

# Estimation of Complex Anatomical Joint Motions Using a Spatial Goniometer

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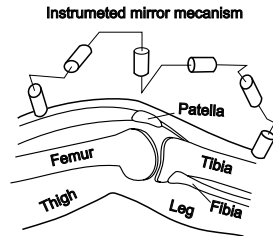
**Abstract** The determination of the instantaneous axis of rotation of human joints has numerous applications. The screw axis can be determined using optical motion capture systems or electromechanical goniometers. In this paper, we introduce a new method for the localization of the instantaneous screw axis in human anatomical joints from the data given by a spatial mechanical goniometer.

## 1 Introduction

The knowledge of the kinematics of human joints is needed in numerous instances, such as in prostheses or orthoses design, or to help the treatment of diseased and injured joints. The knee joint kinematics is known as having a complex motion resulting from a combination of rolling and sliding movements of the femur onto the tibia. The complex, spatial motion of the knee joint varies from one individual to another. It also changes with load conditions and, of course, as the result of certain pathologies (Markolf et al., 1984; Winsman et al., 1980).

To date, the kinematics of a human anatomical joint is identified by functional methods employing standardized movements of extension-flexion, adduction-abduction, and of circumduction. The movement data are acquired with various measurement devices including motion capture systems with cameras and optical markers, or medical imaging modalities to measure the movement of bones. Different kinematic analysis methods have been developed for these different categories of devices (Woltring et al., 1985; Ehrig et al., 2006; Blankevoort et al., 1990). Simple goniometers are

also used for the measurement of rotational amplitude, but this technique has several restrictions such as attachment problems or the lack of device mobility.



**Figure 1.** Spatial linkage as a goniometer for an anatomical knee joint.

Another approach for the identification of a six degrees-of-freedom motion is to employ an instrumented spatial linkage, i.e. a goniometer, attached to the skeletal segments of interest and to measure the motion between adjacent fixations, viz. the femur and the tibia, that primarily define the movement of the joint. Please see Fig. 1 for a diagram of the concept. Several of these systems have been designed by the past that have at least six degrees-of-freedom (Townsend et al., 1977). Indeed six degrees of freedom are required to track limbs motions without any constraint, as the knee joint and mechanism can be misaligned. Numerous goniometers have been so far developed

for use in clinical joint tests, or for the measure of knee kinematics. They were used to analyze continuously the joint motion based on different models, essentially finite transformation models (Kinzel et al., 1972).

In this paper, the use of a spatial linkage goniometer with integrated angle sensors to estimate the anatomical joint motion in real time is investigated. Since the total relative motion between the upper and the lower parts of a human leg belong to the group of rigid motions  $SE(3)$ , the notion of screw axis can be used along with methods for the identification of the infinitesimal screw axis and others kinematic parameters. An example of design of a knee goniometer is shown and validated by simulations.

## 2 Estimation of the Anatomical Joint Screw Axis

### 2.1 Approach Using the Velocities of Points

In this method, the positions and velocities of different points on the moving segment are measured. The localization of the instantaneous screw axis can be then described by the orthogonal projections  $A'_i$  of the points  $A_i$ , belonging to the moving segment, onto the screw axis (Bru and Pasqui, 2009). These projections are defined to be

$$\overrightarrow{A_i A'_i} = (\mathbf{v}_{A_i} \wedge \boldsymbol{\omega}) / (|\boldsymbol{\omega}|^2), \quad (1)$$

where  $\boldsymbol{\omega}$  and  $\mathbf{v}_{A_i}$  are respectively the angular and linear velocities of the second segment at the point  $A_i$ . For systems using optical markers, these values can be measured and determined by using different transformation methods. The relative linear velocity of the two body segments can be estimated from the expression

$$\mathbf{v}_{A'_i} = \mathbf{v}_{A_i} - \boldsymbol{\omega} \wedge \overrightarrow{A_i A'_i}. \quad (2)$$

In the case of a spatial goniometer, the sum of the velocities,  $\boldsymbol{\omega}_{i,i+1}$ , provides the relative angular velocity between the two segments, that is,

$$\boldsymbol{\omega}_{06} = \sum_{i=0}^5 \boldsymbol{\omega}_{i,i+1}. \quad (3)$$

If  $\mathbf{q}$  is the known vector of joint angles, from the forward kinematic model, the twist expressed at points  $A_i$  of the moving segment is

$$\begin{pmatrix} \boldsymbol{\omega}_{06} \\ \mathbf{v}_{A_i} \end{pmatrix} = J_{A_i}(\mathbf{q})\dot{\mathbf{q}}, \quad (4)$$

where  $J_{A_i}$  are the Jacobian matrices of the spatial linkage written at points  $A_i$  of the second limb.

## 2.2 Approach by Loop Closure

It is also possible to compute the coordinates of points of the instantaneous screw axis by solving a linear system of equations obtained from loop closure at these points (Cai et al., 2009). If the segments of an anatomical joint are considered to be part of a linkage, then the loop-closure equation gives

$$J_P(\mathbf{q})\dot{\mathbf{q}} - \begin{pmatrix} \boldsymbol{\omega}_{06} \\ \mathbf{v}_P \end{pmatrix} = 0, \quad (5)$$

where  $P$  is a point on the instantaneous screw axis and the twist at this point is

$$\begin{pmatrix} \boldsymbol{\omega}_{06} \\ \mathbf{v}_P \end{pmatrix} = (\omega_x \ \omega_y \ \omega_z \ v_x \ v_y \ v_z)^\top. \quad (6)$$

Let the position of the point  $P$  relative to the reference body be

$$\overrightarrow{PO_0} = (a \ b \ c)^\top. \quad (7)$$

From (5)–(7), the loop-closure equations give a system of equations

$$\begin{cases} K_1 b + K_2 c + K_3 = v_x, \\ K_4 a + K_5 c + K_6 = v_y, \\ K_7 a + K_8 b + K_9 = v_z, \end{cases} \quad (8)$$

where the  $K_i$ 's depend on the configuration variables (joint angles,  $\theta_i$ 's, and joint positions,  $r_i$ 's, for revolute and prismatic joints respectively), as well as design parameters,  $l_i$ 's and  $d_i$ 's. Moreover, it can be verified that several relationships exist between the  $K_i$ 's, including

$$K_1 = -K_4 = \omega_z, \quad K_7 = -K_2 = \omega_y, \quad \text{and} \quad K_5 = -K_8 = \omega_x. \quad (9)$$

When the anatomical joint motion comprises a rotation, at least one of the twist components,  $\omega_x$ ,  $\omega_y$ ,  $\omega_z$ , must differ from zero. Let us suppose that it happens that  $\omega_x$  is different from zero. Since there is an infinity of choices for the point  $P$ , one of its coordinates,  $a$ ,  $b$ , or  $c$ , can be selected arbitrarily in order to compute the two others. Since  $\omega_x \neq 0$ ,  $a$  is selected and the coordinates  $b$ ,  $c$  of the vector  $\overrightarrow{PO_0}$  may be computed as follows,

$$b = \frac{\omega_y a + K_9 - v_z}{\omega_x}, \quad c = \frac{\omega_z a - K_6 + v_y}{\omega_x}. \quad (10)$$

Since an anatomical joint is modeled by a screw joint, all the points on the instantaneous screw axis have a common velocity that is aligned with the axis. Given  $\epsilon$ , a non-zero constant, and another point  $P'$  on the screw axis with coordinates  $(a', b', c')$ , the common velocity property gives

$$\mathbf{v}_P = \mathbf{v}_{P'} = \epsilon \overrightarrow{PP'} = \epsilon \begin{pmatrix} a' - a \\ b' - b \\ c' - c \end{pmatrix}. \quad (11)$$

Two systems of equations are obtained by rewriting (8) for two different points  $P$  and  $P'$ . From (11), subtracting the two systems gives a new one:

$$\begin{cases} K_1(b - b') + K_2(c - c') = 0, \\ K_4(a - a') + K_5(c - c') = 0, \\ K_7(a - a') + K_8(c - c') = 0. \end{cases} \quad (12)$$

The later system, (12), can be rewritten in terms of  $v_x, v_y, v_z$ ,

$$\begin{cases} K_1 v_y + K_2 v_z = 0, \\ K_4 v_x + K_5 v_z = 0, \\ K_7 v_x + K_8 v_y = 0. \end{cases} \quad (13)$$

From (9), if  $\omega_x \neq 0$ ,  $K_5$  and  $K_8$  are also different from zero. The quantities  $v_y$  and  $v_z$  can then be computed as a function of  $v_x$ ,

$$v_y = -\frac{K_7 v_x}{K_8} = \frac{\omega_y v_x}{\omega_x}, \quad v_z = -\frac{K_4 v_x}{K_5} = \frac{\omega_z v_x}{\omega_x}. \quad (14)$$

From (8), (10), (14), the velocity coordinate  $v_x$  can be computed as a function of  $(\theta_i, d_i, \dot{\theta}_i, r_i, l_i)$ . All calculations done,

$$v_x = \frac{\omega_x^2 K_3 + \omega_x \omega_y K_6 + \omega_x \omega_z K_9}{\omega_x^2 + \omega_y^2 + \omega_z^2}. \quad (15)$$

Similar results are obtained when  $\omega_y$  or  $\omega_z$  are different from zero and  $b$  or  $c$  are chosen arbitrarily. In the case of a pure translation of the joint, according to (8) and (9), the linear velocity is expressed as

$$\mathbf{v}_P = (v_x \quad v_y \quad v_z)^\top = (K_3 \quad K_6 \quad K_9)^\top. \quad (16)$$

### 3 Application to a Goniometer for The Knee Joint

#### 3.1 System description



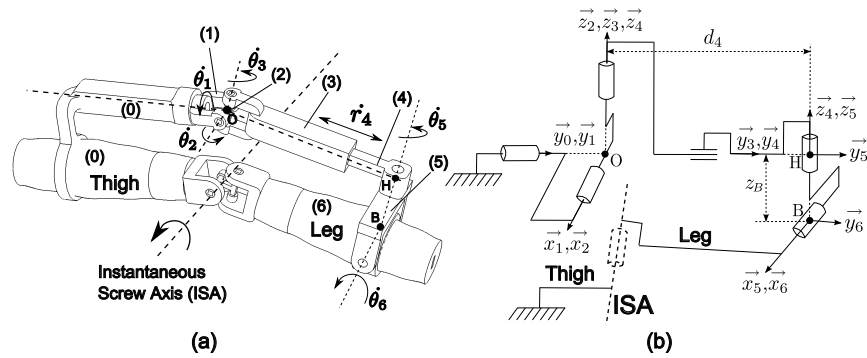
**Figure 2.** The knee goniometer.

Figure 2 shows the overall architecture for a spatial goniometer designed for the knee joint. This mechanism has three intersecting revolute joints, a sliding joint and two others intersecting revolute joints. Like for robot manipulators, intersecting revolute joints simplify the kinematic analysis.

The slider gives to the mechanism the ability to accommodate different limb sizes. The design meets several specifications such as large workspace, no collision with limbs in the operating range and minimum slider displacement during motion. Sensors give the displacements of the six different joints of the mechanism. A computer model, which is illustrated in Fig. 3, is used for the simulation. For testing purposes, the anatomical joint is represented by orthogonal, non-intersecting axes that produce complex motions in three dimensions.

#### 3.2 Simulation results

A simulation was set up to validate these methods. The instantaneous angular and linear velocities are computed by a second-order derivation method from predetermined position input data. The time step was set to be 3ms. The knee joint was modeled as a moving screw joint. Knee amplitude



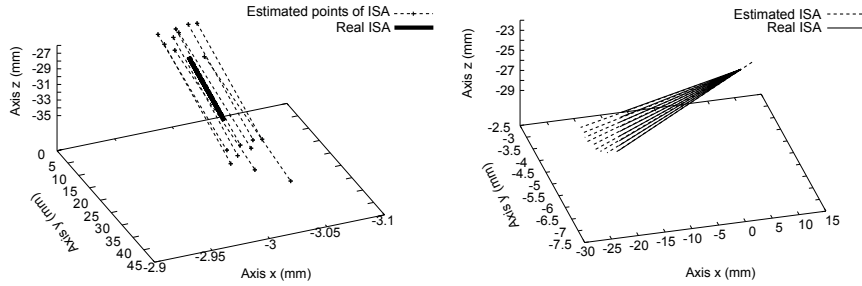
**Figure 3.** The computer model for the simulation of the knee electrogoniometer (a) and its frame assignment (b).

is set to 90 degree for flexion and 0 degree for extension. The maximum knee internal/external rotation is 9 degree. The simulations lasted for 3 seconds (for a total of 1000 samples). Offset errors were set to 2 degree for revolute joints and to 2 mm for the slider joint. Errors of 0.001 degree (for smooth angular value) or 0.01 degree (for discontinuous angular values) were added to each sample. For the slider, this error was set to 0.001 mm or to 0.01 mm. When there was no simulated measurement error, the localization of the instantaneous screw axis was exact. Then measurement errors were added in order to test the sensibility of the system to inaccuracies. All results are computed in the frame  $R_0 = (O, \mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$ .

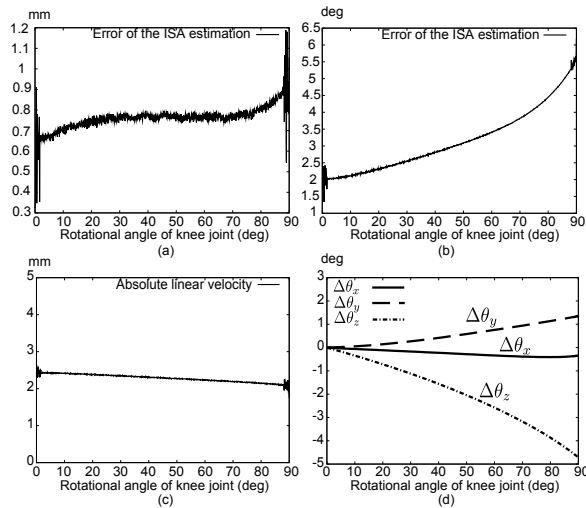
The first simulation was carried out with a fixed instantaneous screw axis. The angular velocity was set to 30 deg/s and the linear velocity was set to 2 mm/s. Figures 4–5 show the results of the simulation with measurement and offset errors due to calibration. In spite of important offset errors, the estimated axis is still located near to the real screw axis with a small angular deviation of 6 degree), see Fig. 5.b. Figure 5.c shows the estimated instantaneous linear velocity of the knee joint. The estimated values are also near to the real velocity in spite of large offset errors. Angular values, however, could differ significantly from the real values due to calibration errors. According to Fig. 5.d, the estimated absolute angular error of the second segment can reach 5 degree for offset errors of 2 degree. Hence, the use of high-performance calibration procedures is required for applications in which the estimation of angles is needed.

A second simulation was run to simulate a moving instantaneous screw axis. The knee motion was a compound motion of two simultaneous rota-

tions, the first at angular velocity of 3 degrees/s, and the second to 30 degree/s. Figure 4.b shows the evolution of the moving screw axis during the knee movement.



**Figure 4.** (a) First simulation: Localization of a fixed ISA. (b) Second simulation: Localization of a mobile ISA - Real and estimated ISA fall graphically on top of each other.



**Figure 5.** Result of the first simulation. (a) ISA estimation error: Distance between the two axes. (b) ISA estimation error: Angular deviation error. (c) Estimated linear velocity. (d) Estimated errors of the rotational angles of the knee joint.

## 4 Conclusion

In this paper, we proposed a new method for the estimation of the instantaneous human joint kinematics based on the use of a mechanical goniometer. This method allows the system to estimate all kinematic parameters of the user's joint, including the angular and linear displacements, angular and linear velocities of the second limb, and the localization of the instantaneous screw axis. The knowledge of the screw axis and of the velocities of the anatomical joint is the most interesting result because it makes it possible to design rehabilitation devices that could conform to the physiological movements of patients.

In future works, this technique will be validated by comparison with other methods using motion capture systems.

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