Privacy-optimal strategies for smart metering systems with a rechargeable battery

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Smart Meters empower smart grids

Fine grained consumption measurements are needed for:

- Time-of-use pricing
- Demand response
- . . .

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Comprehensive report: How smart meters invade privacy

August 29, 2014 by K. T. Weaver
big data, democracy, Fourth Amendment, privacy, rights, smart meters, spying

by K.T. Weaver, for Take Back Your Power

6 Comments

Last week, SkyVision Solutions released an updated **report** entitled, "A Perspective on How Smart Meters Invade Individual Privacy."

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big data, democracy

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Doctor, Bully, Futurist, Spy: The 'Smart Meter'

By MATTHEW L. WALD MAY 20, 2010 2:00 PM

Living

Will future energy and health researchers get useful data from "smart meters"?

As utilities around the country install meters that can

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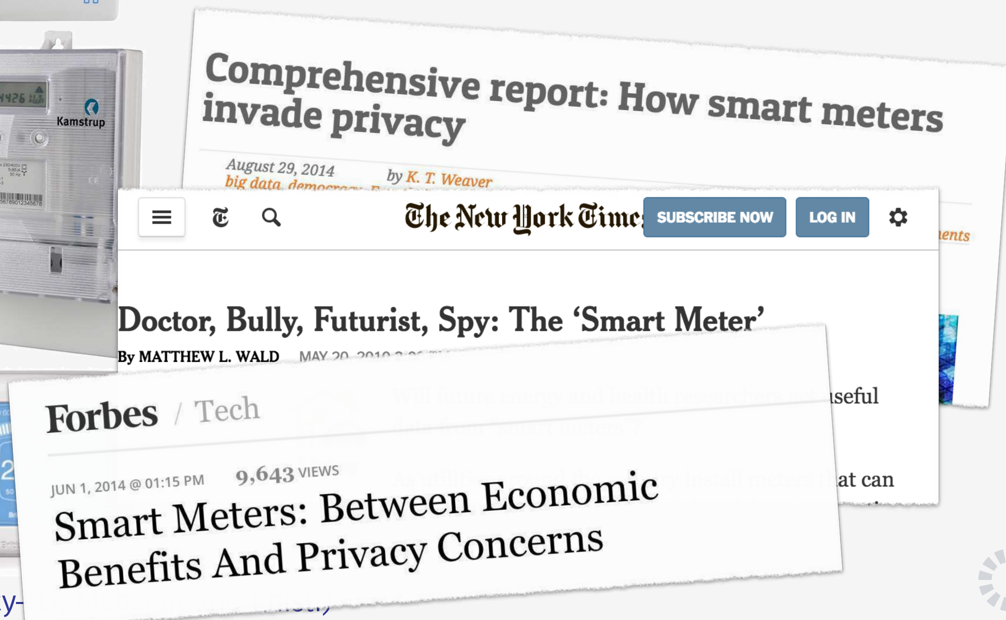

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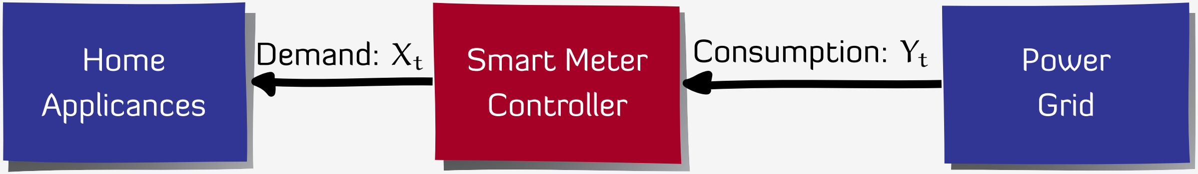
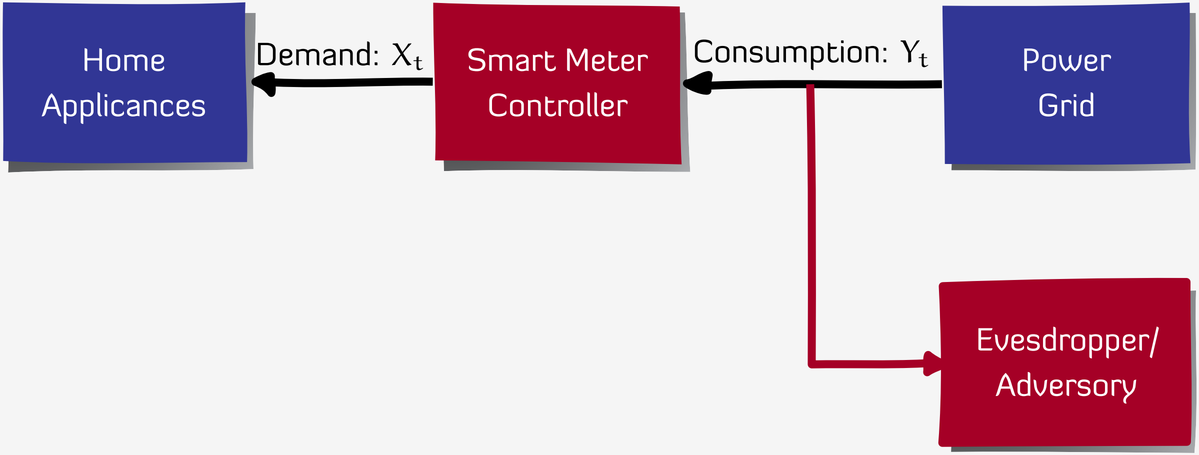
Smart-meter privacy-

What is the minimum information leakage rate if consumers obfuscate consumption using a rechargeable battery?

What are privacy-optimal battery charging strategies?


Home Applicances
Demand: X_t
Smart Meter Controller
Consumption: Y_t
Power Grid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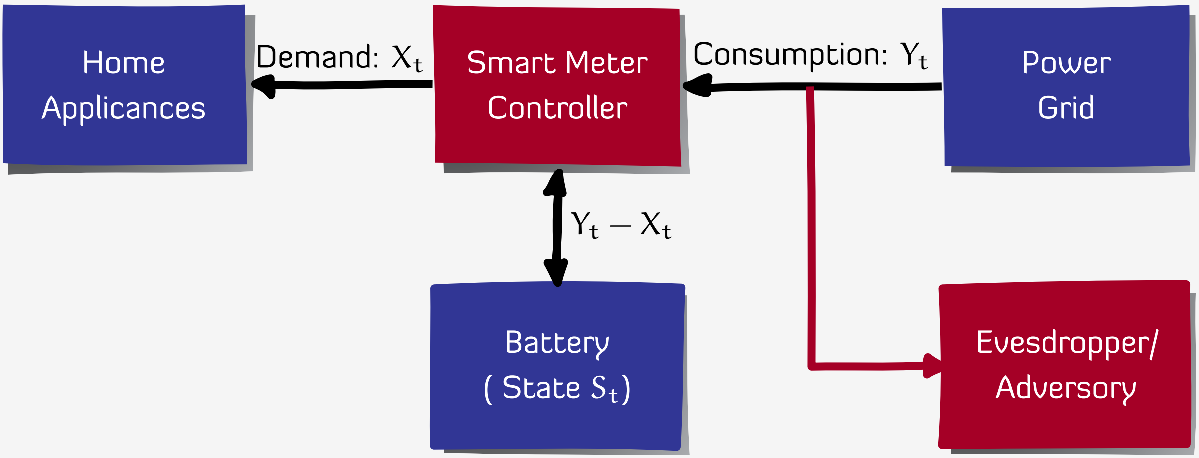
Home
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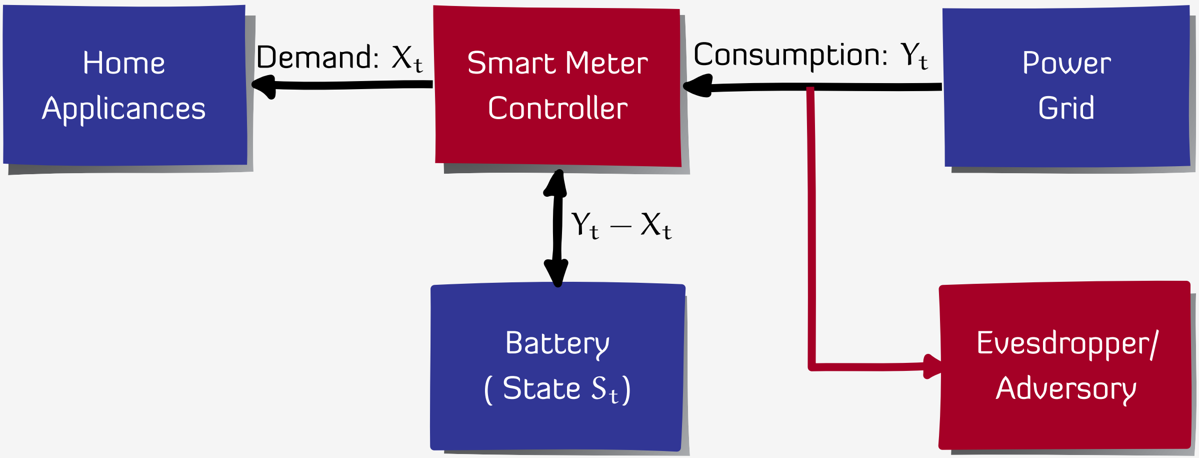
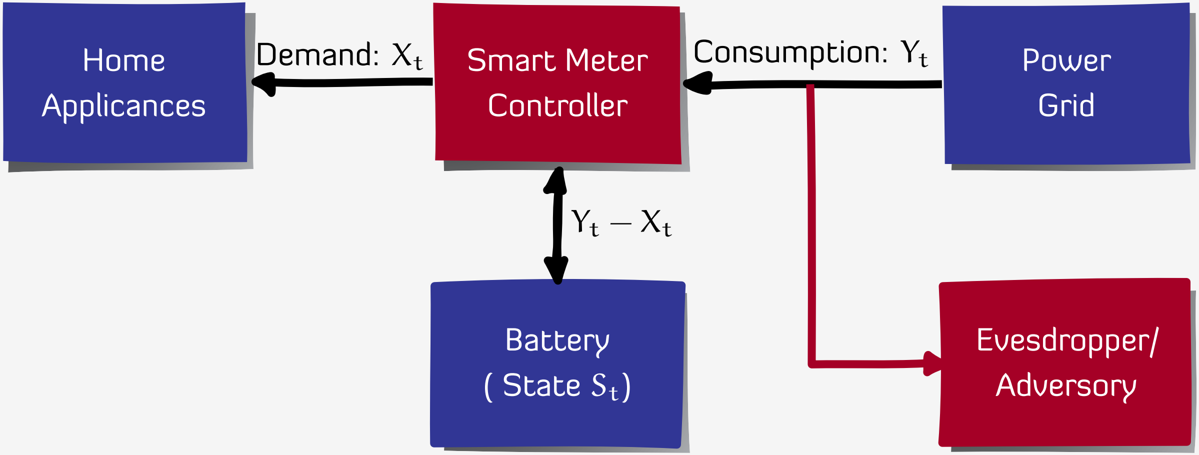


Energy conservation $S_{t+1} = S_t + Y_t - X_t, \quad S_t \in \mathcal{S}$ (Size of battery)



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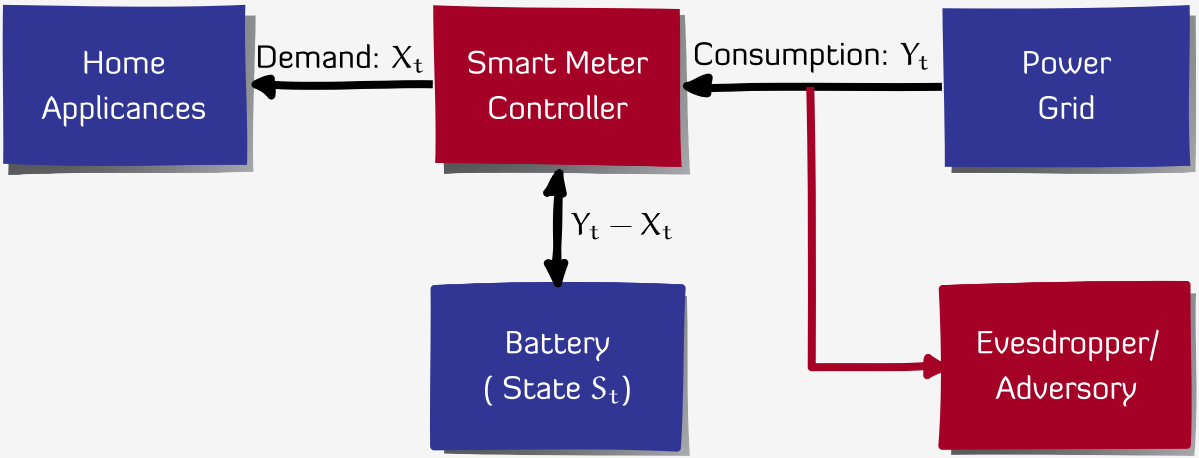

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Objective Choose battery charging strategy $\mathbf{q} = \{q_t\}_{t \geqslant 1}$ to

$$\min \lim_{T \to \infty} \frac{1}{T} I^{\mathbf{q}}(X^T; Y^T) \quad \text{(mutual information rate)}$$

Why is the problem non-trivial?

$\mathcal{X} = \mathcal{Y} = \mathcal{S} = \{0, 1\}$, $P_X = [0.5, \ \ 0.5]$ (Binary model) Consv: $S_t + Y_t - X_t \in \mathcal{S}$

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Deterministic Memoryless Policy

- $P(Y|X = 0, S = 0) = [1 \;\; 0]$; $P(Y|X = 1, S = 1) = [0 \;\; 1]$: Leakage $= 1$ ($\because Y_t = X_t$).
- $P(Y|X = 0, S = 0) = [0 \;\; 1]$; $P(Y|X = 1, S = 1) = [1 \;\; 0]$: Leakage ≈ 1 ($\because Y_t = 1 - S_t$).

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Randomized Memoryless Policy

- $P(Y|X = 0, S = 0) = [0.5 \ \ 0.5]$; $P(Y|X = 1, S = 1) = [0.5 \ \ 0.5]$: Leakage $= 0.5$.
- Is this the best memoryless policy?
- Is this the optimal policy?
- How do we evaluate the performance of an arbitrary policy? Need $\mathbb{P}(X^T, Y^T)$?

Literature overview

Evaluate privacy of specific battery management policies

- [Kalogridis et al., 2010] Monte-Carlo evaluation of best-effort policy
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- [Tan Gündüz Poor, 2012] Generalized results of [Varodayan Khisti] to include models with energy harvesting.
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Many results restrict to the binary battery model

Main results: Markovian demand

Structure of optimal strategies

Define belief state $\pi_t(x, s) = \mathbb{P}(X_t = x, S_t = s | Y^{t-1})$

Charging strategies of the form $q_t(y_t | x_t, s_t, \pi_t)$ are optimal.

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Dynamic programming decomposition

Let $\mathcal{A}$ denote the class of conditional distributions on $\mathcal{Y}$ given $(\mathcal{X}, \mathcal{S})$.

Suppose there exists a $J \in \mathbb{R}$ and $v \colon \mathcal{P}_{X,S} \to \mathbb{R}$ that satisfies the following:

$$J^* + v(\pi) = \inf_{a \in \mathcal{A}} \left\{ I(a; \pi) + \sum_{x,s,y} \pi(x, s) a(y|x, s) v(\varphi(\pi, y, a)) \right\}$$

Then,

- J^* is the minimum leakage rate
- Let $f^*(\pi)$ denote the arg min of the RHS and $a^* = f^*(\pi)$.
  Then, J^* is achieved by the charging policy
  $$q^*(y|x_t, s_t, \pi_t) = a^*(y|x_t, s_t) \quad \text{(note } a^* \text{ depends on } \pi_t\text{)}$$

Main results: Markovian demand

Structure of optimal strategies

- Similar to DP for POMDP.
- Per-step cost is concave rather than linear.
- However, $v(\pi)$ is still concave.

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Main results: i.i.d. demand

Solution of the dynamic program

$$J^* := \min_{\theta \in \mathcal{P}_S} I(S - X; X)$$

where $X \sim P_X$ and $S \sim \theta$. Let θ^* denote the arg min of the RHS.

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Optimal strategies

$$\text{Define } b^*(y|w) = \begin{cases} \dfrac{P_X(y)\theta^*(y+w)}{\displaystyle\sum_{(x,s):x-s=w} P_X(x)\theta^*(s)}, & \text{if } y \in \mathcal{X} \text{ and } y + w \in \mathcal{S} \\ 0, & \text{otherwise} \end{cases}.$$

Then, J^* is achieved by time-invariant action

$$q_t^*(y|x_t, s_t, \pi_t) = b^*(y|s_t - x_t) \quad \text{(note } b^* \text{ does not depend on } \pi_t\text{)}$$

Salient features of the solution

$I(S - X; X)$ is concave in $\mathcal{P}_{\mathcal{S}}$

J^* and θ^* may be computed using Blahut-Arimoto algorithm.

Optimal policy is stationary and memoryless

$q^*_t(y|x^t, s^t) = b^*(y|s_t - x_t)$ (note b^* does not depend on π_t)

If $S_t \sim \theta^*$, then $S_{t+1} \sim \theta^*$ and $S_{t+1} \perp Y^t$.

Support of consumptions

Even if $\mathcal{Y} \supset \mathcal{X}$, under the optimal policy the support of P_Y is $\mathcal{X}$.

Structure of the solution

If P_X is symmetric (and unimodal), so is θ^*.

For binary model, $\theta^* = [0.5 \;\; 0.5]$ is optimal!

Example

$$P_X \sim \mathrm{Bin}(n, 0.5)$$

Corresponds to the situation when there are n devices where each device is ON or OFF with equal probability.

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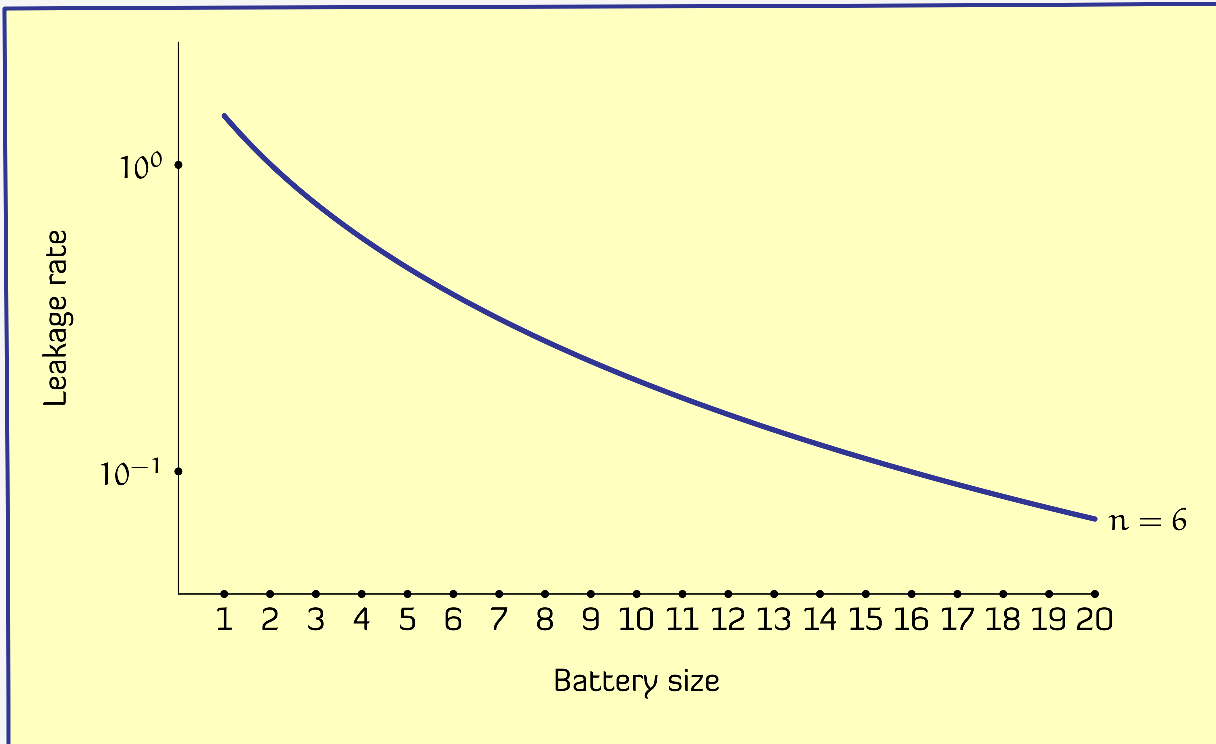
For $n = 6$, and $\mathcal{X} = \mathcal{Y} = \mathcal{S} = \{0, \ldots, 6\}$, we get

$J^* = 0.1638$

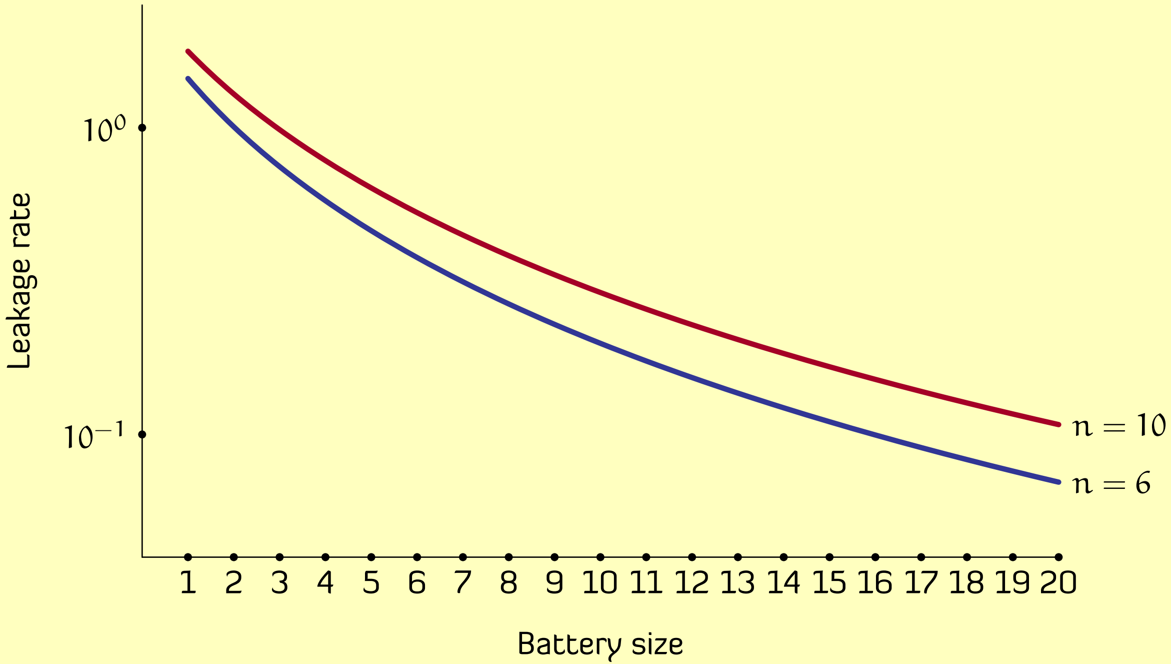
$\theta^* = \{0.0586, 0.1332, 0.1972, 0.2220, 0.1972, 0.1332, 0.0586\}$

Example



evice

Example



vice

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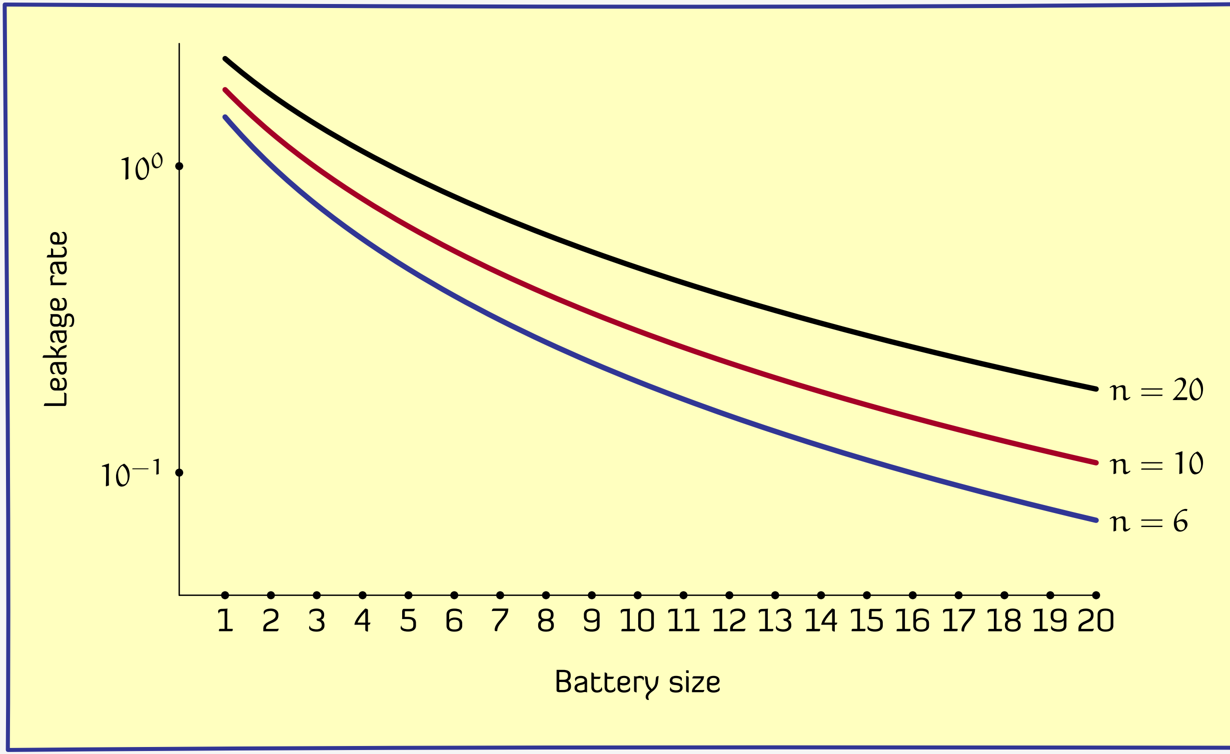

vice

Proof outlines

Proof outline for Markovian demand

Conceptual difficulty

Let $\mathcal{Q}_A$ denote all admissible policies. For any policy $\mathbf{q} \in \mathcal{Q}_A$,

$$I^{\mathbf{q}}(S_1, X^T; Y^T) = \sum_{t=1}^{T} I^{\mathbf{q}}(S_1, X^t; Y_t | Y^{t-1})$$

The cost is additive, but per-step cost depends on $\mathbb{P}(S_1, X^t, Y_t | Y^{t-1})$.

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Lemma

Let $\mathcal{Q}_B \subset \mathcal{Q}_A$ denote randomized charging policies of the form $q(y_t | x^t, s^t, y^{t-1}) = q(y_t | x_t, s_t, y^{t-1})$. Then,

1. For any policy $\mathbf{q}_a \in \mathcal{Q}_A$, there exists a policy $\mathbf{q}_b \in \mathcal{Q}_B$ such that

$$I^{\mathbf{q}_a}(S_1, X^T; Y^T) \geqslant I^{\mathbf{q}_b}(S_1, X^T; Y^T)$$

Thus, we may restrict attention to charging policies in $\mathcal{Q}_B$.

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2. For any policy $\mathbf{q}_b \in \mathcal{Q}_B$,

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Thus, for policies in $\mathcal{Q}_B$, the cost is additive and the per-step cost depends on $\mathbb{P}(\mathbf{S}_t, \mathbf{X}_t, Y_t | Y^{t-1})$.

Proof outline for Markovian demand (cont.)

Equivalent controlled Markov process

[Inspired by Tatikonda Mitter 2009, Capacity of channels with feedback]

State Space : $\mathcal{P}_{X,S}$

Action Space: $\{a \in \mathcal{P}_{Y|X,S}$ such that energy conservation is satisfied.$\}$

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Dynamics : $\pi_{t+1} = \varphi(\pi_t, y_t, a_t)$ where φ is a non-linear filter.

Per-step cost: $I^q(X_t, S_t; Y_t | y^{t-1}) = I(a_t; \pi_t)$, where

$$I(a_t, \pi_t) = \sum_{(x,s,y)} \pi_t(x, s) a_t(y|x, s) \log \frac{a_t(y|x, s)}{\sum_{(\tilde{x},\tilde{s})} \pi_t(\tilde{x}, \tilde{a}) a_t(y|\tilde{x}, \tilde{s})}$$

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Proof outline for i.i.d. demand

Simplifying state space

Let $W_t = S_t - W_t$ and $\xi_t(w) = \mathbb{P}(W_t = w | Y^{t-1} = y^{t-1})$. Then,

1. $\xi_t(w) = \sum_{(x,s):s-x=w} \pi_t(x, s)$.
2. $\pi_t(x, s) = P_X(x)\theta(s)$, where $\theta = P_X * \xi$.

Thus, ξ_t is equivalent to π_t

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Simplifying action space

Let $\mathcal{B} = \{b \in \mathcal{P}_{Y|W}$ s.t. energy consv. is satisfied$\}$. For $a \in \mathcal{A}$ and $\pi \in \mathcal{P}_{X,S}$

Define $b(y|w) = \dfrac{\sum_{(\tilde{x},\tilde{s}): \tilde{s}-\tilde{x}=w} a(y|\tilde{x},\tilde{s})\pi(\tilde{x},\tilde{s})}{\sum_{(\tilde{x},\tilde{s}): \tilde{s}-\tilde{x}=w} \pi(\tilde{x},\tilde{s})}$, $\quad \tilde{a}(y|x,s) = b(y|s-x)$.

Then,
1. Invariant transitions: $\varphi(\pi, y, a) = \varphi(\pi, y, \tilde{a})$.
2. Lower cost: $I(a;\pi) \geqslant I(\tilde{a};\pi) = I(b;\xi)$.

Thus, we may restrict attention to $\mathcal{B}$.

Proof outline for i.i.d. demand

Simplified DP:

$$J^* + v(\xi) = \inf_{b \in \mathcal{B}} \left\{ I(b;\xi) + \sum_{w,y} \xi(w) b(y|w) v(\tilde{\varphi}(\xi, y, b)) \right\}$$

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Simplifying action space

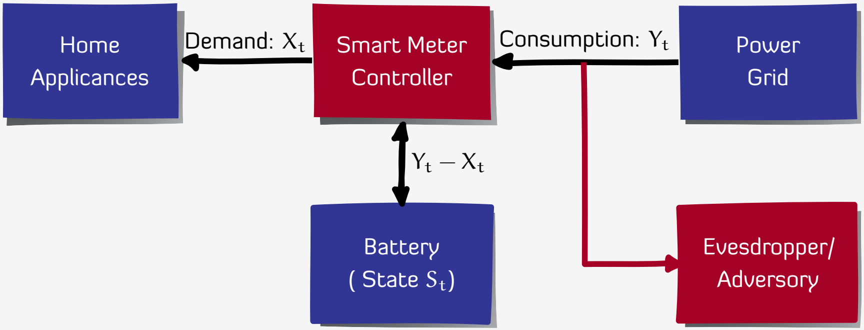
We show that $J^* = \min_{\theta \in \mathcal{P}_S} I(S - X; X)$ and b^* given in the Theorem satisfy the above DP.

Then, 1. Invariant transitions: $\varphi(\pi, y, a) = \varphi(\pi, y, \tilde{a})$.

2. Lower cost: $I(a;\pi) \geqslant I(\tilde{a};\pi) = I(b;\xi)$.

Thus, we may restrict attention to $\mathcal{B}$.

Summary



Energy conservation $S_{t+1} = S_t + Y_t - X_t, \quad S_t \in \mathcal{S}$ (Size of battery)

Randomized charging strategy $q_t(y_t | x^t, s^t, y^{t-1})$: Probability that the consumption $Y_t = y_t$ given history of demand, battery charge, and consumption,

Objective Choose battery charging strategy $\mathbf{q} = \{q_t\}_{t \geqslant 1}$ to

$$\min \lim_{T\to\infty} \frac{1}{T} I^{\mathbf{q}}(X^T; Y^T) \quad \text{(mutual information rate)}$$

Summary

Main results: Markovian demand

Structure of optimal strategies

- Similar to DP for POMDP.
- Per-step cost is concave rather than linear.
- However, $v(\pi)$ is still concave.

Dynamic programming decomposition

Let $\mathcal{A}$ denote the class of conditional distributions on $\mathcal{Y}$ given $(\mathcal{X}, \mathcal{S})$.

Suppose there exists a $J \in \mathbb{R}$ and $v \colon \mathcal{P}_{X,S} \to \mathbb{R}$ that satisfies the following:

$$J^* + v(\pi) = \inf_{a \in \mathcal{A}} \Big\{ I(a;\pi) + \sum_{x,s,y} \pi(x,s) a(y|x,s) v(\varphi(\pi, y, a)) \Big\}$$

Then,

- J^* is the minimum leakage rate
- Let $f^*(\pi)$ denote the arg min of the RHS and $a^* = f^*(\pi)$.
  Then, J^* is achieved by the charging policy
  $q^*(y|x_t, s_t, \pi_t) = a^*(y|x_t, s_t)$ (note a^* depends on π_t)

Summary

Main results: i.i.d. demand

Solution of the dynamic program

$$J^* \coloneqq \min_{\theta \in \mathcal{P}_S} I(S - X; X)$$

where $X \sim P_X$ and $S \sim \theta$. Let θ^* denote the arg min of the RHS.

Then, J^* is the minimum leakage rate

Optimal strategies

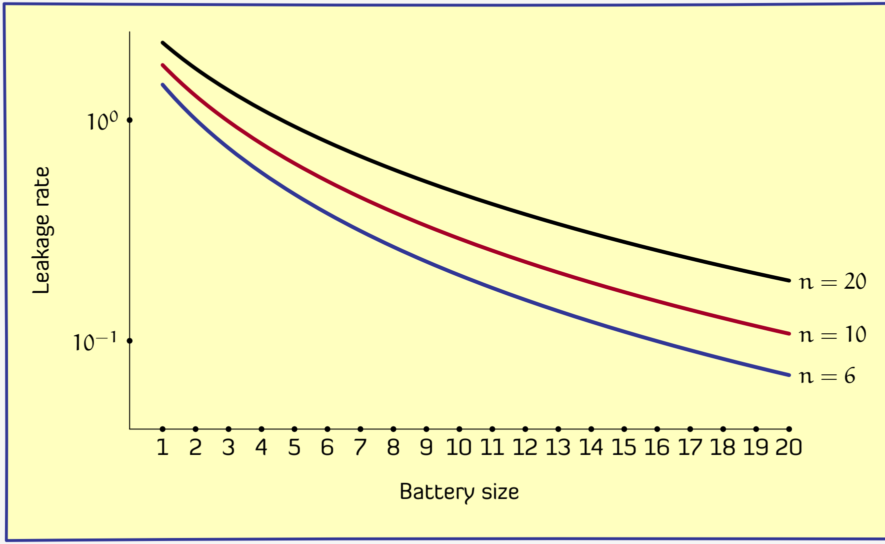
Define
$$b^*(y|w) = \begin{cases} \dfrac{P_X(y)\theta^*(y+w)}{\sum_{(x,s):x-s=w} P_X(x)\theta^*(s)}, & \text{if } y \in \mathcal{X} \text{ and } y+w \in \mathcal{S} \\ 0, & \text{otherwise} \end{cases}.$$

Then, J^* is achieved by time-invariant action

$$q_t^*(y|x_t, s_t, \pi_t) = b^*(y|s_t - x_t) \quad \text{(note } b^* \text{ does not depend on } \pi_t\text{)}$$

Summary

Example



Conclusion

Dynamic programming characterization of optimal privacy in smart meters

Identify structure of optimal strategies

For i.i.d. demand, identify optimal charging strategies and a single letter characterization of optimal leakage rate.

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The results generalize to higher order Markov demands

The results generalize to continuous state spaces

The results are applicable if the demand is modeled as a deterministic process + noise, where the noise is Markov or i.i.d.

Conclusion

Dynamic programming characterization of optimal privacy in smart meters

- Identify structure of optimal strategies
- For i.i.d. demand, identify optimal charging strategies and a single letter characterization of optimal leakage rate.

Remark on modeling assumptions

- The results generalize to higher order Markov demands
- The results generalize to continuous state spaces
- The results are applicable if the demand is modeled as a deterministic process + noise, where the noise is Markov or i.i.d.

Future directions

- Optimal leakage rate in the presence of local energy harvesting devices?