), except that the full workspace has been zoomed in.

Before the lengths $a = 10$ and $b = 12$ can be deemed nearly optimal, the variation of the manipulability must be investigated. Figure 2(h) displays the normalized measure for this workspace and it does not vary by more than 30%, which is excellent.

C. Note on the Significance of Norms: Diamonds and Ellipses

So far, the discussion could be carried out without any reference to a particular norm.

An alternative (and more ornate) method for displaying the sets of achievable acceleration consists of mapping the sets $\|\boldsymbol{\tau}\|_\infty = \max(|\tau_1|, |\tau_2|) \leq \tau_{max}$, which may graphically be represented as diamonds. The infinity norm is the norm that really counts **instantaneously** since the actuators have their torques bounded by the demagnetization current. The 2-norm $\|\boldsymbol{\tau}\|_2 = (\tau_1^2 + \tau_2^2)^{1/2} \leq \tau_{max}$, which yields portraits made of ellipses, can be important too **on average** (i.e. RMS) since it is related to the dissipation in the windings. However, even its weighted version to account for dissimilar motors and torque amplification transmissions does not have a clear physical significance since one actuator could be going up in smoke while the others remain cold; the torques still being inside the unit ball.

At any rate, it is would be very difficult to appreciate the results of the optimization using these graphical techniques because from these representations it is hard to perceptually appreciate the various merits of a particular design [19]. It was found that the isoline method employed in this paper conveyed more concisely and accurately the crucial information.

When a manipulator at isotropic points, the norm does not matter, but globally speaking, consideration of various norms averaged over the operating conditions—in space and time—would allow to further optimize the design, see [11] for additional such global considerations.

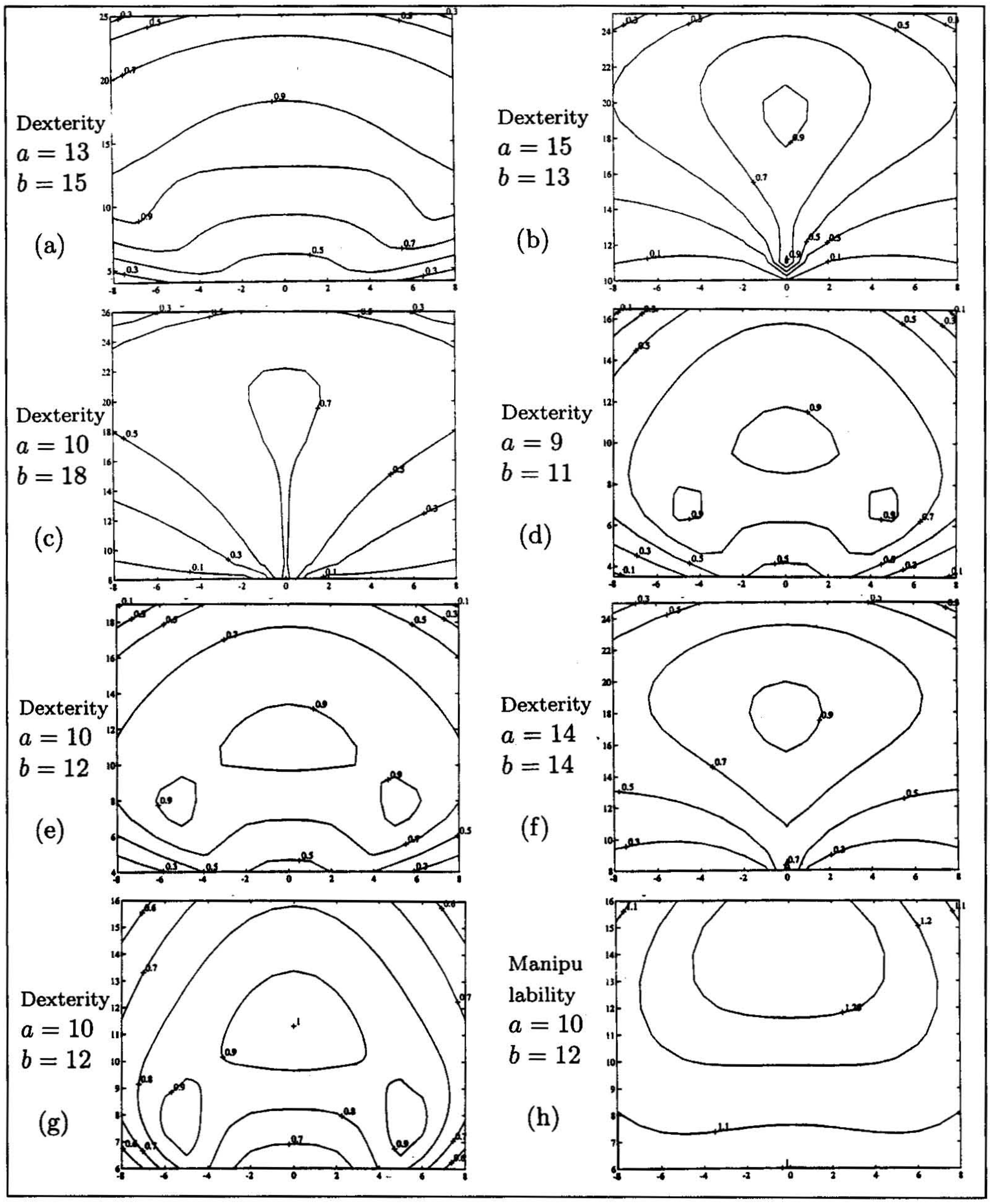

Figure 2: Isolines of the dexterity and the manipulability for various designs. See text.

IV. Conclusion

The choice of a structure, of its kinematic parameters and of a scale has impact on most requirements: workspace, inertial dynamics, structural dynamics, and uniformity of response. We selected for this project a very simple structure: a five-bar linkage with two grounded actuators driving a single knob and examined the effect of the kinematic parameters selection on the requirements in order to determine a preferred design in the presence of many conflicting objectives.

The most surprising result of this study is the **high sensitivity** of the dynamic performance of the linkage with respect to its kinematic parameters, whereas a **low sensitivity** to kinematic performance was observed in an earlier project [6]. The other surprising result was indeed that such a device could be made within the specifications outlined in the introduction.

One of the several existing prototypes was exhibited at the Conference on Human Factors in Computing Systems (ACM-SIGCHI'94) held in Boston in May 1994. Figure 3 shows a picture of this prototype.

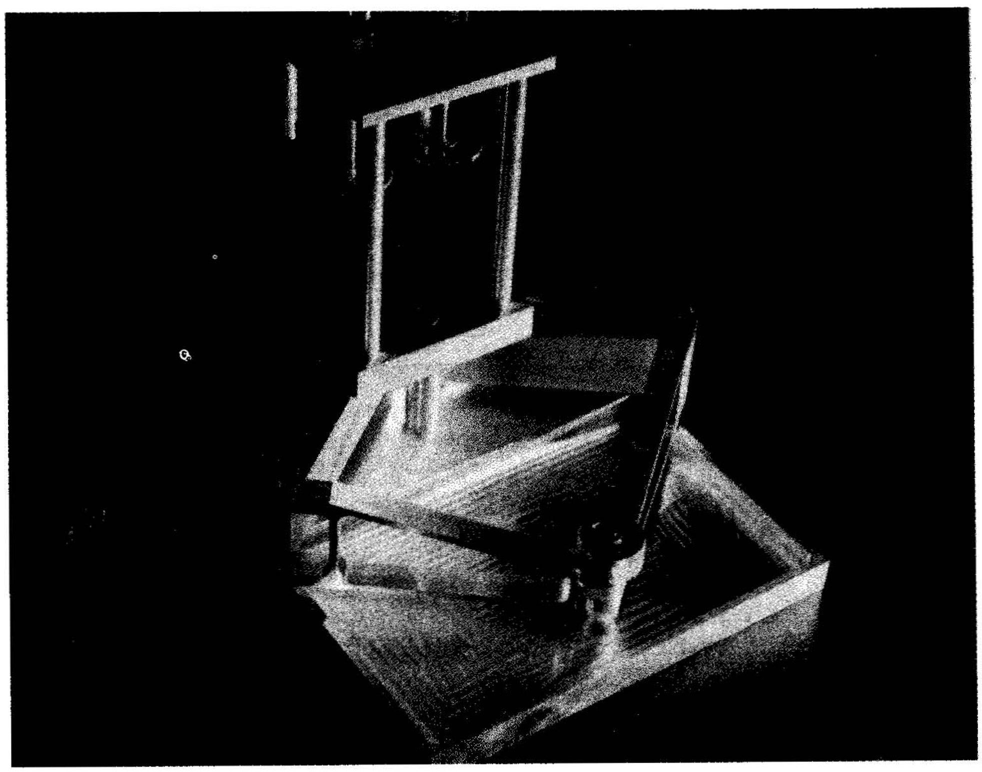

Figure 3: Resulting device.

V. Acknowledgements

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