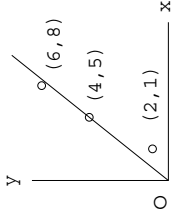


1. Sometimes we need a linear fit that minimizes the sum of squares of y-deviations subject to the line passing through the origin. Write the relations for the coefficients in the equation $y=ax+b$.

$$a = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2} \quad b = \frac{\sum_{i=1}^n y_i}{n}$$

For the data at right

$$a = \frac{5}{4} \quad b = 0$$


2. 180 seats are sold for an aircraft with 175 seats. If 1 in 20 passengers are on average "no shows" what is the most likely number of empty seats that will remain? 4
What is the probability of that occurring? 13.5% Hint:- 180Cr

3. Tellers in a bank must, on average, serve 20 customers per hour. What is the probability that a) Exactly 25 customers arrive in a given hour? 4.46%
b) There will be less than 10 customers arriving during a given hour? 0.50%
Hint:- Poisson; did you try 6.29?

(MECH261-2) MSTMt73b
Name: - key
Student Number: - _____
MECH 262

Midterm Test 07-03-02, 40min.

Figure 1: Midterm Test 07-03-02

```
> restart:with(combinat):
```

```
Warning, the protected name Chi has been redefined and unprotected
```

```
> a:=(2*1+4*5+6*8)/(2^2+4^2+6^2);
```

$$a := \frac{5}{4}$$

A least squares linear fit that is forced to go through the origin requires only one degree of freedom, i.e., a , the slope of the line. This is computed as the sum of the products of the point coordinates divided by the sum of the squares of the x-coordinates. This is question 1 on the 2007 MECH 262 midterm test.

```

> numbcmb(180,7)*.05^7*.95^173;
                                0.1180760941
> numbcmb(180,8)*.05^8*.95^172;
                                0.1343892387
> numbcmb(180,9)*.05^9*.95^171;
                                0.1351751406
> numbcmb(180,10)*.05^10*.95^170;
                                0.1216576265

```

This is an iterative solution to the "rotten egg" problem to find the most likely (highest probability) number in a binomially distributed sample of 180 "events" with a probability of 1/20. This is a unimodal distribution function, i.e., it has a single peak or maximum. We see this is the probability of 9 "no-shows" at about 13.5%. Sorry, I subtracted 175-(180-9)=2, the wrong answer. It should have been 4 empty seats. This is question 2 on the 2007 MECH 262 midterm test.

```

> P25:=exp(-20)*20^25/25!;evalf(P25);
                                
$$P_{25} := \frac{5120000000000000000000}{236682282155319} e^{(-20)}$$

                                0.04458764909
> P0:=evalf(exp(-20)*20^0/0!);P1:=evalf(exp(-20)*20^1/1!);P2:=evalf(exp
> (-20)*20^2/2!);P3:=evalf(exp(-20)*20^3/3!);P4:=evalf(exp(-20)*20^4/4!)
> ;P5:=evalf(exp(-20)*20^5/5!);P6:=evalf(exp(-20)*20^6/6!);P7:=evalf(exp
> (-20)*20^7/7!);P8:=evalf(exp(-20)*20^8/8!);P9:=evalf(exp(-20)*20^9/9!)
> ;
                                P0 := 0.2061153622 10-8
                                P1 := 0.4122307244 10-7
                                P2 := 0.4122307244 10-6
                                P3 := 0.2748204829 10-5
                                P4 := 0.00001374102415
                                P5 := 0.00005496409659
                                P6 := 0.0001832136553
                                P7 := 0.0005234675866
                                P8 := 0.001308668966
                                P9 := 0.002908153258
> P0to9:=P0+P1+P2+P3+P4+P5+P6+P7+P8+P9;
                                P0to9 := 0.004995412306
> P10:=evalf(exp(-20)*20^10/10!);
                                P10 := 0.005816306518
> P0to9+P10;
                                0.01081171882

```

The bank problem is suited to a Poisson distribution model. We get about 4.46% probability that exactly 25 customers will arrive in an hour long period if the average number of arrivals is 20 per hour. The probability that less than 10 will arrive is computed as the sum of the probability of exactly 0,1,2,3,4,5,6,7,8,9 arrivals. Unfortunately added the probability of exactly 10 arrivals as well to get about 1.08% probability. The answer should have been about 0.5%.(MECH261-2)MStMt73b.mws. Recomputed 07-03-03.