MECH 261/262 Midterm Questions 7 & 8

February 17, 2004

1 Question 7

Recall Eq. 1, the differential equation of the first order response of a spring and dash-pot mechanical system excited by the time function $F(t)$.

$$F - kx - c\dot{x} = 0 \tag{1}$$

In the example $F(t) = F$ where F was a constant representing a suddenly applied force. More generally, F represents a sudden change in measurement system *input* variable, say, a temperature change ΔT. Similarly, x represents the system *output* variable, not displacement like in the mechanical system but say, voltage of a linearized and amplified thermocouple signal Δv when the thermocouple is, *e.g.*, initially at room temperature and suddenly plunged in boiling water. Not surprisingly k is called the zeroth order coefficient and c is called the first order coefficient of the *linear* system modeled by the first order *linear* differential equation, Eq. 1. k is also called *static sensitivity* and may be evaluated with Eq. 1 by setting $\dot{x} = 0$ which would occur, with a step input, at time $t \to \infty$ after the step. Question 7 pertains to static or zeroth order sensitivity.

2 Question 8

Now recall the simplification of first order response equation solution for the mechanical first order system, responding to a step force input, which originally appeared as

$$\left(\frac{k}{F}x - 1\right) e^{\frac{k}{c}t} + 1 = 0 \tag{2}$$

and simplified to

$$X = \frac{e^T - 1}{e^T} = 1 - e^{-T} \tag{3}$$

Now, in the context of the sudden temperature change ΔT, we get

$$\frac{\Delta v}{\Delta V} = 1 - e^{-\frac{k}{c}t} \tag{4}$$

where Δv is the output change at time t and ΔV is the static or long term output change and where $\frac{k}{c}$ is called the *time constant* of the first order system. Given results from Question 7 and a bit more information from a so-called "system identification" experiment we can determine

- The time constant, $\frac{k}{c}$, and
- The first order system coefficient, c.

Question 8 pertains to these two items.

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